Innovation and the Long-Run Elasticity of Total Taxable Income

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The elasticity of taxable income determines revenue and welfare responses to taxes. Measurement of this elasticity is an ongoing focus of tax policy research. Empirical studies report short-run elasticities. However, general equilibrium relationships can cause short-run and long-run elasticities to diverge. This paper uses a Computable General Equilibrium simulation model to construct long-run taxable income elasticities. The model differs from most previous simulation analyses of tax policy by its inclusion of endogenous, profit-motivated Research and Development and innovation. The results indicate that (i) taxable income elasticities can be relatively large, even if tax rate changes initially have only modest effects on labor supply and saving; (ii) total taxable income could be much more responsive to the corporate tax than to the individual tax; and (iii) the elasticities are substantially larger under endogenous innovation than under exogenous innovation. Together the results indicate an increase in the likelihood that the incidence of capital income taxes shifts away from capital as economies evolve toward higher levels of innovation.

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1. Introduction

The elasticity of taxable income with respect to the income tax determines revenue responses to changes in the tax rate and influences the welfare effects of the tax. As a result, measurement of this elasticity is a continuing focus of research on tax policy.1 Most empirical estimates of taxable income elasticities are based on short-run analyses. However, long-run fiscal policy effects may differ from short-run effects in sign as well as in size.2 The sign reversals arise from long-run general equilibrium relationships. The empirical literature has often cited the desirability for long-run measurement.3 However, in this regard nearly no work has been reported. This paper derives long-run general equilibrium elasticities of taxable income with respect to income taxes and discusses policy implications.
Before the advent of the concept "taxable income elasticity" economists often estimated labor supply and saving elasticities in an attempt to quantify the behavioral and efficiency effects of taxes. Many empirical estimates indicate labor supply and saving responses to after-tax wage rates and interest rates may be relatively small. However, Lindsey (1987) argues that the important behavioral responses to tax rates include more than just changes in labor supply. For example, a change in the tax rate may lead to changes in capital gains realizations and itemized deductions. Feldstein (1995) argues that the behavioral responses to taxes also include changes in portfolio investment in the short run and changes in the location of work and types of jobs in the long run. These authors suggest that changes in taxable income reflect behavioral responses to taxes more reliably than does labor supply, and they develop the concept of the elasticity of taxable income to quantify such changes.

The literature on empirical estimation of taxable income elasticities has generated many important insights into behavioral responses to income taxes. Nevertheless, factors exist that could cause empirical estimates to overstate or understate true tax responsiveness. For example, some tax avoidance schemes can cause shifts in the timing of economic decisions without affecting real economic activity (Slemrod 1996). In this case, empirical estimates of taxable income elasticities could overstate tax responsiveness. In addition, if long-run general equilibrium relationships are not captured by empirical data, empirical estimates could understate tax responsiveness. To see this, note that in general equilibrium initial tax responses in one market can be reinforced by responses elsewhere. Suppose, for example, that a change in tax rates increases labor supply and saving, initially, by small amounts. Over time, larger saving increases the capital stock and, therefore, the marginal product of labor. Higher wages, in turn, could stimulate labor supply. Transactions and adjustment costs, as well as uncertainty, could cause these inter-relationships to take a long time to develop, so it may be difficult to capture them in standard empirical analyses and in event studies in particular.

In this paper, I attempt to avoid some of the difficult issues raised by Slemrod (1996) and, at the same time, capture long-run general equilibrium relationships that seem likely to affect tax responsiveness. To these ends, I use a Computable General Equilibrium (CGE) simulation model to construct long-run taxable income elasticities.

A mainstay of previous CGE tax analysis is a model in which physical capital investment is motivated by profits, but in which innovation is exogenous. However, innovative activity must vary with monopoly profits, to some extent, because profits are required to repay the large up-front costs of Research and Development (R&D). And it seems reasonable to assume that some, if not many, entrepreneurs expect to earn economic profits from technical knowledge their firms create. Technical knowledge is at least partly non-rival, so the marginal cost of providing an innovative good to an additional user tends to be small. Therefore, the extent of the market for innovative goods can have a large effect on after-tax profits and the incentive to

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5 Feldstein states "High marginal tax rates may induce taxpayers to take more 'aggressive' interpretations of tax rules (e.g. claiming questionable deductions) or even to evade taxes by understating income or claiming unjustified deductions" (1995, p. 555). Note that these important types of tax avoidance are not included in the model used in the current paper.
7 Endogenous innovation has explanatory power (Aghion and Howitt 1998; Bernanke and Gürkaynak 2001).
innovate. This indicates that the economy could be more tax responsive if innovation is endogenous rather than exogenous. In addition, saving and innovation share a complementary relationship under endogenous innovation: An initial tax-induced increase in innovation tends to encourage more saving, which in turn could reinforce the initial increase in innovation (Lin and Russo 2002). This possibility also indicates that the economy would tend to be more tax responsive under endogenous, rather than exogenous, innovation. This paper models endogenous innovation and then compares responsiveness under the two regimes.

While empirical analysis has some weaknesses in terms of estimating taxable income elasticities, I am mindful that CGE models also have important limitations. They are highly stylized, and their solutions require strong assumptions. CGEs are calibrated to mimic actual economies by culling parameter values from the empirical literature, so they inherit some of the problems from empirical estimates. No single technique appears capable of producing complete answers to questions about the effects of tax policy. It is important to note, also, that I study only steady states. Behavior along transition paths can produce important insights, especially when relationships are highly non-linear, as they are in the model used here. The analysis of transitions under endogenous innovation is difficult and is left for future work.

With these qualifications in mind, the simulation results reported below are suggestive of the following points regarding taxable income elasticities:

(i) Long-run elasticities can be relatively large, even if initial impacts of taxes on labor supply and saving are small;
(ii) Long-run elasticities with respect to the corporate tax can exceed uncompensated elasticities with respect to the individual tax by very large margins; and
(iii) Elasticities are much larger under endogenous innovation than under exogenous innovation.

