Directions: Answer each question as completely as possible. You may work in a group consisting of up to 3 members – for each group please turn in only 1 set of answers and make sure all group member names are on that set of answers. All group members will receive the same grade.

1. Producing output $y$ requires only input $x$. The production function is

$$y = 70\sqrt{x}$$

Let $w$ denote the price of input $x$. Compute the marginal cost and the average cost of producing $y$. Verify that the average cost is less than the marginal cost for all values of $y$. Explain why this is so.

2. Consider a firm with the cost function

$$c(y, w_1, w_2) = y^2(w_1 + w_2),$$

where $w_i$ denotes the price of input $i$ for $i = 1, 2$. Let $p$ denote the output price. Derive the output supply function $y(p, w_1, w_2)$, and the input demand functions $x_i(p, w_1, w_2)$ for $i = 1, 2$.

3. Consider a firm with production function

$$y = (x_1^\rho + x_2^\rho)^\alpha,$$

where $0 < \rho < 1$, and $\alpha > 0$.

- a For what value of $\rho$ and $\alpha$ are there (i) increasing returns to scale; (ii) constant return to scales; (iii) decreasing returns to scale?

- b Suppose that there are decreasing return to scale. Find the long run cost function. Derive the output supply function and the input demand functions for this long-run cost function.

4. Consider a firm with a linear production function:

$$y = \sum_{i=1}^{n} \alpha_i x_i,$$

with input prices $w_1, ..., w_n$ for each of the $n$ inputs, with each $w_i > 0$.

- a Assume that input prices are such that there are no two inputs $i$ and $j$ where:

$$\frac{w_i}{w_j} = \frac{\alpha_i}{\alpha_j}$$

Find the combination of inputs that minimize the firm’s cost of producing a particular amount of output $y$.

- b Find the cost function and conditional input demand functions for all possible values of input prices.