These notes essentially correspond to chapter 5 of the text.

1 Applying Consumer Theory

In this section we will apply indifference curve analysis to answer questions about how consumers behave.

2 Deriving Demand Curves

We should be able to connect an individual’s optimal bundle purchase to his demand curve. Recall that when creating a demand curve we hold everything in the world fixed except the price and quantity demanded of the good. In our two-good “world” with a fixed budget, “everything else” consists of the price of the other good and the individual’s income. So we need to hold those two factors constant when deriving an individual’s demand curve for a product.

Look at the picture below. There are three budget constraints, all starting from the same point on the good A axis. These 3 budget constraints correspond to a high price for good B ($P_H$), a medium price for good B ($P_M$), and a low price for good B ($P_L$).

Notice that when the price of good B is $P_H$ that the consumer only purchases 10 units of good B at his optimal bundle. When the price of good B is $P_M$, the consumer now purchases 35 units of good B, and when the price of good B
is $P^L$ the consumer purchases 65 units of good B. Notice that this is exactly what we would need to construct a demand curve for good B—the price and quantity pairs for the good. Plotting these points on a price-quantity axis gives us:

![Graph](image)

Notice that we get a nice, intuitive downward-sloping demand curve.

### 2.1 Income changes and demand curves

We know from principles of micro that an increase in income (at least for normal goods) will cause the demand curve to increase (shift to the right). We can also use indifference curve analysis to show this fact. The picture below shows three budget constraints, each with a different income level ($Y_L, Y_M, Y_H$). We then find the consumers optimal consumption bundle for each income level. Notice that we will be keeping the prices of goods A and B the same.
Now, suppose that we were to plot the points on a price-quantity graph for Good B. The key is to realize that the price of Good B has NOT changed. Suppose the price is some price $P^*$. Then we would get the following graph, with three distinct demand curves corresponding to $Y^L$, $Y^M$, and $Y^H$. Note that we only have one point on each of the demand curves – if we wanted to get more points we would need to look at a fixed income level (either $Y^L$ or $Y^M$ or $Y^H$) and change the price of Good B.
Since the demand for Good B increases when we increase income, we have a normal good. We can also show the relationship between income and quantity demanded on a separate graph. Define an Engel curve as the graphical representation of the relationship between a consumer’s income and his quantity demanded. We will place income on the y-axis and quantity demanded on the x-axis. The Engel curve for Good B is shown below. Note that if the Engel curve is upward-sloping then the good is a normal good; if the Engel curve is downward-sloping the good is an inferior good.
2.1.1 Goods that are normal and inferior

It is highly possible that some goods may be both normal goods and inferior goods, depending on the range of income. To someone who has very little income, Ramen noodles may be a normal good if that person is given more income – they can now consume one additional meal. But if that same consumer receives a large enough increase in income, then he may consume less Ramen noodles and switch to consuming higher quality foods. In a case like this, the Engel curve for the good will be backward-bending. Over the part of the curve with the increasing slope the good is a normal good, but when income becomes high enough (which is at the income level $Y^M$ in the picture), the consumer starts to shift away from purchasing the good when he receives more income, which means that the good is now inferior.
3 Effects of a price change

We know that a price change will cause the budget constraint to pivot on the intercept of the good for which price does not change. However, we can also decompose the effects of price changes into two pieces, the substitution effect and the income effect. The substitution effect is the part of the quantity purchased increase that occurs from the price of the good being now relatively lower. To find the substitution effect we need to find the bundle that the consumer WOULD have bought had he faced the same relative prices (after the price change) but been forced to remain on his initial indifference curve. The income effect is the part of the quantity purchased increase that occurs from the consumer's income being expanded due to a relatively lower price. The two effects combined give us the total effect, which is the actual amount that the quantity purchased of a good increases or decreases by when the price of the good changes. Thus, we have:

\[
\text{Total effect} = \text{Substitution effect} + \text{Income effect}
\]

