# Assignment 2 Answers

ECON 3161, Game Theory

Due: By the end of class on Thursday Sept. 18th

**Directions:** Answer all questions completely. Note the due date of the assignment. Late assignments will be accepted at the cost of 10 points per day, up until 11am on Sept. 20. At that time I will return the graded assignments and post the answers online. You may turn in assignments to me after that time so that I can check your work for you, but please realize that you will not receive a grade for the assignment. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. (60 points) Consider the following three simultaneous games, denoted by their game number in the upper left hand corner of each matrix:

<table>
<thead>
<tr>
<th>Game 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>Player 1 A</td>
<td>-3, -1</td>
</tr>
<tr>
<td>B</td>
<td>0, 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Game 2</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Player 1 E</td>
<td>6, 3</td>
</tr>
<tr>
<td>F</td>
<td>7, 2</td>
</tr>
<tr>
<td>G</td>
<td>2, 2</td>
</tr>
<tr>
<td>H</td>
<td>4, 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Game 3</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
</tr>
<tr>
<td>Player 1 M</td>
<td>7, 6</td>
</tr>
<tr>
<td>N</td>
<td>4, 5</td>
</tr>
<tr>
<td>O</td>
<td>3, 1</td>
</tr>
</tbody>
</table>

For each game:

**a** (15 points – 5 for each game): Find all the pure strategy Nash equilibria (PSNE). **Hint:** It is possible for a game not to have a PSNE.

**Answer:**

We can see in Game 1 that there are two PSNE: (1) Player 1 chooses B and Player 2 chooses C and (2) Player 1 chooses A and Player 2 chooses D

We can see in Game 2 that there are two PSNE: (1) Player 1 chooses E and Player 2 chooses J and (2) Player 1 chooses G and Player 2 chooses L

We can see in Game 3 that there are zero PSNE.

**b** (30 points – 10 for each game): Find the mixed strategy Nash equilibrium (MSNE). **Hint:** All games have a MSNE, but not all strategies are used in the MSNE in each game.

**Answer:**

**Game 1**
In Game 1 there is an odd type of MSNE. Let \( a \) be the probability that Player 1 chooses A and \( b = 1 - a \) be the probability that Player 1 chooses B. We now want to calculate the expected value of Player 2 choosing C and the expected value of Player 2 choosing D and set them equal to each other.

\[
E_2[C] = -1 * a + 2 * (1 - a) = -a + 2 - 2a = 2 - 3a \\
E_2[D] = 1 * a + 2 * (1 - a) = a + 2 - 2a = 2 - a
\]

Setting these equal to each other we find:

\[
2 - 3a = 2 - a \\
0 = 2a \\
0 = a
\]

This is fine – it just says that Player 1’s mixed strategy is a pure strategy where he chooses strategy A with 0% probability and strategy B with 100% probability. Note that this is the only way that Player 1 can make Player 2 indifferent over his pure strategies because D is a weakly dominant strategy. Now to find Player 2’s mixed strategy we will let \( c \) be the probability that Player 2 chooses C and \( d = 1 - c \) be the probability that Player 2 chooses D. We now want to calculate the expected value of Player 1 choosing A and Player 1 choosing B and set those equal to each other.

\[
E_1[A] = -3 * c + 2 * (1 - c) = -3c + 2 - 2c = 2 - 5c \\
E_1[B] = 0 * c + 0 * (1 - c) = 0
\]

Setting these values equal to each other we find:

\[
2 - 5c = 0 \\
2 = 5c \\
\frac{2}{5} = c
\]

We see that Player 2 should choose strategy C with probability \( \frac{2}{5} \) and strategy D with probability \( \frac{3}{5} \). So there is a "mixed strategy" Nash equilibrium (even though 1 player is using a pure strategy) where Player 1 chooses B 100% of the time and Player 2 chooses C 40% of the time and D 60% of the time.