The second point indicates that reducing the corporate income tax by a percentage point could increase the efficiency of the tax system more than reducing the individual income tax by a percentage point, even if the initial tax rates are the same. If true, policy makers concerned with improving the efficiency of the tax system may obtain better results by reducing the corporate income tax rate than by adjusting the individual income tax rate. The third point indicates that elasticities tend to grow as value added in the innovative sector grows as a share of national income. Relatively large elasticities with respect to the corporate tax, together with the complementary relationship between saving and innovation, could lead to substantial shifting of tax incidence away from capital in the long run. If this occurs, capital income could become harder to tax, and progressive tax structures may become more difficult to sustain as economies evolve toward higher levels of innovation.

The simulations indicate at least two additional results worth mentioning. First, the R&D tax credit could magnify taxable income elasticities. Second, a form of tax avoidance with no apparent real short-run effects, namely, a tax-induced shift from corporate to non-corporate structure, can produce real long-run effects. In particular, a change in the structure of taxes that encourages producers to shift from corporate to non-corporate form may reduce the elasticity with respect to the corporate tax and increase the elasticity with respect to the individual tax.

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8 A shift in tax incidence away from capital also occurs under exogenous innovation (Fullerton et al. 1981) but could be more pronounced under endogenous innovation.
The next section reviews literature that is relevant to the issues I study and the techniques I use. Slemrod (1998) argues the elasticity of total taxable income should be the central concern in studies of tax elasticity. This point seems particularly relevant here because in general, equilibrium behavioral responses to individual taxes can affect corporate income, and behavioral responses to corporate taxes can affect individual income. Therefore, I focus on the elasticity of total (individual plus corporate) taxable income. Section 3 describes the model used in the simulations. Section 4 describes the results of benchmark numerical simulations of elasticities of total taxable income with respect to the individual income tax and, separately, with respect to the corporate income tax. Compensated and uncompensated elasticities are reported for the individual income tax. I also compare responsiveness under endogenous and exogenous innovation and report on an extensive sensitivity analysis. Section 5 summarizes the paper. Appendix A defines all symbols used in the text.

2. Literature

The original papers in the taxable income elasticity literature argue that income can be responsive to taxes even if saving and labor supply are relatively unresponsive (Lindsey 1987; Feldstein 1995). Although early precursors of the elasticity literature did not study taxable income per se, their work has a similar implication.

Harberger (1962) uses a general equilibrium model with fixed capital stock to show how the incidence of the corporate income tax may fall on capital, on labor, or on both. The final resting place of the tax depends on relative capital intensities and capital/labor substitution elasticities in the corporate and non-corporate sectors. This is a crucial economic possibility per se. However, the implication relevant to the current paper is that if the tax causes capital to migrate to a relatively unproductive sector, the corporate income tax could tend to reduce taxable income.

Feldstein (1974) extends Harberger's analysis to a long-run setting, with exogenous saving and labor supply. The incidence of a tax on capital income may not rest fully on capital, even if the personal saving rate is fixed. In this case, the capital income tax reduces disposable income, reducing private saving. If the government does not invest capital income tax revenue in productive capital, national saving and the aggregate capital stock decline, reducing the marginal product of labor; thus, some of the tax is shifted to labor in the long run. This implies that production and taxable income would decline, even if the saving rate does not respond to the tax.

Feldstein (1974) assumes the saving rate and labor supply are exogenous. More recent papers model endogenous saving. Summers (1981) formulates an Overlapping Generations Model (OLG) of life-cycle consumers with 50-year planning horizons. Simulations using this model indicate that an increase in a capital income tax can reduce the capital stock and national income by large amounts in the long run, even if the intertemporal elasticity of substitution is small. Altig et al. (2001) use an OLG with a 55-year planning horizon and a much more detailed tax structure than Summers to derive similar qualitative, albeit less dramatic, results. In both papers, an interest rate–induced human wealth effect contributes greatly to the large long-run

 Examples are discussed below.
decline in capital and income. The wealth effect occurs because a higher capital income tax rate reduces the after-tax interest rate, increasing the present value of life-cycle wage income. This windfall increases life-cycle consumption in all planning periods, so current saving declines reducing the capital stock and income. I model an infinite-horizon life-cycle saver, so the interest rate–induced human wealth effect contributes to tax responsiveness here also. Nonetheless, the simulations below indicate that endogenous innovation makes a large contribution to tax responsiveness, independent of the interest rate–induced wealth effect.

Ballard et al. (1985) use a CGE model to derive a relationship between labor income tax rates and tax revenue. Unless marginal tax rates are extremely high, an increase in the income tax rate generates substantial tax revenue, which implies small elasticities of taxable income. Ballard et al. assume uncompensated saving and labor supply elasticities equal to 0.4 and 0.15, respectively. In contrast, I set parameters of utility and production functions, which is somewhat less restrictive because labor supply and saving elasticities can change in response to tax rates. In this model, the impact effect of taxes on the personal saving rate is controlled by the intertemporal elasticity of substitution. To be consistent with the relatively small saving elasticities reported in the empirical literature, I set the reciprocal of the intertemporal elasticity of substitution equal to 2.0. I set the parameter that controls the wage elasticity of labor supply equal to 0.5, which implies that the wage elasticity is less than 0.1. Appendix B explains the sources of all parameter settings.

Gravelle and Kotlikoff (1989) extend Harberger’s (1962) analysis to the case in which corporate and non-corporate firms operate in the same industry. They show that the corporate income tax can reduce productive efficiency—and, by implication, taxable income—if it causes production to shift to less efficient non-corporate firms within an industry.

Lindsey’s (1987) is the first study to report estimates of elasticities of individual taxable income. He uses successive cross sections of Internal Revenue Service (IRS) data to perform an event study of the Economic Recovery and Tax Act of 1981 (ERTA). Lindsey ranks taxpayers in different income fractiles before ERTA and again after ERTA. He approximates a difference-in-differences estimator by subtracting incomes in fractiles across the two periods. The measure of the elasticity of taxable income is the ratio of the percent change in individual taxable income before and after ERTA to the percent change in the net-of-tax rate (1 minus the tax rate) before and after ERTA. Lindsey’s estimates of the elasticity of individual taxable income exceed 1.


