Directions: Answer each question completely. If you cannot determine the answer, explaining how you would arrive at the answer may earn you some points.

1. (40 points) Consider the following Sender-Receiver game, where the probability that the Sender is a type $t_1$ is $\frac{2}{3}$ and the probability that the Sender is a type $t_2$ is $\frac{1}{3}$:

   a. (20 points) Find all pure strategy perfect Bayesian separating equilibria in this game.

   Answer:
   Suppose that $t_1$ chooses $L$ and $t_2$ chooses $R$. The Sender’s strategy is:
   \[
   \begin{align*}
   &t_1 \text{ choose } L \\
   &t_2 \text{ choose } R
   \end{align*}
   \]
   Sender’s strategy

   The Receiver’s beliefs are:
   \[
   \begin{align*}
   \Pr(t_1|L) &= 1 \\
   \Pr(t_1|R) &= 0 \\
   \Pr(t_2|L) &= 0 \\
   \Pr(t_2|R) &= 1
   \end{align*}
   \]
   Receiver’s beliefs
The Receiver’s strategy (best responses) would be:

\[
\begin{align*}
U & \text{ if } L \\
D & \text{ if } R
\end{align*}
\]

Receiver’s strategy

Is the Sender’s strategy a best response to the Receiver’s strategy? If \( t_1 \) switches to \( R \) he would receive 4 because the Receiver is choosing \( D \) if \( R \). But this is less than the 5 the \( t_1 \) type receives by choosing \( L \) so he would not switch. If \( t_2 \) switches to \( L \) he would receive 2. But this is the same amount he receives if he chooses \( R \) so he has no incentive to switch. Thus, this is a separating equilibrium.

Now consider the other potential separating equilibrium in which type \( t_1 \) chooses \( R \) and type \( t_2 \) chooses \( L \). The Sender’s strategy is:

\[
\begin{align*}
 t_1 & \text{ choose } R \\
 t_2 & \text{ choose } L
\end{align*}
\]

Sender’s strategy

The Receiver’s beliefs are:

\[
\begin{align*}
\Pr(t_1|L) & = 0 \\
\Pr(t_2|L) & = 1 \\
\Pr(t_1|R) & = 1 \\
\Pr(t_2|R) & = 0
\end{align*}
\]

Receiver’s beliefs

The Receiver’s strategy (best responses) would be:

\[
\begin{align*}
D & \text{ if } L \\
D & \text{ if } R
\end{align*}
\]

Receiver’s strategy

Is the Sender’s strategy a best response to the Receiver’s strategy? If \( t_1 \) switches to \( L \) he would receive 2 because the Receiver is choosing \( D \) if \( L \). But this is less than the 6 the \( t_1 \) type receives by choosing \( R \) so he would not switch. If \( t_2 \) switches to \( R \) he would receive 2. But this is less than the 6 he receives if he chooses \( L \) so he has no incentive to switch. Thus, this is also a separating equilibrium.

b (20 points) Find all pure strategy perfect Bayesian pooling equilibria in this game.

Answer:

Considering pooling equilibria where both types choose \( R \)

Consider the other potential pooling equilibrium in which both types \( t_1 \) and \( t_2 \) choose \( R \). The Sender’s strategy is:

\[
\begin{align*}
 t_1 & \text{ choose } R \\
 t_2 & \text{ choose } R
\end{align*}
\]

Sender’s strategy

The Receiver’s beliefs are:

\[
\begin{align*}
\Pr(t_1|L) & = q \\
\Pr(t_2|L) & = 1 - q \\
\Pr(t_1|R) & = \frac{2}{3} \\
\Pr(t_2|R) & = \frac{3}{3}
\end{align*}
\]

Receiver’s beliefs

If the Receiver observes \( R \) then he can either choose \( U \) or \( D \). If he chooses \( U \) he expects to receive:

\[
E[U|R] = \frac{2}{3} \times 1 + \frac{1}{3} \times 3 = \frac{5}{3}
\]

If he chooses \( D \) he expects to receive:

\[
E[D|R] = \frac{2}{3} \times 2 + \frac{1}{3} \times 5 = \frac{9}{3}
\]