The easiest way to describe income and substitution effects is to look at how a consumer’s optimal decision changes when the price of one of the goods changes. Consider a decrease in the price of good B, as shown on the graph below.
Now, to explain the graph. This consumer’s initial optimal bundle was $Q_1$. After the price of Good B decreased his new optimal bundle became $Q_2$. Thus, the total effect is: $Q_2 - Q_1$. We now want to decompose this total effect into the substitution and income effects. Recall that the substitution effect is given by the amount by which quantity purchased would change IF the consumer had initially faced the relative prices of the goods after the price change (so if the slope of the budget constraint was the same as it is AFTER the prices change) AND he was held at his initial utility level (so we keep him on $I_1$). In order to find the substitution effect we simply shift the new (after the price change) budget constraint back until we find the optimal bundle that the consumer would have purchased on $I_1$ had he faced these relative prices. This is point $Q_3$ in the graph. The substitution effect is then given by $Q_3 - Q_1$, since that is the amount by which quantity purchased would increase if the consumer faced the same relative prices after the price change and was forced to remain on $I_1$. The income effect is then the remaining piece of the total effect, which in this case is $Q_2 - Q_3$. Notice that if we add the income and substitution effects we get:

$$Q_3 - Q_1 + Q_2 - Q_3 = Q_2 - Q_1$$

which is just the total effect.
3.1 Giffen goods

A Giffen good is a good that “disobeys” the Law of Demand. For Giffen goods, a decrease in the price of a good will actually cause LESS of the good to be purchased. We can use income and substitution effects to show that this is due to a negative income effect. The picture below uses indifference curve analysis and the decomposition of the total effect into substitution and income effects to show this.

Notice that the consumer purchases $Q_1$ when facing the initial prices. After the price of Good B falls, the consumer then purchases $Q_2$. Notice that the total effect in this instance, which is still $Q_2 - Q_1$, is negative because $Q_2 < Q_1$. To find the substitution effect we create the hypothetical budget constraint and find $Q_3$. Note that the substitution effect is still $Q_3 - Q_1$, which is still positive\(^1\). However, the income effect, which is still $Q_2 - Q_3$, is now negative. In the case of these particular goods and this price change, the negative income effect dominates the positive substitution effect, and the result is that as the price of Good B falls the consumer purchases less of that good.

The existence of such good is debateable, although anyone who wants to discuss the possibility of collectible items being Giffen goods is free to stop by my office.

\(^1\)The substitution effect will ALWAYS be positive for price decreases, and will ALWAYS be negative for price increases.
4 Labor-Leisure decisions

Although “work” may not seem like a good (in fact, most people probably believe it is an economic bad, as in something that usually lowers utility), we can use indifference curve analysis to determine how much people will work. This is because the opposite of work, which is leisure, is an economic good. We will place the leisure good on the x-axis of our indifference curve analysis. The other good we will consider is income, which we can use to buy all other goods. Thus the consumer will have a utility function over leisure and income. This implies that the consumer will have preferences over leisure and income, and will have indifference curves for these two goods. However, our consumer may only receive income if he works (we will not use the nice endowment/allowance economy here). We will consider a situation where the worker earns an hourly wage of \( w \).

**Time Constraints** One other factor that the worker must consider is that he has a quantity restriction on the amount of time that he can work or take leisure (not even those named Gates and Hilton can change that fact). We will call the amount of hours of leisure a person takes \( N \). The amount of hours worked will be \( H \). Adding them together, we get that \( N + H = 24 \) or \( H = 24 - N \).

**Income and the cost of leisure** Since the consumer can only earn income through the hourly wage \( w \), the consumer’s income is the product of the amount of hours worked and the hourly wage, or \( Y = wH \). Notice that in addition to partially determining income, the wage also serves an important function as it determines the opportunity cost of leisure. For each hour of leisure that you take you must give up one hour of work, which means you are giving up one \( w \) for each hour of leisure.