The seminal work of Lindsey (1987) and Feldstein (1995) shows the importance of looking beyond labor supply and saving in the attempt to understand and estimate tax responsiveness. They brought the concept of taxable income elasticity to the fore and showed how it could be measured empirically. It remains true, however, that empirical estimation of elasticities of taxable income is difficult. For example, according to the life-cycle/permanent income

10 Mankiw and Weinzierl (2006) simulate revenue responses to tax cuts and find that replacing the infinite horizon with a realistic finite horizon reduces revenue responses somewhat but not by a large amount. This may occur because discount factors for wages expected more than 50 years in the future tend to be small, so discounted wages expected in the distant future do not contribute greatly to the interest rate–induced wealth effect.
hypothesis, the incomes of taxpayers experiencing temporarily low or high incomes will, respectively, increase or decrease as incomes revert to their permanent levels. This will occur apart from any influence taxes may have on income. The estimates could be affected by other non-tax factors, such as income trends that differ across income groups. It is difficult, empirically, to correct for this.

For example, Slemrod (1992, 1996, 1998), Gordon and Slemrod (2000), and Saez (2004) argue that income shifting between corporate and non-corporate business could give a false appearance that income is responsive to taxes. In 1985, the highest individual marginal tax rate exceeded the highest corporate marginal tax rate on Subchapter C corporations. TRA86 reversed the inequality. Slemrod argues the change motivated a shift in income from firms organized under Subchapter C to partnerships. This shift could cause individual income tax revenue to increase and corporate income tax revenue to decrease without having an economically significant effect on total taxable income. Knowing how the income of a particular group responds to a tax change “is not a sufficient statistic for evaluating adequately the revenue consequences” (Slemrod 1996, p. 188). Slemrod (1998) documents changes in the timing of capital gains income toward periods when tax rates are expected to be relatively low. This may produce an appearance of income responsiveness, even if the effect is only temporary. The simulation model rules out changes such as these. Simulated tax-induced changes in income represent “real” economic decisions of the agents in the model.

Empirical estimation of long-run tax responsiveness may be even more problematic. Goolsbee says “taxes have many potentially important long-run impacts .... Using tax return data to identify the magnitude of the longer-term effects is almost impossible ....” (1999, p. 9). Laibson (1999) argues that even sophisticated taxpayers may take a long time to understand the implications of tax policy changes. The notoriously high error rate in answers provided to taxpayers calling IRS hotlines indicates that it is difficult not only for taxpayers, but for tax authorities as well, to adapt to changing tax rules.11

However, even if taxpayers respond quickly to changing tax law, the economy may not. Investment incurs adjustment costs that slow capital formation. Barro and Sala-i-Martin (2004) estimate convergence speeds in the United States economy to be on the order of 2% per year. This indicates that the half-life of the economy could be on the order of 35 years. The full response to tax policy could take a long time and may be missed by data used in short-run analyses. Saez (2004) points out that comparison of years just before and after a tax policy change reveals a short-run elasticity. As well, changes in definitions of taxable income often accompany changes in tax law, making it difficult to isolate behavioral responses (Slemrod 1996). Increases in taxable income among high income groups in the 1980s could have resulted from changes in wage structure or education premia rather than behavioral responses to lower taxes, biasing elasticities upward. Increases in taxable income in the 1990s, when tax rates were increased, could have biased elasticities downward.

Gruber and Rauh (2007) estimate elasticities of corporate taxable income with respect to the effective, as well as statutory, corporate net-of-tax rate. They find evidence of small elasticities. A difficulty with this important contribution stems from diminishing marginal returns with respect to capital used in production. If a tax increase leads to a decline in capital, the marginal product of capital tends to rise, which would offset some of the initial decline in

11 Slemrod states “Perhaps not enough time elapsed after the tax changes for individuals and firms to exhibit their long-term reaction to the changed environment” (2001, p. 120).
capital income (Mulligan 2007). The larger the falloff in capital, the larger the increase in marginal product will be. Moreover, a reduction in capital reduces the marginal product of labor and wages, so part of the effect transfers to individual taxable income and would be missed by focusing on corporate income. Total—corporate plus individual—taxable income provides a more reliable indicator of the long-run tax effects I wish to understand, so I report elasticities of total taxable income.

The CGE model has the potential to avoid some of the difficulties mentioned above and to provide insights not captured by empirical analysis. But simulation results must be qualified because the computer model is also restrictive. I believe the safe approach is to consider a portfolio of techniques, including both empirical specifications and computer models.

3. The Model

Overview

Until relatively recently, CGE models used for fiscal policy analysis assumed that physical capital is a profit-driven choice variable, but that innovation is exogenous. With the advent of solvable endogenous innovation models (Romer 1990) there appears to be no methodological justification for disparate treatment of the decision to save and the decision to innovate. The model used here assumes monopoly profit motivates private investment in R&D, so innovation is endogenous.12

In this model, the production side of the economy consists of an intermediate goods sector and a final output sector. Intermediate goods producers purchase foregone consumption from the final output producer to use in the manufacture of innovative capital goods. Intermediate goods producers then rent these innovative capital goods to the final output producer. Innovations (technical knowledge) are created by R&D conducted in the intermediate goods sector. The majority of R&D investment consists of up-front costs, namely, salaries of scientists and engineers. Thus, the model assumes that the cost of innovation consists of wages paid to skilled labor. Monopoly profits are necessary to repay these costs, so the intermediate goods sector is monopolistically competitive. Intermediate goods producers secure patents to protect the right to profit from innovation. On the economy's long-run balanced growth path, the stream of discounted monopoly profit is just sufficient to cover the cost of creating a patentable innovation. Thus, the market value of a patent also is the cost of creating technical knowledge. The final output producer rents innovative capital goods from intermediate goods producers, hires labor, and sells final output to consumers and intermediate goods producers. The markets for final output and labor are perfectly competitive.