**Game 2**

For Game 2 we need to do a little work before calculating the MSNE. If you try to use all 4 strategies you will not get a reasonable answer, and this is because you would be trying to create an MSNE using a strictly dominated strategy. Looking at the game, we can see that strategy H is strictly dominated by strategy E. Once H is removed, strategy I is strictly dominated by strategy J. Once strategy I is removed, strategy F is strictly dominated by strategy G. Once strategy F is removed, strategy K is strictly dominated by J and L and we are left with the 2x2 matrix below. The matrices show the progression as strategies are eliminated.

<table>
<thead>
<tr>
<th>Game 2 Player 2</th>
<th>Game 2 Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>I J K L</td>
<td>I J K L</td>
</tr>
<tr>
<td>Player 1</td>
<td>Player 1</td>
</tr>
<tr>
<td>E 6.3 7.6 7.1 3.5</td>
<td>E 7.6 7.1 3.5</td>
</tr>
<tr>
<td>F 7.2 3.6 0.6 4.2</td>
<td>F 3.6 0.6 4.2</td>
</tr>
<tr>
<td>G 2.2 6.3 4.0 5.4</td>
<td>G 6.3 4.0 5.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Game 2 Player 2</th>
<th>Game 2 Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>J K L</td>
<td>J L</td>
</tr>
<tr>
<td>Player 1</td>
<td>Player 1</td>
</tr>
<tr>
<td>E 7.6 7.1 3.5</td>
<td>E 7.6 3.5</td>
</tr>
<tr>
<td>G 6.3 4.0 5.4</td>
<td>G 6.3 5.4</td>
</tr>
</tbody>
</table>

Now that there are no more strictly dominated strategies we can look for an MSNE. Let \( e \) be the probability that Player 1 uses E and \( g = 1 - e \) be the probability that Player 1 uses G. Setting the
expected value of J equal to the expected value of L we have:

\[ E_2 [J] = E_2 [L] \]

\[ 6e + 3 * (1 - e) = 5e + 4 * (1 - e) \]

\[ 6e + 3 - 3e = 5e + 4 - 4e \]

\[ 3e + 3 = e + 4 \]

\[ 2e = 1 \]

\[ e = \frac{1}{2} \]

So Player 1 would choose strategy E with probability \( \frac{1}{2} \) and strategy G with probability \( \frac{1}{2} \) to make Player 2 indifferent between using J and L (note that Player 2 does NOT want to use I or K if Player 1 chooses this strategy).

Now let \( j \) be the probability that Player 2 chooses J and \( l = 1 - j \) be the probability that Player 2 chooses L. Setting the expected value of Player 1 choosing E equal to the expected value of Player 1 choosing G we have:

\[ E_1 [E] = E_1 [G] \]

\[ 7j + 3 * (1 - j) = 6j + 5 * (1 - j) \]

\[ 7j + 3 - 3j = 6j + 5 - 5j \]

\[ 4j + 3 = j + 5 \]

\[ 3j = 2 \]

\[ j = \frac{2}{3} \]

So Player 2 would choose strategy J with probability \( \frac{2}{3} \) and strategy L with probability \( \frac{1}{2} \) to make Player 1 indifferent between using E and G (note that Player 1 does NOT want to use F or H if Player 2 chooses this strategy).

So the MSNE to this game is: Player 1 chooses E with probability \( \frac{1}{2} \) and G with probability \( \frac{1}{2} \) while Player 2 chooses J with probability \( \frac{2}{3} \) and L with probability \( \frac{1}{3} \).