So the Receiver would choose \( D \) if he observes \( R \). If he observes \( L \) and chooses \( U \) he expects to receive:

\[
E[U|L] = q \times 9 + (1-q) \times 3 = 9q + 3 - 3q = 6q + 3
\]
while if he chooses $D$ he expects to receive:

$$E[D|L] = q \times 6 + (1-q) \times 4 = 6q + 4 - 4q = 2q + 4$$

So the Receiver will choose $U$ if:

$$\begin{align*}
6q + 3 & \geq 2q + 4 \\
4q & \geq 1 \\
q & \geq \frac{1}{4}
\end{align*}$$

and $D$ otherwise. Considering the case when the Receiver chooses $U$ if $L$ we have the following potential equilibrium:

$$\begin{align*}
t_1 \text{ choose } R \\
t_2 \text{ choose } R
\end{align*}$$

Sender’s strategy

The Receiver’s beliefs are:

$$\begin{align*}
\Pr(t_1|L) = q & \geq \frac{1}{3} \\
\Pr(t_2|L) = 1 - q \\
\Pr(t_1|R) = \frac{2}{3} \\
\Pr(t_2|R) = \frac{1}{3}
\end{align*}$$

Receiver’s beliefs

The Receiver’s strategy (best responses) would be:

$$\begin{align*}
U & \text{ if } L \\
D & \text{ if } R
\end{align*}$$

Receiver’s strategy

Would either Sender type like to switch? Type $t_1$ would like to switch – $t_1$ currently gets 4 from choosing $R$ and would get 5 from choosing $L$ so this is not an equilibrium.

Now consider the case where the Receiver chooses $D$ if $L$ we have the following potential equilibrium:

$$\begin{align*}
t_1 \text{ choose } R \\
t_2 \text{ choose } R
\end{align*}$$

Sender’s strategy

The Receiver’s beliefs are:

$$\begin{align*}
\Pr(t_1|L) = q & \leq \frac{1}{3} \\
\Pr(t_2|L) = 1 - q \\
\Pr(t_1|R) = \frac{2}{3} \\
\Pr(t_2|R) = \frac{1}{3}
\end{align*}$$

Receiver’s beliefs

The Receiver’s strategy (best responses) would be:

$$\begin{align*}
D & \text{ if } L \\
D & \text{ if } R
\end{align*}$$

Receiver’s strategy

But here the type $t_2$ would like to switch to $L$ because the $t_2$ type receives 2 if it chooses $R$ and would receive 6 if it switched to $L$. Thus, there are no pooling equilibria where both sender types choose $R$.

Considering pooling equilibria where both types choose $L$

Now we need to check to see if there are any pooling equilibria when both types choose $L$. The Sender’s strategy is:

$$\begin{align*}
t_1 \text{ choose } L \\
t_2 \text{ choose } L
\end{align*}$$

Sender’s strategy

The Receiver’s beliefs are:

$$\begin{align*}
\Pr(t_1|L) = \frac{2}{3} \\
\Pr(t_2|L) = \frac{1}{3} \\
\Pr(t_1|R) = p \\
\Pr(t_2|R) = 1 - p
\end{align*}$$

Receiver’s beliefs
If the Receiver observes $L$ then he can either choose $U$ or $D$. If he chooses $U$ he expects to receive:

$$E[U|L] = \frac{2}{3} \cdot 9 + \frac{1}{3} \cdot 3 = \frac{21}{3}$$

If he chooses $D$ he expects to receive:

$$E[D|L] = \frac{2}{3} \cdot 6 + \frac{1}{3} \cdot 4 = \frac{16}{3}$$

The Receiver chooses $U$ if $L$ is observed because $\frac{21}{3} > \frac{16}{3}$. If the Receiver observes $R$ and he chooses $U$ he expects to receive:

$$E[U|R] = 1 \cdot p + 3 \cdot (1-p) = p + 3 - 3p = 3 - 2p$$

and if he observes $R$ and he chooses $D$ he expects to receive:

$$E[D|R] = 2 \cdot p + 5 \cdot (1-p) = 2p + 5 - 5p = 5 - 3p$$

The Receiver will choose $U$ if:

$$3 - 2p \geq 5 - 3p$$

$$p \geq 2$$

But this cannot be true because $0 \leq p \leq 1$, so the Receiver must choose $D$. We already knew this though because the Receiver always chose $D$ in the separating equilibria regardless of which player chose $R$. Thus there is only one potential pooling equilibrium to check when both Sender types choose $L$: The Sender’s strategy is:

- $t_1$ choose $L$
- $t_2$ choose $L$

Sender’s strategy

The Receiver’s beliefs are:

- $Pr(t_1|L) = \frac{2}{3}$
- $Pr(t_2|L) = \frac{1}{3}$
- $Pr(t_1|R) = p \leq 1$
- $Pr(t_2|R) = 1 - p$

Receiver’s beliefs

The Receiver’s strategy (best responses) would be:

- $U$ if $L$
- $D$ if $R$

Receiver’s strategy

Does the $t_1$ type want to switch? No, because it currently receives 5 from choosing $L$ and would receive 4 if it switched to $R$. Does the $t_2$ type want to switch? No, because it currently receives 2 from choosing $L$ and would still receive 2 if it chose $R$. Thus, this is the only pooling equilibrium, where both sender types choose $L$.

2. (15 points) There are 2 envelopes, each containing an amount of money; the amount of money is either $5, $10, $20, $40, $80, or $160. All players know this. Furthermore, all players know that one envelope contains exactly twice as much money as the other envelope. This means that if one player receives the $20 envelope, the other player will receive either the $40 envelope or the $10 envelope, etc. The two envelopes are shuffled and one is given to Player 1 and the other to Player 2. After both the envelopes are opened (but the amounts inside are kept as private information) the players are given the opportunity to switch envelopes with each other. If both players agree to switch, then the envelopes are switched. If one (or both) player does not agree to switch, then the players keep their original envelopes. A Bayes-Nash equilibrium to this game consists of each player choosing either “Keep” or “Switch” for each type the player could be.

a (5 points) What should a player who receives the $5 envelope choose as his action? What should a player who receives the $160 envelope choose as his action?
Answer:
A player who receives the $5 envelope should choose to switch as there is no risk – that player knows the other player has the $10 envelope. A player who receives the $160 envelope should choose to keep as there is no benefit in switching – if he chooses to switch then he can only receive less money (the $80 envelope).

b (10 points) Find a Bayes-Nash equilibrium to this game.

Answer:
We need to specify an action for each possible envelope each player could see. A Bayes-Nash equilibrium to this game is Player 1 chooses Keep for any envelope, Player 2 chooses Keep for any envelope. Another one would be that Player 1 chooses Keep for all envelopes but the $5 envelope, and Player 2 chooses Switch for the $5 envelope and Keep for the other envelopes. Basically, both players need to Keep any envelope above $5, and can either Keep or Switch the $5 envelope.

Why must the players Keep all the envelopes with amounts greater than $5? Think about a player who receives $160 – that player is going to Keep because choosing Switch will make that player worse off. Now think about a player who receives $80 – that player is also going to choose Keep. The player would like to choose Switch in hopes of getting the $160 envelope, but the player with the $80 envelope knows that if a switch is actually made, it will be because the other player has the $40 envelope, not the $160 envelope (because the $160 envelopes will all be kept). So the player with the $80 envelope will choose Keep. Using this logic we can see that the $40 envelope, the $20 envelope, and the $10 envelope will all also be kept. With the $5 envelope, it does not matter if the player chooses Keep or Switch, because no switch will actually be made, and the person with the $5 envelope cannot be made worse off by switching.

Now, suppose one player used a strategy like:
Keep $5, Switch $10, Keep $20, $40, $80, $160
The other player would then like to use a strategy like:
Switch $5, Keep $10, $20, $40, $80, $160
This is not a Nash equilibrium because the player who is using the strategy Switch $10 has a lower expected value from Switch $10 than from Keep $10. If the Player chooses Keep $10 then the player receives $10 for certain. If the player chooses Switch $10, then half the time the player receives $10 (when the other player has $20 no switch is made) and the other half of the time the player receives $5 (when the other player has $5 a switch is made), so the player ends up with $7.50 on average.

3. (25 points) Consider a Cournot game of incomplete information. There are 2 firms in this market. Firms face the following inverse demand function, \( P(Q) = 194 - Q \), where \( Q = q_1 + q_2 \). Firms 1 and 2 may have high or low cost and while each firm knows its own cost the other firm only knows the distribution of costs for its competitor. With probability \( \alpha \) Firm 1 has total cost \( TC_{1L} = 16q_{1L} \), where \( q_{1L} \) is the amount Firm 1 produces when it has low cost, and with probability \( 1 - \alpha \) Firm 1 has total cost \( TC_{1H} = 32q_{1H} \), , where \( q_{1H} \) is the amount Firm 1 produces when it has high cost. With probability \( \theta \) Firm 2 has total cost \( TC_{2L} = 24q_{2L} \), where \( q_{2L} \) is the amount Firm 2 produces when it has low cost and with probability \( 1 - \theta \) Firm 2 has total cost \( TC_{2H} = 40q_{2H} \), where \( q_{2H} \) is the amount Firm 2 produces when it has high cost. Let \( \alpha = \frac{3}{4} \) and \( \theta = \frac{1}{2} \). Both firms simultaneously choose a quantity of production in this market. The profit function for each type of firm is:

For Firm 1 with cost \( TC_{1L} \) we have:

\[
\Pi_{1L} = 178q_{1L} - (q_{1L})^2 - \frac{1}{2}q_{2L}q_{1L} - \frac{1}{2}q_{2H}q_{1L}
\]

For Firm 1 with cost \( TC_{1H} \) we have:

\[
\Pi_{1H} = 162q_{1H} - (q_{1H})^2 - \frac{1}{2}q_{2L}q_{1H} - \frac{1}{2}q_{2H}q_{1H}
\]
For Firm 2 with cost $TC_{2L}$ we have:

$$
\Pi_{2L} = 170q_{2L} - (q_{2L})^2 - \frac{3}{4}q_{1L}q_{2L} - \frac{1}{4}q_{1H}q_{2L}
$$

For Firm 2 with cost $TC_{2H}$ we have:

$$
\Pi_{2H} = 154q_{2H} - (q_{2H})^2 - \frac{3}{4}q_{1L}q_{2H} - \frac{1}{4}q_{1H}q_{2H}
$$

(a) (10 points) Find the best response functions for each type.

**Answer:**

To find the best response for each type we take the derivative of the profit function with respect to the choice variable and then set the derivative equal to zero and solve for the choice variable.

For Firm 1 with cost $TC_{1L}$ we have:

$$
\Pi_{1L} = 178q_{1L} - (q_{1L})^2 - \frac{1}{2}q_{2L}q_{1L} - \frac{1}{2}q_{2H}q_{1L}
$$

$$
\frac{\partial \Pi_{1L}}{\partial q_{1L}} = 178 - 2q_{1L} - \frac{1}{2}q_{2L} - \frac{1}{2}q_{2H}
$$

$$
0 = 178 - 2q_{1L} - \frac{1}{2}q_{2L} - \frac{1}{2}q_{2H}
$$

$$
q_{1L} = \frac{178 - \frac{1}{2}q_{2L} - \frac{1}{2}q_{2H}}{2}
$$

For Firm 1 with cost $TC_{1H}$ we have:

$$
\Pi_{1H} = 162q_{1H} - (q_{1H})^2 - \frac{1}{2}q_{2L}q_{1H} - \frac{1}{2}q_{2H}q_{1H}
$$

$$
\frac{\partial \Pi_{1H}}{\partial q_{1H}} = 162 - 2q_{1H} - \frac{1}{2}q_{2L} - \frac{1}{2}q_{2H}
$$

$$
0 = 162 - 2q_{1H} - \frac{1}{2}q_{2L} - \frac{1}{2}q_{2H}
$$

$$
q_{1H} = \frac{162 - \frac{1}{2}q_{2L} - \frac{1}{2}q_{2H}}{2}
$$

For Firm 2 with cost $TC_{2L}$ we have:

$$
\Pi_{2L} = 170q_{2L} - (q_{2L})^2 - \frac{3}{4}q_{1L}q_{2L} - \frac{1}{4}q_{1H}q_{2L}
$$

$$
\frac{\partial \Pi_{2L}}{\partial q_{2L}} = 170 - 2q_{2L} - \frac{3}{4}q_{1L} - \frac{1}{4}q_{1H}
$$

$$
0 = 170 - 2q_{2L} - \frac{3}{4}q_{1L} - \frac{1}{4}q_{1H}
$$

$$
q_{2L} = \frac{170 - \frac{3}{4}q_{1L} - \frac{1}{4}q_{1H}}{2}
$$

For Firm 2 with cost $TC_{2H}$ we have:

$$
\Pi_{2H} = 154q_{2H} - (q_{2H})^2 - \frac{3}{4}q_{1L}q_{2H} - \frac{1}{4}q_{1H}q_{2H}
$$

$$
\frac{\partial \Pi_{2H}}{\partial q_{2H}} = 154 - 2q_{2H} - \frac{3}{4}q_{1L} - \frac{1}{4}q_{1H}
$$

$$
0 = 154 - 2q_{2H} - \frac{3}{4}q_{1L} - \frac{1}{4}q_{1H}
$$

$$
q_{2H} = \frac{154 - \frac{3}{4}q_{1L} - \frac{1}{4}q_{1H}}{2}
$$

Technically, all of these should be either the function we found or 0 if the quantity is too low.
b (10 points) Find the Bayes-Nash equilibrium to this game.

**Answer:**

To find the Nash equilibrium to this game I will begin by substituting the best response functions for $q_2H$ and $q_2L$ into the best response functions for $q_1L$ and $q_1H$, so that $q_1L$ is solely a function of $q_1H$ and $q_1H$ is solely a function of $q_1L$.

\[
q_{1L} = 178 - \frac{1}{2} \left( \frac{170 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right) - \frac{1}{2} \left( \frac{154 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right)
\]

\[
2q_{1L} = 178 - \frac{1}{2} \left( \frac{170 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right) - \frac{1}{2} \left( \frac{154 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right)
\]

\[
8q_{1L} = 712 - 170 + \frac{3}{4} q_1L + \frac{1}{4} q_1H - 154 + \frac{3}{4} q_1L + \frac{1}{4} q_1H
\]

\[
8q_{1L} = 388 + \frac{3}{2} q_1L + \frac{1}{2} q_1H
\]

\[
16q_{1L} = 776 + 3q_1L + q_1H
\]

\[
13q_{1L} = 776 + q_1H
\]

For $q_1H$ we have:

\[
q_{1H} = 162 - \frac{1}{2} \left( \frac{170 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right) - \frac{1}{2} \left( \frac{154 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right)
\]

\[
2q_{1H} = 162 - \frac{1}{2} \left( \frac{170 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right) - \frac{1}{2} \left( \frac{154 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2} \right)
\]

\[
8q_{1H} = 648 - 170 + \frac{3}{4} q_1L + \frac{1}{4} q_1H - 154 + \frac{3}{4} q_1L + \frac{1}{4} q_1H
\]

\[
8q_{1H} = 324 + \frac{3}{2} q_1L + \frac{1}{2} q_1H
\]

\[
16q_{1H} = 648 + 3q_1L + q_1H
\]

\[
15q_{1H} = 648 + 3q_1L
\]

\[
q_{1H} = \frac{648 + 3q_1L}{15}
\]

Now, substituting in for $q_{1H}$ we have:

\[
13q_{1L} = 776 + \frac{648 + 3q_1L}{15}
\]

\[
195q_{1L} = 11640 + 648 + 3q_1L
\]

\[
192q_{1L} = 12288
\]

\[
q_{1L} = 64
\]

Now that we have $q_{1L} = 64$, we know $q_{1H} = 56$ because $q_{1H} = \frac{648 + 3q_1L}{15}$. Now that we have $q_{1H}$ and $q_{1L}$, we know that:

\[
q_{2L} = \frac{170 - \frac{3}{4} q_1L - \frac{1}{4} q_1H}{2}
\]

\[
q_{2L} = \frac{170 - \frac{3}{4} \times 64 - \frac{1}{4} \times 56}{2}
\]

\[
q_{2L} = \frac{170 - 48 - 14}{2}
\]

\[
q_{2L} = 54
\]
and:

\[ q_{2H} = \frac{154 - \frac{3}{4}q_{1L} - \frac{1}{4}q_{1H}}{2} \]
\[ q_{2H} = \frac{154 - \frac{3}{4} \times 64 - \frac{1}{4} \times 56}{2} \]
\[ q_{2H} = \frac{154 - 48 - 14}{2} \]
\[ q_{2H} = 46 \]

So the Nash equilibrium to this game is: \( q_{1L} = 64 \), \( q_{1H} = 56 \), \( q_{2L} = 54 \), and \( q_{2H} = 46 \).

There were a few people who used matrices to solve for these quantities – either way gets the same answer.

c. (5 points) Suppose that Firm 1 has low cost and Firm 2 has high cost. What is the resulting market quantity and price, and what is each firm’s profit?

Answer:
If Firm 1 has low cost and Firm 2 has high cost then we know that Firm 1 will produce \( q_{1L} = 64 \) and that Firm 2 will produce \( q_{2H} = 46 \). This means that \( Q = 110 \) so that \( P(Q) = 194 - 110 = 84 \). Firm 1’s profit is:

\[ \Pi_1 = 84 \times 64 - 16 \times 64 = 4352 \]

while Firm 2’s profit is:

\[ \Pi_2 = 84 \times 46 - 40 \times 46 = 2024 \]

Once the firms have actually produced their profit is based on actual production levels, not expected production levels (this was a mistake a few people made).

4. (20 points) Consider the following simultaneous game of incomplete information between two players. Player 1 is a known type to all players, whereas Player 2 knows his own type but Player 1 does not. Player 1 believes that Player 2 is a Soft type with probability \( \frac{1}{4} \), a Medium type with probability \( \frac{2}{4} \), and a Tough type with probability \( \frac{1}{4} \).

<table>
<thead>
<tr>
<th>P2 (Soft)</th>
<th>P2 (Medium)</th>
<th>P2 (Tough)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>Left</td>
<td>Left</td>
</tr>
<tr>
<td>Up</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Down</td>
<td>3</td>
<td>4.7</td>
</tr>
<tr>
<td>Right</td>
<td>0.5</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Find all pure strategy Bayes-Nash equilibria to this game.

Answer:
Suppose that P1 chooses Up. What is the best response for each type of P2? The Soft type chooses Left (6 > 5), the Medium type chooses Right (0 < 3), and the Tough type chooses Left (12 > 1). Now, is P1 choosing Up a best response to this strategy by P2?
If P1 chooses Up he expects to receive:

\[ E[Up] = \frac{1}{4} \times 9 + \frac{2}{4} \times 2 + \frac{1}{4} \times 5 = \frac{18}{4} \]

If P1 switches to Down he expects to receive:

\[ E[Down] = \frac{1}{4} \times 3 + \frac{2}{4} \times 1 + \frac{1}{4} \times 10 = \frac{15}{4} \]

So P1 does not want to switch to Down because \( \frac{18}{4} > \frac{15}{4} \). Thus, one pure strategy Bayes-Nash equilibrium is P1 choose Up, P2 choose Left if Soft type, Right if Medium type, and Left if Tough type.
Now, suppose that P1 chooses Down. What is the best response for each type of P2? The Soft type chooses Right (4 < 7), the Medium type chooses Left (9 > 2), and the Tough type chooses Left (11 > 10). Now, is P1 choosing Down a best response to this strategy by P2?

If P1 chooses Down he expects to receive:

\[ E[\text{Down}] = \frac{1}{4} \times 4 + \frac{2}{4} \times 8 + \frac{1}{4} \times 10 = \frac{30}{4} \]

If P1 switches to Up he expects to receive:

\[ E[\text{Up}] = \frac{1}{4} \times 0 + \frac{2}{4} \times 6 + \frac{1}{4} \times 5 = \frac{17}{4} \]

So P1 does not want to switch to Up because \( \frac{30}{4} > \frac{17}{4} \). Thus, a second pure strategy Bayes-Nash equilibrium is P1 choose Down, P2 choose Right if Soft type, Left if Medium type, and Left if Tough type.

And that’s all there can be because there are no points where a P2 type is indifferent given what P1 has chosen.

**Bonus:** (5 points) Consider the following Sender-Receiver game:

In this game the only perfect Bayesian equilibrium is a separating equilibrium when type \( t_1 \) chooses \( L \) and type \( t_2 \) chooses \( R \) and the Receiver chooses \( U \) if \( L \) and \( U \) if \( R \). Explain why this is the only PBE to this game.  

**Note:** You do not have to show that any other potential equilibrium is not an equilibrium – there is a very basic reason why the only equilibrium is when type \( t_1 \) chooses \( L \) and type \( t_2 \) chooses \( R \).

**Answer:**

It should (hopefully) be straightforward to see that type \( t_1 \) will never choose \( R \) regardless of what the Receiver chooses because the worst payoff from \( L \) is 95 whereas the best payoff from \( R \) is 1; similarly, type \( t_2 \) will never choose \( L \) because the worst payoff from \( R \) is 71 while the best payoff from \( L \) is 9. So the first mover has a strictly dominant strategy of choosing \( L \) if type \( t_1 \) and \( R \) if type \( t_2 \).