It is important to emphasize a distinction between endogenous innovation and endogenous growth. Under endogenous growth, the long-run growth rate of the economy is determined within the model and is, therefore, affected by policy variables. This is true in Romer (1990) because his model assumed Constant Returns to Scale (CRS) in technical knowledge used to

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12 Endogenous innovation has no bearing on many questions of tax policy and economic growth. The principle of parsimony recommends assuming exogenous innovation whenever doing so is simpler and appropriate. We argue that tax responsiveness is a primary example where exogenous innovation is restrictive. Simulation results reported in subsection 4.5 support this viewpoint.
produce new technology. CRS in production of technical knowledge generates a scale effect—that is, the long-run growth rate of output per person increases with the scale of the economy. Scale effects appear to be at odds with empirical evidence. The model used here follows Jones (1995a) and Jones and Williams (2000) and avoids scale effects by assuming diminishing marginal returns to technical knowledge used to produce technology. This is explained in section 3.4. Although innovation is endogenous in this model, the long-run growth rate is exogenous. In this case, policy variables affect the balanced growth path levels of production and income, so they can affect tax responsiveness. However, policy variables cannot affect the long-run growth rate.

The model used here essentially is Romer's (1990) model of endogenous innovation, amended—per Jones and Williams (2000)—to eliminate scale effects. I add income taxes, capital depreciation, and endogenous labor supply. Most privately financed R&D in the United States is conducted by large manufacturing corporations. I assume that all firms in the intermediate goods manufacturing sector pay the corporate income tax. About 20% of value added in the United States is produced by small firms (see Appendix B). Therefore, I assume that 20% of firms in the final output sector pay the individual income tax. The remaining 80% pay the corporate income tax. Households pay the individual income tax. The computer model assumes all income taxes are proportional.

The Household\(^{14}\)

This subsection models endogenous labor supply and optimal consumption growth and introduces taxes. The household maximizes lifetime utility defined over consumption and leisure:\(^{15}\)

\[
U = \int_0^\infty e^{-(\rho - g_L)t} \left\{ c(t) \left[ 1 - L(t) \right] \right\}^{1-\sigma} \left[ \frac{1}{1 - \sigma} \right] dt; \ 0 < \sigma, \ 0 \leq \eta,
\]

where \(\rho\) is the rate of time preference; \(c(t)\) is private expenditure at time \(t\), measured per person; \(\sigma\) is the reciprocal of the intertemporal elasticity of substitution; and a fraction \(k(t)\) of the unit time endowment is supplied as skilled labor at time \(t\). \(\eta\) determines the wage elasticity of labor supply: The larger the \(\eta\), the more responsive is labor supply to the after-tax wage rate. \(g_L\) is the exogenous population growth rate, and \(L\) is the size of the population and the number of workers. Assuming \(L(0) = 1\), the workforce is \(L(t) = e^{\eta t}\) at time \(t\). To simplify presentation, I drop the \(t\) index hereafter.\(^{16}\)

The household budget constraint, Equation 2 below, is complicated by the fact that it includes debt and equity rather than a general asset subsuming both. This complication is necessary because debt and equity are treated differently for tax purposes: Firms can deduct the cost of debt finance but not the cost of equity finance. The difference is an important feature of the tax system and is modeled in subsections 3.3 and 3.4. Thus, the household flow budget

\(^{13}\) See Backus, Kehoe, and Kehoe (1992) and Jones (1995b).

\(^{14}\) Each symbol used in the model is defined in Appendix A.

\(^{15}\) This utility function is used in Lucas (1990) and in Jones, Manuelli, and Rossi (1993).

\(^{16}\) Starrett (1988) points out features of the utility function in Equation 1 that contribute to tax responsiveness. In particular, the interest rate-induced human wealth effect increases with the length of the planning horizon, so the infinite horizon could cause tax effects to be overstated. The results in Alog et al. (2001) and Mankiw and Weinzierl (2006) indicate that finite horizons do not necessarily reduce tax effects by very large amounts.
constraint in per-person units is
\[
\sum_j b_j + \sum_j p_\eta e_j = (1 - \tau^p) \left[ bw + r \sum_j b_j + \sum_j \left( d_j + \frac{\dot{p}_\eta}{p_\eta} \right) p_\eta e_j \right]
+ LST - c - \left( \sum_j b_j + \sum_j p_\eta e_j \right) g_L,
\]
(2)
where \( b_j \) represents bond holdings per person, \( e_j \) represents equity share holdings per person, \( d_j \) is dividend yield on shares, and the \( j \) subscript indexes the production sector (final output and intermediate goods). A dot above a variable indicates the time derivative, \( \tau^p \) is the flat rate individual income tax rate, \( w \) is the real wage per worker, and \( r \) is the before-tax short-term interest rate. \( LST \) represents a per capita lump-sum transfer of funds. When compensated elasticities are constructed below, we assume all tax revenue is returned to the household, and so \( LST \) is set equal to per capita government tax revenue. The transfer eliminates income effects. When uncompensated elasticities are constructed, we assume \( LST \) is 0, so tax rate changes have income effects. This is explained further below.

Pontryagin’s Maximum Principle gives the first-order conditions for consumption and leisure (time away from work), which are, respectively,
\[
(c(1 - l)^\eta - \sigma (1 - l)^\eta = \Psi, \tag{3a}
\]
\[
(c(1 - l)^\eta - \sigma c \eta (1 - l)^{\eta - 1} = \Psi (1 - \tau^p) w. \tag{3b}
\]
Equation 3a shows that the life-cycle consumer maximizes utility by choosing consumption (and saving), such that the time \( t \) marginal utility of an increment to consumption is equal to the time \( t \) marginal cost. An incremental increase in consumption, \textit{ceteris paribus}, requires a reduction in the stock of assets. Therefore, the marginal cost, \( \psi \), is the shadow price of an incremental change in assets at \( t \). Equation 3b shows that the consumer chooses leisure (and work effort), such that the time \( t \) marginal utility of leisure is equal to the time \( t \) after-tax wage, valued at \( \psi \).

There are two assets, bonds and equity, so there are two co-state equations. Respectively, these are
\[
\frac{\dot{\psi}}{\psi} = - \left[ (1 - \tau^p) r - \rho \right], \tag{3c}
\]
\[
\frac{\dot{\psi}}{\psi} = - \left[ (1 - \tau^p) \left( d_j + \frac{\dot{p}_\eta}{p_\eta} \right) - \rho \right]. \tag{3d}
\]
Equations 3c and 3d show that the consumer chooses bonds and equity such that the rate of change in the shadow value of assets equals the after-tax \textit{net} rates of return on the two assets. The returns are net of the rate of time preference.