**Game 3**

In Game 3 let \( p \) be the probability Player 2 chooses strategy P, \( q \) be the probability that Player 2 chooses strategy Q, and \( r = 1 - p - q \) be the probability that Player 2 chooses strategy R. First find the expected value of each of Player 1’s strategies:

\[ E_1 [M] = 7p + 3q + 4 (1 - p - q) = 7p + 3q + 4 - 4p - 4q \]

\[ E_1 [M] = 3p - q + 4 \]

\[ E_1 [N] = 4p + 7q + 2 (1 - p - q) = 4p + 7q + 2 - 2p - 2q \]

\[ E_1 [N] = 2p + 5q + 2 \]

\[ E_1 [O] = 3p + 2q + 8 (1 - p - q) = 3p + 2q + 8 - 8p - 8q \]

\[ E_1 [O] = -5p - 6q + 8 \]

There will need to be 2 equations – I will choose \( E_1 [M] = E_1 [N] \) and \( E_1 [N] = E_1 [O] \). First \( E_1 [M] = E_1 [N] \):

\[ E_1 [M] = E_1 [N] \]

\[ 3p - q + 4 = 2p + 5q + 2 \]

\[ p = 6q - 2 \]

Now we know \( p \) in terms of \( q \). Next use \( E_1 [N] = E_1 [O] \):

\[ E_1 [N] = E_1 [O] \]

\[ 2p + 5q + 2 = -5p - 6q + 8 \]

\[ 7p + 11q = 6 \]
Substituting in for $p$ we have:

\[ 7p + 11q = 6 \]
\[ 7(6q - 2) + 11q = 6 \]
\[ 42q - 14 + 11q = 6 \]
\[ 53q = 20 \]
\[ q = \frac{20}{53} \]

So we know that $q = \frac{20}{53}$ (I will leave this as a fraction – easier to deal with and more exact than converting it to a decimal). We know that:

\[ p = 6q - 2 \]
\[ p = 6 \times \frac{20}{53} - \frac{106}{53} \]
\[ p = \frac{120}{53} - \frac{106}{53} = \frac{14}{53} \]

Since $p = \frac{14}{53}$ and $q = \frac{20}{53}$, we know that $r = 1 - \frac{14}{53} - \frac{20}{53} = \frac{19}{53}$.

Does this mixed strategy make Player 1 indifferent over his pure strategies? Let’s check:

\[ E_1 [M] = 7p + 3q + 4r = 7 \times \frac{14}{53} + 3 \times \frac{20}{53} + 4 \times \frac{19}{53} = \frac{234}{53} \]
\[ E_1 [N] = 4p + 7q + 2r = 4 \times \frac{14}{53} + 7 \times \frac{20}{53} + 2 \times \frac{19}{53} = \frac{234}{53} \]
\[ E_1 [O] = 3p + 2q + 8r = 3 \times \frac{14}{53} + 2 \times \frac{20}{53} + 8 \times \frac{19}{53} = \frac{234}{53} \]

So the mixed strategy for Player 2 of using $p = \frac{14}{53}$, $q = \frac{20}{53}$, and $r = \frac{19}{53}$ does make Player 1 indifferent over his pure strategies.

Now ... time to find Player 1’s mixed strategy. Let $m$ be the probability that Player 1 chooses $M$, $n$ be the probability that Player 1 chooses $N$, and $o = 1 - m - n$ be the probability that Player 1 chooses $O$. First find the expected value of each of Player 2’s strategies:

\[ E_2 [P] = 6m + 5n + 1(1 - m - n) = 6m + 5n + 1 - m - n \]
\[ E_2 [P] = 5m + 4n + 1 \]
\[ E_2 [Q] = 2m + 3n + 4(1 - m - n) = 2m + 3n + 4 - 4m - 4n \]
\[ E_2 [Q] = -2m - n + 4 \]
\[ E_2 [R] = 8m + 4n + 2(1 - m - n) = 8m + 4n + 2 - 2m - 2n \]
\[ E_2 [R] = 6m + 2n + 2 \]

