Assignment 4

ECON 3161, Game Theory

Due: By the end of class on Tuesday, October 30th

Directions: Answer all questions completely. Note the due date of the assignment. Late assignments will be accepted at the cost of 10 points per day, up until 11am on Thursday November 1st. At that time I will return the graded assignments and post the answers online. You may turn in assignments to me after that time so that I can check your work for you, but please realize that you will not receive a grade for the assignment. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. (50 points) Consider a simultaneous quantity choice (Cournot) game between 2 firms. Each firm chooses a quantity, \( q_1 \) and \( q_2 \) respectively. The inverse market demand function is given by \( P(Q) = 1434 - 2Q \), where \( Q = q_1 + q_2 \). Firm 1 has total cost function \( TC(q_1) = 3q_1^2 \) and Firm 2 has total cost function \( TC(q_2) = 12q_2^2 - 12q_2 \). Each firm wishes to maximize profit.

a (10 points) Set up the profit function for Firm 1 and Firm 2. Remember, this is a quantity choice game.

b (10 points) Find the best response functions for Firms 1 and 2.

c (10 points) Find the Nash equilibrium to this game.

d (10 points) Find the (1) total market quantity, (2) price, and (3) profit for each firm.

e (10 points) Assume Firm 1 is the only producer in the market now. Find the Firm 1’s monopoly (1) quantity, (2) price, and (3) profit.

2. (25 points) The citizens of Circleburg live in a city that is laid out in a perfect circle. The circumference of the circle is 12 miles. Residents live in houses which are evenly spaced (or uniformly distributed) over the 12 miles. There are three competing gas stations, Chi Station, Delta Station, and Tau Station. They are attempting to determine where to locate their respective stations. They know that residents of Circleburg will go to the gas station closest to their home. Assume that gas stations are concerned with maximizing the number of customers who visit their station. You may want to use a diagram to aid you when answering the questions. A Nash Equilibrium for this game is a set of locations for the gas stations. Note that gas stations may locate at the same point on the circle. Note: Customers and gas stations can only locate on the perimeter of the circle. Assume that the interior of the circle is a huge chasm or a steep mountain. Someone always tries to locate stations/customers in the middle of the circle - do NOT do that. Hint: It may help to think of the circle as the face of a clock.

a (5 points) Find a pure strategy Nash equilibrium to this game where all firms receive the same number of customers. Explain why this is a PSNE.

b (10 points) Find a pure strategy Nash equilibrium to this game where all firms do NOT receive the same number of customers. Explain why this is a PSNE.

c (10 points) The citizens of Circleburg have decided to outlaw backward thinking, which includes counterclockwise driving. Now, residents of Circleburg will stop at the first gas station they see when they leave their home. If there is a PSNE to this game, find it. If there are multiple PSNE, describe the set of equilibria. If there are no PSNE, explain why there are none.
3. (25 points) Consider two firms engaged in price (Bertrand) competition, such that the firm that charges the lowest price produces the entire market quantity. If the 2 firms charge the same price they split the market quantity evenly. This means that each firm has the following demand function:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Firm 1 Demand</th>
<th>Firm 2 Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 &gt; p_2 )</td>
<td>0</td>
<td>( \frac{\text{min}(500-p_2)}{2} )</td>
</tr>
<tr>
<td>( p_1 = p_2 )</td>
<td>( \frac{1}{2} \times \frac{500-p_2}{2} )</td>
<td>( \frac{1}{2} \times \frac{500-p_2}{2} )</td>
</tr>
<tr>
<td>( p_1 &lt; p_2 )</td>
<td>( \frac{\text{min}(500-p_1)}{2} )</td>
<td>0</td>
</tr>
</tbody>
</table>

Each firm has total cost equal to \( TC(q_i) = 20q_i \), so that each firm has a constant marginal and average cost of production of 20.

a (10 points) Suppose that the pricing choices are made simultaneously. Find the pure strategy Nash equilibrium to this game.

Now suppose that Firm 1 announces the following policy: We are going to charge $260 for our product. If any customer finds a lower price for this product than $260 then tell us and we will not only match that price but offer a refund equal to the difference in the two prices. For instance, if another firm charges $240, we will only charge $220 (take $260 - $240 = $20 and then deduct another $20 so that the total amount deducted from our price of $260 is $40). This is known as a price-beating policy.

b (10 points) Find Firm 2’s best response to Firm 1’s policy announcement. Reminder: This is only a one period game.

c (5 points) Given the best response you found for Firm 2 in part b, is that best response and Firm 1’s strategy (announced policy and price choice of $260) a pure strategy Nash equilibrium to the game? Explain.

Bonus:

(5 points) In game 3, suppose that Firm 1 announced a policy where they merely met Firm 2’s price (a price-matching policy). Does this affect the equilibrium to the game when compared to the price-beating policy? Explain why or why not.