\textsuperscript{17} The later in life an asset is received, the less time it has to contribute to consumption, so \( \psi \) declines over the planning horizon. If asset purchases were such that the rate of decline in \( \psi \) exceeded asset rates of return, utility could be increased by reducing asset purchases. This would reduce the rate of decline in \( \psi \), so the consumer’s choice problem has a solution.
Because incomes from bonds and equity are taxed at the same rate and there is no uncertainty in the model, the before-tax rates of return on bonds and equity must be equal. Thus, we can use Equations 3a and 3c to derive the growth rate of consumption:

\[ g_C = \frac{1}{\sigma} \left[ (1 - \tau^p)r - \rho + \eta(\sigma - 1)g_I \right], \]  

where \( g_I \) is the growth rate of the fraction of a worker’s unit time endowment supplied as skilled labor.\(^{18}\)

The Final Output Sector

This section derives first-order conditions for labor and the innovative capital good used in production of final output. The production function in the final output sector is

\[ y = \left[(1 - \zeta)\int_0^A x(i)^{(1 - \omega)} di\right]^{1/\omega} 0 < \alpha, \zeta < 1, \]  

where \( y \) is the flow of final output per person, \( (1 - \zeta) \) is the fraction of skilled labor employed in this sector, \( \alpha \) is the elasticity of final output with respect to labor, \( x(i) \) is the quantity of the \( i \)th innovative capital good per person, and \( A \) is an index of the level of non-rival technical knowledge. Equation 5 imposes the restriction that output is additively separable in the \( x(i) \).\(^{19}\)

Final output can be used either for consumption or as foregone consumption employed in the manufacture of capital goods. I assume that one unit of foregone consumption can be transformed into one capital good. Following Romer (1990), each intermediate goods producer creates a single innovation and a single capital good, \( x(i) \). Because each firm has title to a single innovation, the total number of intermediate goods producers equals the level of technology, \( A \). Summing over \( x(i) \) gives the aggregate capital stock in this model, \( k = \int_0^A x(i)di \).\(^{20}\)

The final output producer is a price taker and maximizes after-tax profit per person, \( \pi_y \), given by

\[ \pi_y = (1 - \tau^C) \left[ y - (1 - \zeta)hw - f \int_0^A p(i)x(i)di \right] - (1 - f) \int_0^A p(i)x(i)di, \]  

where \( \tau^C \) is the proportional corporate income tax rate, \( (1 - \zeta)hw \) is the tax-deductible cost of labor in this sector, \( p(i) \) is the rental price of the innovative capital good, \( x(i) \), and \( p(i)x(i) \) is the cost per unit of the \( i \)th innovative capital good. Integrating this cost over all \( i \) gives the total cost

\(^{18}\) Note that in growth models with endogenous saving, equilibrium in the capital market is determined by household preferences and the economy’s production function. The supply of saving is determined by Equation 2. This is clearer in Appendix C, where Equation 2 is transformed into growth in the capital stock. Parameters of the production function enter through the wage rate and interest rate. The demand for saving is determined in Equation 4. Because there is no uncertainty in the model, the market for financial claims on income from capital equilibrates whenever the market for capital equilibrates.

\(^{19}\) Equation 5 is a version of Romer’s (1990) Equation 1’, but it is written in per-person terms. Romer used separate symbols for labor and for skill level.

\(^{20}\) On the balanced growth path, each \( x(i) \) is produced in the same constant quantity. Thus, \( k = Ax \), so capital per person grows at the same rate as technology as in the Solow growth model.
of innovative capital goods used to produce final output. \( f \) is the fraction of capital financed by debt, so \( \int_0^A p(i) x(i) di \) is the tax-deductible part of capital costs in this sector.

The next two equations are the final output producer's first-order conditions for labor and capital, respectively. Writing marginal benefits on the left-hand side and marginal costs on the right-hand side,

\[
\alpha [(1 - \zeta) f]^{\gamma-1} \int_0^A x(i)^{1-\gamma} di = w, \quad (7a)
\]

\[
(1 - \tau^c)(1 - \alpha) [(1 - \zeta) f]^{\gamma} x(i)^{-\gamma} = p(i)(1 - f \tau^c). \quad (7b)
\]

Because labor costs are fully deductible, \( \tau^c \) affects the marginal benefits and costs of labor equally, and it does not affect the firm's labor choice; thus, \( \tau^c \) does not appear in Equation 7a. In contrast, because the fraction of capital financed by debt, \( f \), is tax deductible, \( \tau^c \) reduces the marginal after-tax cost of capital by less than it reduces the marginal after-tax benefit. Thus, \( \tau^c \) reduces demand for the innovative capital good and the size of the market for innovative goods, and this, in turn, reduces the incentive to innovate. Because technical knowledge is non-rival, changes in the size of the market have large effects on the level of innovative activity in the long run. Thus, \( \tau^c \) can have a large effect on tax responsiveness in this model.

The Intermediate Goods Sector

R&D incurs large up-front costs before technical innovations generate any cash flow, so the intermediate goods sector is monopolistically competitive. Intermediate goods producers hire skilled labor to produce technical innovations. They embed their innovations in foregone consumption to produce innovative capital goods and charge a mark-up rental price. Because there are many different innovative capital goods, behavior cannot be summarized in terms of the decisions of a single representative firm. To simplify, assume each intermediate goods producer creates a single innovation, which it incorporates into a single capital good. Therefore, the number of intermediate goods, the number of intermediate goods producers, and the level of technical knowledge all equal \( A \).