There will need to be 2 equations – I will choose $E_2 [P] = E_2 [Q]$ and $E_2 [Q] = E_2 [R]$. First $E_2 [P] = E_2 [Q]$:

\[ E_2 [P] = E_2 [Q] \]
\[ 5m + 4n + 1 = -2m - n + 4 \]
\[ 7m = 3 - 5n \]
\[ m = \frac{3 - 5n}{7} \]
Now we know $m$ in terms of $n$. Next use $E_2 [Q] = E_2 [R]$: 

\[
\begin{align*}
E_2 [Q] &= E_2 [R] \\
-2m - n + 4 &= 6m + 2n + 2 \\
-8m &= 3n - 2 \\
8m + 3n &= 2 \\
8 \left( \frac{3 - 5n}{7} \right) + 3n &= 2 \\
8 (3 - 5n) + 21n &= 14 \\
24 - 40n + 21n &= 14 \\
-19n &= -10 \\
n &= \frac{10}{19}
\end{align*}
\]

So $n = \frac{10}{19}$. We know that:

\[
\begin{align*}
m &= \frac{3 - 5n}{7} \\
7m &= 3 - 5n \\
7m &= 3 - 5 \cdot \frac{10}{19} \\
133m &= 57 - 50 \\
m &= \frac{7}{133}
\end{align*}
\]

Since $m = \frac{7}{133}$ and $n = \frac{10}{19} = \frac{70}{133}$, $o = 1 - \frac{7}{133} - \frac{56}{133} = \frac{56}{133}$

Does this mixed strategy make Player 2 indifferent over his pure strategies? Let’s check:

\[
\begin{align*}
E_2 [P] &= 6m + 5n + 1o = 6 \cdot \frac{7}{133} + 5 \cdot \frac{70}{133} + 1 \cdot \frac{56}{133} = \frac{448}{133} \\
E_2 [Q] &= 2m + 3n + 4o = 2 \cdot \frac{7}{133} + 3 \cdot \frac{70}{133} + 4 \cdot \frac{56}{133} = \frac{448}{133} \\
E_2 [R] &= 8m + 4n + 2o = 8 \cdot \frac{7}{133} + 4 \cdot \frac{70}{133} + 2 \cdot \frac{56}{133} = \frac{448}{133}
\end{align*}
\]

So the mixed strategy for Player 2 of using $m = \frac{7}{133}$, $n = \frac{70}{133}$, and $o = \frac{56}{133}$ does make Player 2 indifferent over his pure strategies.

So, after all that work, the MSNE to this game is:

Player 1 plays M with probability $\frac{7}{133}$, N with probability $\frac{70}{133}$, and O with probability $\frac{56}{133}$ while Player 2 plays P with probability $\frac{14}{53}$, Q with probability $\frac{20}{53}$, and R with probability $\frac{19}{53}$.

c (15 points – 5 for each game): State whether any of the equilibria can be eliminated using either the equilibrium payoff dominance or undominated Nash equilibrium criteria and explain why the equilibrium can be eliminated.

**Answer:**

In Game 1 there are 3 equilibria: PSNE 1 where Player 1 chooses A and Player 2 chooses D (payoffs 2 and 1 respectively); PSNE 2 where Player 1 chooses B and Player 2 chooses C (payoffs 0 and 2 respectively); and the MSNE (payoffs also 0 and 2 respectively since Player 1 always chooses B). Note that no equilibrium can be ruled out using the equilibrium payoff dominance criterion as Player 1 does better in PSNE 1 than in PSNE 2 or the MSNE while Player 2 is the opposite. However, Player 2 does have a weakly dominant strategy of using D, so we can rule out both PSNE 2 (when Player 2 chooses C) and the MSNE by the undominated Nash equilibrium criterion.
In Game 2 there are 3 equilibria: PSNE 1 where Player 1 chooses E and Player 2 chooses J (payoffs 7 and 6 respectively); PSNE 2 where Player 1 chooses G and Player 2 chooses L (payoffs of 5 and 4 respectively) and the MSNE (payoffs \(7 \frac{4}{2} \) and \( \frac{5}{2} \) respectively). None of these equilibria involve a weakly dominated strategy, but both PSNE 2 and the MSNE can be eliminated using the equilibrium payoff dominance criterion since both Players 1 and 2 have a strictly higher payoff in PSNE 1 than in the other 2 equilibria.