Now, let’s use indifference curve analysis to determine how much the consumer will work if we consider a given wage \( w \). We assume that the consumer has normal convex indifference curves for leisure and income. However, we need a budget constraint in order to determine the consumer’s optimal bundle of leisure and income. The budget constraint in this problem is the line with slope \((-w)\) that passes through the point \((24,0)\), assuming that we are counting time in hours. The way to construct the budget constraint is as follows: Suppose that a person works 0 hours (or takes 24 hours of leisure) – then the person will have 0 income, and will be at the point \((24,0)\). If the person works one hour, then the person will be at the point \((23,w)\), because he gives up one hour of leisure for one hour of work, and one hour of work is worth \( w \) to him. If the person works 2 hours then he is at \((22,2w)\), if 3 hours then \((21,3w)\), ..., if 24 hours then \((0,24w)\). Thus the wage is the rate at which the market will let a person trade leisure for work. Since it is the rate at which the market allows trades to occur, the (negative of) wage rate is the Marginal Rate of Transformation, which we all know is also the slope of the budget constraint.
The picture shows the consumer's optimal bundle of leisure time and income under two different wages, \( w_1 \) and \( w_2 \), where \( w_2 > w_1 \). When the wage is low (at \( w_1 \)), the consumer chooses to take 16 hours of leisure (or work 8 hours). The income level at that bundle is \( 8w_1 \). When the wage rises to \( w_2 \) the worker chooses to take 12 hours of leisure (or work 12 hours). The income level at that bundle is \( 12w_2 \).

4.1 Deriving leisure demand and labor supply curves

We can derive leisure demand curves as well as labor supply curves from our indifference curve analysis of the consumer's labor-leisure decision. The derivation of the leisure demand curve is a simple application of deriving demand curves from indifference curve analysis when the price of a good changes. We can exploit the fact that \( H = 24 - N \) to derive a labor supply curve from this analysis as well.

4.1.1 Leisure Demand Curves

Using the figure above we know that when the wage is \( w_1 \) the consumer will choose to 16 hours of leisure and when the wage is \( w_2 \) the consumer will choose to 12 hours of leisure. Since the wage is the price of leisure we can construct a demand curve for leisure.
4.1.2 Labor supply curves

We also know how many hours of work a consumer will supply at each wage level. When the wage is $w_1$, the consumer takes 16 hours of leisure, which means that he is supplying 8 hours of work. When the wage is $w_2$, the consumer takes 12 hours of leisure, which means that he is supplying 12 hours of work. Using these results we can construct the consumer’s labor supply curve. Note that it is upward-sloping as we typically assume of supply curves.
4.2 Backward-bending labor supply curve

It is possible for a labor supply curve to be backwards-bending. An intuitive discussion of this result is as follows. Suppose a person receives a raise in his wage rate. The substitution effect would make that person work more hours (take less leisure hours) because leisure is now relatively more expensive. However, a wage raise also increases the person’s income. It may be the case that the person strongly prefers leisure time to working, or that the person was previously at a sufficient income level to sustain his lifestyle. With the raise, the person will be able to move to a higher indifference curve if he takes more leisure time than he did before, making leisure a Giffen good (as the price of leisure rises, he consumes more of it). This is because the income effect dominates the substitution effect in this case. Now, this does not happen for all wage rates — at very low wage rates increases in wages are likely to cause increases in hours worked. Only at very high wages will increases in wages begin to cause a decrease in hours worked. Of course, “very low” and “very high” are subjective terms, determined by the individual. Thus, there will be a certain wage, call it $w^*$, for EACH individual such that the labor supply curve becomes backward bending at that wage, as shown in the picture below.
4.2.1 Taxes

The backward-bending labor supply curve can have important implications for tax policy. We know that at a certain point people will begin cutting back their hours worked if they receive more income. Thus, if we have reached that point \( w^* \) in the picture above, then increasing income cuts down on hours worked. Suppose that we currently have the tax rate set such that the after-tax wage rate is exactly \( w^* \). What will happen if we lower taxes? People will receive a higher after-tax wage rate, \( w^H \). At \( w^H \) they will work less than they did at \( w^* \). This will cause tax revenues to fall as people will not be working as much. If we increase the tax rate, we will be reducing the wage rate even further, to some point below \( w^* \). This is also unlikely to increase tax revenue, as people will be working less.

If the tax rate is such that the after-tax wage rate is on the downward-sloping portion of the labor supply curve, then the government can generate more tax revenue and stimulate production in the economy by increasing the tax rate (which lowers the wage and causes the individual to work more). However, if the tax rate is such that the after-tax wage rate is on the upward-sloping portion of the labor supply curve, then the government can attempt to stimulate production in the economy by lowering the tax rate, which will cause individuals to work more hours.