Under U.S. tax law R&D is expensible. Therefore, as seen below, the corporate tax rate drops out of the first-order condition governing R&D. This is not true of equity-financed investment in capital. But this important difference would be obscured if I were to write a single profit function covering both the decision to conduct R&D and the decision about the quantity of intermediate goods to produce. Therefore, I assume the intermediate goods producer has two separate divisions: a research division that conducts R&D and a manufacturing division that produces the innovative capital good.\(^{21}\) I study the firm's profit maximizing decision in two stages. In the first stage, the research division chooses how much to invest in R&D. This is equivalent to deciding how much skilled labor to hire. The research division transfers the

\(^{21}\) Romer imposes a stricter separation. He assumes innovations and capital goods are produced by separate firms: "it is easier to describe the equilibrium if the research and development department is treated as a separate firm and designs (innovations) are transferred for an explicit price" (1990, p. S82).
Innovation and the Elasticity of Income

Innovation it creates to the manufacturing division for the price, $P^*$. In the second stage, the manufacturing division chooses the rental price of its innovative capital good and how much to produce.

In firm $i$'s research division, the production function for its (single) innovation is

$$
\dot{A}_i = \delta(\zeta L)A^\lambda; \quad 0 \leq \lambda < 1,
$$

(8)

where $\delta$ determines the productivity of labor in research, $\zeta$ is the fraction of skilled labor employed in the intermediate goods sector, $(\zeta L)$ is the quantity of skilled labor employed by the $i$th firm, and $\lambda$ governs technical knowledge’s contribution to new technology. If $\lambda > 0$, technical knowledge is at least partly non-rival and provides externalities that contribute to all intermediate goods firms’ abilities to innovate.

Two restrictions in Equation 8 require explanation. First, in contrast to Romer’s (1990) specification of the production function for technology, Equation 8 precludes scale effects. Jones and Williams (2000) argue that the absence of scale effects in the data can be explained by “fishing out” and congestion in research. Fishing out occurs if researchers discover the most productive ideas first. In this case, there are diminishing returns to technical knowledge in production of technical knowledge, so $\lambda$ must be less than 1. Second, congestion in research arises when firms in similar industries duplicate one another’s research effort in the competition to create innovations. Following Jones (1995a), congestion is accounted for in the definition $\delta = \delta(\zeta L)^{(1-\theta)}$ and in the restriction $\theta \leq 1$. If congestion does not occur, $\theta$ is equal to 1, and productivity in technology production is given by $\delta$, as in Romer (1990). Ceteris paribus, the larger the quantity of labor employed in the intermediate goods sector, the larger is congestion, the smaller is $\theta$, and the lower is the productivity of the individual intermediate goods producer, $\delta$. The individual firm does not internalize the congestion externality and treats $\delta$ as a parameter, so it is treated as a constant when deriving the first-order condition.

Following Romer (1990), I assume each innovator has the same technology production function and sum across firms (and drop the $i$ index hereafter) to obtain aggregate technology production,

$$
\dot{A} = \delta(\zeta L)A^\lambda.
$$

(9)

The research division hires labor to conduct R&D in order to maximize profit. U.S. firms can expense R&D, so after-tax profit in the research division (per person) is

$$
\pi_A = \left(1 - \tau^C\right)\left[P_A \delta(\zeta L)A^\lambda - w\zeta L\right],
$$

(10)

where $P_A$ is the value of a technical innovation. In Romer’s (1990) terminology, $P_A$ is the value of a patent on a new design. The corresponding first-order condition for skilled labor, $\zeta L$, is

$$
P_A \delta A^\lambda = w.
$$

(11)

This shows, first, that all of the research division’s revenue accrues to skilled workers. Second, $\tau^C$ does not appear in Equation 11: Because all R&D costs are expensed, $\tau^C$ equally affects the marginal benefit and cost of skilled labor used to produce technology and therefore drops out.
In the next stage of production, each manufacturing division chooses the quantity of the innovative capital good, $x$, to produce and its rental price, $p$, in order to maximize after-tax profit. $P_A$ is the cost of technology, so manufacturing’s after-tax profit (per person) is

$$\pi_x = (1 - \tau^C) \left[ px - (hr + \Delta)x - rh \frac{P_A}{L} \right] - (1 - h)r \left[ x + \frac{P_A}{L} \right],$$

(12)

where $px$ is taxable revenue, $r$ is the cost of capital funds, $\Delta$ is the rate of economic depreciation, and $h$ is the tax-deductible fraction of capital costs financed by debt. Because final consumption is transformed into capital, one for one, $(hr + \Delta)x$ is the tax-deductible cost of foregone consumption used to produce an innovative capital good. $P_A$ is the cost the manufacturing division pays for the innovation created by the research division, so $rhP_A$ is the tax-deductible cost of the innovation. $P_A$ is divided by $L$ in Equation 12 because the equation is written in per-person terms.

Use the inverse demand for innovative capital goods in Equation 7b to replace $p$ in Equation 12, take the derivative with respect to $x$, and use $p$ again. This gives the rental price the manufacturer charges the final output producer for the innovative intermediate good:

$$p = \frac{(1 - hr^C)r + (1 - \tau^C)\Delta}{(1 - \alpha)(1 - \tau^C)}.$$

(13)

Equation 13 shows that the rental price of capital increases with increases in $r, \Delta, 1/(1 - \alpha)$ (the before-tax mark-up over marginal cost), and $\tau^C$. Thus, part of any increase in $\tau^C$ is passed on to the final output producer via an increase in $p$. This provides a second avenue by which $\tau^C$ reduces the demand for innovative goods and the incentive to innovate.

The no-arbitrage condition for the value of an innovation is necessary to close the model (Romer 1990; Grossman and Helpman 1992). The no-arbitrage condition implies that the cost of innovation must equal the discounted stream of profits earned from innovation. If the cost of innovation, $P_A/L$, were to exceed the discounted stream of profits, there would be no incentive to innovate. If the discounted stream of profits were to exceed $P_A/L$, free entry into the intermediate goods sector would drive profits down. Thus, the no-arbitrage condition requires

$$\frac{P_A}{L} = \int_{t}^{\infty} e^{-r(s-t)} \left[ \left( 1 - \tau^C \right) \left( px - (hr + \Delta)x \right) - (1 - h)rx \right] ds.$$

(14)

Equation 14 can be simplified because $r, x, p$ must be constant on the balanced growth path. To see this, note that the left side of Equation 4 is the growth rate of consumption per person, which is constant and equal to the growth rate of technical knowledge in the long run. $\ell$, the fraction of time devoted to work, must be constant in the long run because it is bounded by 0 and 1. Thus, $g_{\ell}$ is 0, and so $r$ is constant on the balanced growth path. Equation 13 then shows that $p$ is constant, and so Equation 7b shows that $x$ is constant. Given constant values of $r, x, p$ on the balanced growth path, $P_A/L$ simplifies to

$$\frac{P_A}{L} = x \frac{(1 - \tau^C)p - (1 - hr^C)r - (1 - \tau^C)\Delta}{(1 - hr^C)r}.$$

(15)
Equation 15 says the value the market places on innovation, relative to the scale of the market (measured here by $L$), must equal the discounted stream of after-tax profit from producing innovative capital goods. This condition is necessary to find the steady-state solution of the model.

The Government Flow Budget Constraint

On the balanced growth path the government's flow budget constraint is balanced, and so government outlays per person are equal to tax revenue per person, $R$:

$$R = \tau_p \left[ hw + r \sum_j b_j + \sum_j \left( d_j + \frac{\dot{p}_e}{p_e} \right) p_e e_j \right] + \tau_c \left[ y - (1 - \zeta)hw - fAp_x \right] + A \tau_c \left[ px - (hr + \Delta)x - hr \frac{P_A}{L} \right].$$

The terms in square brackets on the right-hand side of Equation 16 are taken from Equations 2, 6, and 12. Note that when compensated elasticities are calculated below, all government outlays are returned to the household as a lump sum: That is, $R = LST$. However, when uncompensated elasticities are calculated, all government outlays are spent on government consumption: That is, $R = G$, where $G$ is government consumption per person.

4. Numerical Simulations of Total Taxable Income Elasticities

Overview

This section reports on the results of numerical simulations. Because the goal here is to study the effects of sustainable, long-run fiscal policy, the simulations assume the government's budget is balanced. In the previous section, variables are in units per person. With two exceptions, the variables must be transformed into effective labor units in order to solve the model because only variables in effective labor units are constant on balanced growth paths. The two exceptions are the prices $p$ and $P_A$. Earlier, $p$ was shown to be constant on the balanced path. $P_A$ divided by $L$ is constant on the balanced path because, under endogenous innovation, the value of technical knowledge increases with the scale of the economy, which is measured here by $L$. Appendix C provides a detailed description of the adjustments required to transform the model into effective labor units.

The 'FindRoot' instruction in Mathematica's symbolic calculator is used to solve the nonlinear nine-equation system for the nine endogenous variables $\delta, \dot{k}, \dot{l}, \dot{p}, \dot{P}_A, r, \omega, \dot{R},$ and $\zeta$. A tilde above a variable indicates it is measured in effective labor units, except for $\dot{P}_A$, which is defined as $\dot{P}_A = P_A/L$. After a solution is derived, the variables must be transformed back into units per person in order to calculate the elasticities. This is a non-trivial problem because the level of technical knowledge, $A$, is unknown. Appendix D provides a detailed description of the transformation of the solution values into units per person.
<table>
<thead>
<tr>
<th>Panel A: Individual income tax rate (%)</th>
<th>Panel B: Corporate income tax rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>k</strong></td>
<td><strong>r</strong></td>
</tr>
<tr>
<td>10.0</td>
<td>2.18</td>
</tr>
<tr>
<td>20.0</td>
<td>1.95</td>
</tr>
<tr>
<td>30.0</td>
<td>1.68</td>
</tr>
<tr>
<td>40.0</td>
<td>1.41</td>
</tr>
<tr>
<td>50.0</td>
<td>1.19</td>
</tr>
<tr>
<td>60.0</td>
<td>0.92</td>
</tr>
</tbody>
</table>

* k is capital per person, r is the interest rate on bonds, l is the fraction of time workers allocate to work, ζ is the fraction of the labor force employed in research, TI is taxable income per person, and R is tax revenue per person. The table shows the steady-state values of these variables when parameters are set to the benchmark values listed in Appendix A.1 and explained in Appendix B. In Panel A, the corporate income tax rate is fixed at the benchmark value 0.35, and the individual income tax rate takes the values shown in the first column. In Panel B, the individual income tax rate is fixed at the benchmark value 0.21, and the corporate income tax rate takes the values shown in the first column.

**Simulated Benchmark Values of Endogenous Variables along the Balanced Growth Path**

This subsection reports benchmark values of endogenous variables along the economy’s balanced growth path. The benchmark is defined here to be the set of simulations calculated on the basis of initial parameter settings. The initial parameter settings are σ = 2.0, p = 0.034, η = 0.5, α = 0.7, Δ = 0.06, gₐ = 0.018, gₑ = 0.0144, f = 0.33, h = 0.2, τᶜ = 0.35, and τᵣ = 0.21. Also, intermediate goods firms pay the corporate income tax, while 80% of the value added in the final goods sector is taxed by the corporate income tax. The remaining 20% is taxed by the individual income tax. These parameter settings are based on the empirical literature and on computer analyses previously reported in the literature. For example, α, the elasticity of final output with respect to labor, is set equal to 0.7, which is a commonly used value for this parameter. Appendix B explains the sources of the benchmark parameter settings. Appendix A.1 lists the numerical values.

To provide some quantitative perspective on the workings of the model, Table 1 reports benchmark balanced growth path values of capital per person, k, the before-tax interest rate, r, the fraction of workers’ unit time endowment allocated to work, l, the fraction of the labor force employed in research, ζ, taxable income per person, TI, and tax revenue per person, R. In Panel A of the table, the corporate income tax rate is fixed at the benchmark value 0.35, and the individual income tax rate takes the values shown in the first column of the table. In Panel B, the individual income tax rate is fixed at the benchmark value 0.21, and the corporate income tax rate takes the values shown in the first column.

Table 1 indicates that increasing the individual (corporate) income tax rate, holding the corporate (individual) tax rate constant, results in a large decline in k. The corporate income tax has the larger effect. An increase in the individual income tax increases r, but the corporate...
income tax does not affect $r$. This occurs because the balanced growth solution for $r$ must satisfy household preferences for consumption growth. The solution is found by setting both growth rates ($g_A$ and $g_I$) in Equation 4 equal to 0. Intuitively, the individual income tax increases the return investors require on saving (the cost of capital funds), so $r$ increases. However, corporations treat the required return as given and adjust the stock of capital and the marginal product of capital to deliver that return. Although the corporate income tax is correlated with $l$, neither tax has a large effect on its value in balanced growth.

