Assignment 6

ECON 3161, Game Theory

Due: By the end of class on Tuesday, December 4th

Directions: Answer all questions completely. Note the due date of the assignment. Late assignments will be accepted at the cost of 10 points per day, up until 11am on Wednesday December 5th. At that time I will return the graded assignments and post the answers online. You may turn in assignments to me after that time so that I can check your work for you, but please realize that you will not receive a grade for the assignment. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. (20 points) Suppose that an instructor stands in front of his class and says, “I am going to auction off this $1 bill”. There is nothing special about this bill (it is not some rare error or some 200 year-old bill), it is just a regular $1 bill. The format of the auction is as follows:

This auction is an ascending oral outcry auction (in this type of auction bidders call out bids that must be greater than the standing high bid; the auction ends after the instructor hears no more bids, or you can think about it as the instructor uses the phrase “Going once ... going twice ... sold!” as the stopping rule of the auction). Students are able to call out bids. The bid space is discrete, which means that students can only call out amounts in increments of a penny. So the only allowable bids are [$0.01; $0.02; $0.03; ...]. The winning bidder will be the one who calls out the highest bid. The winning bidder will pay an amount equal to the last amount that he called out – so if the last amount a winning bidder calls out is 50 cents, the winning bidder pays 50 cents. However, any other bidder who calls out at least one bid must also pay an amount equal to the highest bid he called out. So if you are a losing bidder and your highest bid was 25 cents, you must also pay 25 cents.

a (5 points) Suppose that there are 4 students bidding for the item: The bids are currently 50 cents, 49 cents, 10 cents and 1 cent for students 1-4 respectively. What are the current payoffs to the bidders? Can anyone do any better by increasing their bid? If so, who?

b (5 points) Suppose that now there are still those same 4 students and that the bids are now 1 dollar, 99 cents, 10 cents, and 1 cent for students 1-4 respectively. Would any student bid more than $1? Explain why or why not.

c (10 points) Consider now the case of two bidders at the start of the game (so no bids have been made yet). Full specification of an equilibrium would be difficult given the sequential nature of the game (if a picture were to be drawn it would be a game tree with perfect information with many different turns for the players – Player 1 bids, then Player 2 bids, then Player 1 bids again (or chooses not to bid), etc.) However, there are two opening bids by the first bidder which would result in the game ending after only one bid, meaning there are no bids the second bidder could make that could possibly lead to a strictly higher payoff than not bidding for the second bidder. What are those two opening bids?
2. (40 points) Consider the following Sender-Receiver game:

a (10 points) Find all pure strategy separating equilibria where type \( t_1 \) chooses \( L \) and type \( t_2 \) chooses \( R \). If there are none explain why not.

b (10 points) Find all pure strategy separating equilibria where type \( t_1 \) chooses \( R \) and type \( t_2 \) chooses \( L \). If there are none explain why not.

c (10 points) Find all pure strategy pooling equilibria where type \( t_1 \) chooses \( L \) and type \( t_2 \) chooses \( L \). If there are none explain why not.

d (10 points) Find all pure strategy pooling equilibria where type \( t_1 \) chooses \( R \) and type \( t_2 \) chooses \( R \). If there are none explain why not.

3. (40 points) Consider the following: There are two types of workers, high ability and low ability. The type of worker is randomly determined by chance with \( \Pr(\text{high}) \) being the probability of a high type and \( \Pr(\text{low}) \) being the probability of a low type. The workers must make a decision regarding school. They can either go to college or not go to college. If a high ability worker attends college it costs him \( C_H \). If a low ability worker attends college it costs him \( C_L \). We will assume that \( C_L > C_H \) because the low ability worker will have to work harder in college than the high ability worker. If workers do not attend college then they obtain a job paying some wage \( M \) regardless of their type. There is an employer who must make a hiring decision. The employer is unable to determine whether a worker is high ability or low ability, but the employer can observe that the worker graduated from college. A high ability worker is worth \( R_H \) to the employer while a low ability worker is worth \( R_L \) to the employer. The employer makes a decision to offer a job or not offer a job. If a job is offered, the employer pays the worker a salary of \( W \). If a job is not offered, the worker returns to his job paying \( M \). Assume that the firm receives a payoff of zero if it does not hire a worker. The game tree is as follows:
a Propose a perfect Bayesian equilibrium that is a pooling equilibrium where all workers attend college.

- (10 points) Write down the equilibrium you propose.
- (5 points) Verify that this is in fact a Bayes-Nash Equilibrium by checking the relevant constraints. (When I write verify I mean write down the relevant constraints, not just say “Yes, I verified it by checking the constraints.”)

b Propose a perfect Bayesian equilibrium that is a separating equilibrium where only high ability workers attend college.

- (10 points) Write down the equilibrium you propose.
- (5 points) Verify that this is in fact a Bayes-Nash Equilibrium by checking the relevant constraints.

c Suppose that $R^H = 20$, $R^L = 15$, $C^L = 12$, $C^H = 8$, $M = 12$, $W = 19$, and $Pr(high) = 0.5$.

- (10 points) Will there be a pooling equilibrium or a separating equilibrium and what will the equilibrium be?
Bonus:
(10 points) Find all pure strategy Nash equilibria, subgame perfect Nash equilibria, and perfect Bayesian equilibria in the following game: