1 Cost minimization

Consider a firm which uses only two inputs in production, capital ($K$) and labor ($L$). The firm has production function $q (K, L) = L^\beta K^\alpha$. Let $w$ be the wage rate for $L$ and $r$ be the rental rate of capital. Suppose the firm wishes to produce 1080 units of the good. Let $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$, and let $w = $4 and $r = $27.

1. Find the marginal product of capital.
2. Find the marginal product of labor.
3. Find the marginal rate of technical substitution.
4. Find the cost-minimizing bundle of capital and labor for this firm, as well as the associated total cost amount.

Suppose that the price of capital increases, so that now $r = $64. The firm still wishes to produce 1080 units.

5. Find the cost-minimizing bundle of capital and labor for this firm given the increase in $r$, as well as the associated total cost amount.

2 Production

Consider the following production functions, where $q$ is the quantity produced of the good, $K$ is the quantity of capital used, and $L$ is the quantity of labor used:

Production function 1

$$q (K, L) = K^\alpha L^\beta$$

Production function 2

$$q (K, L) = K^\alpha + L^\beta$$

1. For production function 1, for what values of $\alpha$ and $\beta$ will this production function exhibit (a) increasing, (b) constant, and (c) decreasing returns to scale?

2. For production function 2, for what values of $\alpha$ and $\beta$ will this production function exhibit (a) increasing, (b) constant, and (c) decreasing returns to scale?
3 Monopoly

Suppose that a monopolist faces the following inverse demand curve, \( P(Q) = 65 - 5Q \). The monopolist’s total cost function is: \( TC = 2.4Q^3 - 19Q^2 + 66.5Q + 40 \).

1. Find the monopolist’s marginal revenue function.

2. Find the monopolist’s marginal cost function.

3. Find the monopolist’s profit-maximizing price and quantity in this market, as well as the monopolist’s profit at this price and quantity.

4 Cournot with Different MC

Assume the following: there are two firms competing in a Cournot (quantity) game. The firms face the following inverse demand function: \( P(Q) = a - bQ = 15000 - 50Q \). Firm 1 has a cost structure such that \( TC_1 = c_1 * q_1 \), so that Firm 1’s marginal cost is \( MC_1 = c_1 \). Firm 2 has a cost structure such that \( TC_2 = c_2 * q_2 \), so that Firm 2’s marginal cost is \( MC_2 = c_2 \). Let \( c_1 = 50 \) and \( c_2 = 100 \), so that we have \( c_1 < c_2 \).


2. Find the Nash equilibrium for this game.

3. Find the market price and resulting firm profits at the Nash equilibrium.

5 Game Trees

Here are 2 extensive form games. Answer the questions below.
1. In Game tree 1, find the subgame perfect NE (SPNE). Also find a pure strategy Nash Equilibrium that is not subgame perfect.

2. In Game tree 2, find the subgame perfect NE (SPNE). Explain why the set of strategies A, D, F, G (which would yield the efficient payoff) will NOT constitute a NE (so not only is that set of strategies not SPNE, it is not even NE).