In Game 3 there is only 1 equilibrium to the game so there are no comparisons to make.

2. (20 points) Suppose that three players are engaged in the following simultaneous game:

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Player 1 A</td>
<td>2,1,2</td>
</tr>
<tr>
<td>B</td>
<td>0,3,1</td>
</tr>
<tr>
<td>C</td>
<td>1,1,1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tr>
<td></td>
<td>X</td>
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<tr>
<td>Player 1 A</td>
<td>2,1,3</td>
</tr>
<tr>
<td>B</td>
<td>1,2,1</td>
</tr>
<tr>
<td>C</td>
<td>1,2,1</td>
</tr>
</tbody>
</table>

| P | Q | P3 |

a (5 points) Do any players have any strategies that are strictly dominated and can be removed from the game? Explain.

Answer:
No players have any strategies that are strictly dominated as all strategies are best responses to some choices of strategies by the other players.

b (15 points) Find all pure strategy Nash equilibria to this game.

Answer:
The matrix above shows that there are 2 PSNE to this game: (1) Player 1 choose C, Player 2 choose Y, and Player 3 choose P and (2) Player 1 choose A, Player 2 choose X, and Player 3 choose Q.

3. (20 points) Suppose that 3 players are playing a simultaneous game. There are two available systems to choose from, Beta and VHS. If all three players choose Beta they all receive a payoff of 2. If all three players choose VHS they all receive a payoff of 1. However, if there is any miscoordination, meaning that one player chooses one system while the other two players choose a different system, then all three players receive a payoff of 0.

a (5 points) Find all pure strategy Nash equilibria to this 3 player game.

Answer:
There are 2 PSNE to this game. One is where all 3 players choose Beta; the other is where all 3 players choose VHS. If one player were to choose differently than the others, then that player could switch to what the others were choosing and be strictly better off because he would go from receiving a zero payoff to a positive payoff. So there cannot be any equilibria with miscoordination.

b (15 points) Now suppose that there are 4 players in this game. The payoffs still work in the same manner, in that if all 4 pick Beta they all receive a payoff of 2 and if all 4 pick VHS they all receive a payoff of 1. However, if there is any miscoordination (meaning at least one player chooses a different system than the others) then all players receive a payoff of 0. Find all pure strategy Nash equilibria in this 4 player game.

Answer:
There are more than 2 PSNE now. The 2 PSNE from part a are still PSNE, as all 4 players choosing Beta and all 4 players choosing VHS are PSNE. However, there are now equilibria in which the players miscoordinate. Any outcome where 2 players choose VHS and 2 players choose Beta is a PSNE to
this game (there are 6 of these outcomes). To understand why, think about what happens if the players are at such an outcome and one player switches – that player still receives zero because we have now moved to an outcome where 3 people have chosen one system and 1 person has chosen the other. Essentially, if the players are at an outcome where 2 people have chosen each system it would take 2 people to change in order to increase a player’s payoff.

**Bonus:** (10 points)
There is a mixed strategy Nash equilibrium in the 3 player Beta/VHS game in question 3 – find it. **Note:** This is not the easiest problem, and it doesn’t have the nicest answer.

**Answer:**
In order to find the MSNE to this game, you follow the same process as in a 2 player game by setting the expected values of each player’s strategies equal to each other. I’m going to use the following notation. Let \( \sigma_{1V} \) be the probability that Player 1 chooses VHS (so the probability he chooses Beta is given by \( 1 - \sigma_{1V} \)). Let \( \sigma_{2V} \) and \( 1 - \sigma_{2V} \) be the probabilities that Player 2 chooses VHS and Beta, respectively, and let \( \sigma_{3V} \) and \( 1 - \sigma_{3V} \) be the probabilities that Player 3 chooses VHS and Beta, respectively.

What is Player 1’s expected value of choosing VHS? He receives 1 if both Players 2 and 3 choose VHS, and 0 otherwise. The probability that both Players 2 and 3 choose VHS is given by \( \sigma_{2V} \cdot \sigma_{3V} \). So:

\[
E_1 [VHS] = 1 \cdot \sigma_{2V} \cdot \sigma_{3V}
\]

What is Player 1’s expected value of choosing Beta? He receives 2 if both Players 2 and 3 choose Beta, and 0 otherwise. The probability that both Players 2 and 3 choose Beta is given by \( (1 - \sigma_{2V}) \cdot (1 - \sigma_{3V}) \). So:

\[
E_1 [Beta] = 2 \cdot (1 - \sigma_{2V}) \cdot (1 - \sigma_{3V})
\]

Setting these equal and solving for \( \sigma_{3V} \) we have:

\[
E_1 [VHS] = E_1 [Beta]
\]
\[
1 \cdot \sigma_{2V} \cdot \sigma_{3V} = 2 \cdot (1 - \sigma_{2V}) \cdot (1 - \sigma_{3V})
\]
\[
\sigma_{2V} \cdot \sigma_{3V} = 2 \cdot (1 - \sigma_{2V} - \sigma_{3V} + \sigma_{2V} \cdot \sigma_{3V})
\]
\[
\sigma_{2V} \cdot \sigma_{3V} = 2 - 2\sigma_{2V} - 2\sigma_{3V} + 4\sigma_{2V} \cdot \sigma_{3V}
\]
\[
0 = 2 - 2\sigma_{2V} - 2\sigma_{3V} + 4\sigma_{2V} \cdot \sigma_{3V}
\]
\[
2\sigma_{2V} - 2 = \sigma_{2V} \cdot \sigma_{3V} - 2\sigma_{3V}
\]
\[
2\sigma_{2V} - 2 = \sigma_{3V} \cdot (\sigma_{2V} - 2)
\]
\[
\frac{2\sigma_{2V} - 2}{\sigma_{2V} - 2} = \sigma_{3V}
\]

Now we know \( \sigma_{3V} \) in terms of \( \sigma_{2V} \).

Now let’s work on setting \( E_2 [VHS] = E_2 [Beta] \). If Player 2 chooses VHS he receives 1 with probability \( \sigma_{1V} \cdot \sigma_{3V} \) and if he chooses Beta he receives 2 with probability \( (1 - \sigma_{1V}) \cdot (1 - \sigma_{3V}) \). So:

\[
E_2 [VHS] = E_2 [Beta]
\]
\[
1\sigma_{1V} \cdot \sigma_{3V} = 2 \cdot (1 - \sigma_{1V}) \cdot (1 - \sigma_{3V})
\]
\[
\sigma_{1V} \cdot \sigma_{3V} = 2 - 2\sigma_{1V} - 2\sigma_{3V} + 2\sigma_{1V} \cdot \sigma_{3V}
\]
\[
0 = 2 - 2\sigma_{1V} - 2\sigma_{3V} + 2\sigma_{1V} \cdot \sigma_{3V}
\]

But we know \( \sigma_{3V} \) in terms of \( \sigma_{2V} \) so substituting in we have:

\[
0 = 2 - 2\sigma_{1V} - 2 \cdot \left( \frac{2\sigma_{2V} - 2}{\sigma_{2V} - 2} \right) + \sigma_{1V} \cdot \left( \frac{2\sigma_{2V} - 2}{\sigma_{2V} - 2} \right)
\]
\[
0 = 2 \cdot (\sigma_{2V} - 2) - 2\sigma_{1V} \cdot (\sigma_{2V} - 2) - 2 \cdot (2\sigma_{2V} - 2) + \sigma_{1V} \cdot (2\sigma_{2V} - 2)
\]
\[
0 = 2\sigma_{2V} - 4 - 2\sigma_{1V} \cdot \sigma_{2V} + \sigma_{1V} - 4\sigma_{2V} + 4 + 2\sigma_{1V} \cdot \sigma_{2V} - 2\sigma_{1V}
\]
\[
0 = 2\sigma_{1V} - 2\sigma_{2V}
\]
\[
\sigma_{2V} = \sigma_{1V}
\]
So that is a nice result, that $\sigma_2V = \sigma_{1V}$. It will make things easier. Now let’s work on setting $E_3[\text{VHS}] = E_3[\text{Beta}]$. If Player 3 chooses VHS he receives 1 with probability $\sigma_{1V} \cdot \sigma_2V$ and if he chooses Beta he receives 2 with probability $(1 - \sigma_{1V}) \cdot (1 - \sigma_2V)$. So:

$$E_3[\text{VHS}] = E_3[\text{Beta}]$$

1. $\sigma_{1V} \cdot \sigma_2V = 2 \cdot (1 - \sigma_{1V}) \cdot (1 - \sigma_2V)$
2. $\sigma_{1V} \cdot \sigma_{3V} = 2 \cdot (1 - \sigma_{1V} - \sigma_2V + \sigma_{1V} \cdot \sigma_2V)$
3. $\sigma_{1V} \cdot \sigma_{2V} = 2 - 2\sigma_{1V} - 2\sigma_2V + 2\sigma_{1V} \cdot \sigma_2V$

$$0 = 2 - 2\sigma_{1V} - 2\sigma_2V + \sigma_{1V} \cdot \sigma_2V$$

But we know that $\sigma_{1V} = \sigma_2V$, so:

$$0 = 2 - 2\sigma_{1V} - 2\sigma_2V + \sigma_{1V} \cdot \sigma_2V$$

Note that you can’t FOIL this to find the solution, but you have to use the quadratic formula. So we would have:

$$\sigma_{1V} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sigma_{1V} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$\sigma_{1V} = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$\sigma_{1V} = \frac{4 \pm \sqrt{8}}{2}$$

$$\sigma_{1V} = \frac{4 \pm 2\sqrt{2}}{2}$$

$$\sigma_{1V} = 2 \pm \sqrt{2}$$

So $\sigma_{1V} = 2 + \sqrt{2}$ or $\sigma_{1V} = 2 - \sqrt{2}$. Since $2 + \sqrt{2} > 1$ this cannot be a solution, which leaves $\sigma_{1V} = 2 - \sqrt{2}$. We already know that $\sigma_{1V} = \sigma_2V$ so $\sigma_2V = 2 - \sqrt{2}$ as well. As for $\sigma_{3V}$, we know that

$$\frac{2\sigma_2V - 2}{\sigma_2V - 2} = \sigma_{3V}$$

$$2\sigma_2V - 2 = \sigma_{3V} (\sigma_2V - 2)$$

$$2 \left(2 - \sqrt{2}\right) - 2 = \sigma_{3V} \left(2 - \sqrt{2} - 2\right)$$

$$4 - 2\sqrt{2} - 2 = -\sigma_{3V} \sqrt{2}$$

$$2 - 2\sqrt{2} = -\sigma_{3V} \sqrt{2}$$

$$\frac{2\sqrt{2} - 2}{\sqrt{2}} = \sigma_{3V}$$

And now I’m going to do something a little tricky:

$$\frac{2^{3/2} - 2^1}{2^{1/2}} = \sigma_{3V}$$

$$2^{1} - 2^{1/2} = \sigma_{3V}$$

$$2 - \sqrt{2} = \sigma_{3V}$$

So all three players choose VHS with probability $2 - \sqrt{2}$ and all three players will choose Beta with probability $(1 - (2 - \sqrt{2})) = \sqrt{2} - 1$. While this is the correct MSNE, there is a problem with implementing it since $2 - \sqrt{2}$ is not a rational number. But that’s a story for a different day.