The most significant result in Table 1 concerns $\zeta$, the fraction of skilled labor employed in research. $\zeta$ depends on business incentives to invest in R&D and determines the rate of innovation. Both taxes reduce $\zeta$ by large percentages. However, the corporate tax has the larger effect. The individual tax reduces saving and increases the interest rate. This increases the price of capital goods (Equation 13), which reduces the market demand for capital goods, the incentive to innovate, and $\zeta$. The corporate tax also increases the price of capital goods. In addition, however, the corporate tax reduces demand for capital goods directly through the final goods producers' first-order condition (Equation 7b). Thus, the corporate tax has the larger effect on the extent of the market for capital goods and a larger effect on $\zeta$. This indicates the corporate tax effect on taxable income elasticities could be larger than the individual tax effect. Consistent with this, Table 1 shows that $T_I$ responds much more to the corporate income tax than to the individual tax. In these simulations, $R$ rises with the individual income tax, to a peak at a value of 44%. However, the peak of the corporate income tax is much lower, at 6.5%.

Simulated Benchmark Elasticities of Total Taxable Income under Endogenous Innovation

Next, I simulate benchmark values of elasticities of total taxable income with respect to the individual and corporate net-of-tax rates. In each experiment, a marginal income tax rate is set, the model is solved, the marginal tax rate is increased, in this case by one basis point, and the model is solved again. Percentage changes in total taxable income and net-of-tax rates are calculated from the solutions. The elasticity of taxable income is the ratio of the former to the latter.

The simulations derive the long-run solution, so I assume the government's budget is balanced. Compensated tax policy responses do not have income effects. Therefore, when compensated elasticities with respect to the individual net-of-tax rate are calculated, budget balance is achieved by adjusting lump-sum transfers to offset changes in the consumer's gross tax liability. When uncompensated elasticities are computed, budget balance is achieved by adjusting government consumption to match tax revenue. In this case, changes in individual income tax rates have income effects as well as substitution effects.

The empirical literature usually focuses on the elasticity of individual taxable income with respect to the individual net-of-tax rate. Computer simulation permits a more general viewpoint. Corporate income can be affected by behavioral responses to individual taxes, and individual income can be affected by behavioral responses to corporate taxes, so the important issue is the way the level of overall (individual plus corporate) taxable income responds to tax rates (Slemrod 1998). I report elasticities of total taxable income.

\[ \frac{\partial p}{\partial c} = \frac{r(1-h)}{(1-\alpha)(1-\tau c)^2} > 0. \]
Table 2. Benchmark Elasticity of Total Taxable Income with Respect to Individual Net-of-Tax Rate

<table>
<thead>
<tr>
<th>Tax Rate (%)</th>
<th>Uncompensated Elasticity</th>
<th>Compensated Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>1.05</td>
<td>1.51</td>
</tr>
<tr>
<td>20.0</td>
<td>1.06</td>
<td>1.55</td>
</tr>
<tr>
<td>30.0</td>
<td>1.07</td>
<td>1.59</td>
</tr>
<tr>
<td>40.0</td>
<td>1.08</td>
<td>1.64</td>
</tr>
<tr>
<td>50.0</td>
<td>1.09</td>
<td>1.69</td>
</tr>
<tr>
<td>60.0</td>
<td>1.09</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Table 2 shows elasticities of total taxable income resulting from a 1% increase in the individual tax rate from the value shown in column 1. These are calculated based on parameter settings listed in Appendix A.1 and explained in Appendix B. In Table 2, the corporate income tax rate is held constant at 0.35. The compensated elasticities are calculated by holding household income constant: That is, any change in tax liability resulting from a change in the tax rate is offset by a lump-sum transfer.

Table 2 reports benchmark elasticities of total taxable income with respect to the individual net-of-tax rate at marginal tax rates ranging from 10% to 60%. For example, the elasticities reported at the 10% rate are computed by initially setting the rate to 10% and increasing it to 10.01%. The corresponding uncompensated elasticity is 1.05. When the tax rate is 60%, the uncompensated elasticity is 1.09. Compensated elasticities are of interest because they reflect the efficiency effects of taxes. Table 2 reports a compensated elasticity of 1.51 at the 10% tax rate, ranging to 1.75 at a tax rate of 60%. The uncompensated elasticities are much smaller than compensated elasticities (the average difference is −0.565), so income effects are relatively large.

Benchmark elasticities with respect to the corporate income net-of-tax rate are reported in Table 3. The elasticities in Table 3 range from 1.41 to 1.55, much larger than the uncompensated elasticities with respect to the individual tax. The larger elasticities with respect to the corporate tax indicate that this tax is likely to be shifted substantially in the long run.

Comparison with Empirical Estimates

The taxable income elasticities reported above are larger than many, though not all, of the elasticities reported in the empirical literature. Lindsey (1987) and Feldstein (1995) report elasticities of individual taxable income with respect to the individual net-of-tax rate that exceed 1. Table 2 reports uncompensated elasticities of total taxable income with respect to the individual net-of-tax rate of between 1.05 and 1.09. However, Gruber and Saez (2002) suggest that their preferred specification produces an estimate of the individual taxable income elasticity of 0.40. Using the log of initial income to control for mean reversion, Kopczuk (2005) estimates an elasticity of individual taxable income of 1.44. However, using Gruber and Saez's

\[ \gamma^C \]

Whenever \( \gamma^P \) is varied, as in Table 2, \( \gamma^C \) is fixed at its benchmark value, 0.35.

\[ \gamma^P \]

A referee points out that the uncompensated elasticities vary much less than the compensated elasticities. Why would this be? As one should expect, the compensated elasticities grow larger with the tax rate. This contrasts strongly with the behavior of the uncompensated elasticities. The differences between the two elasticities result from general equilibrium income effects, so the income effects must grow larger at nearly the same rate as the substitution effects.

\[ \gamma^C \]

Whenever \( \gamma^P \) is varied, as in Table 3, \( \gamma^C \) is fixed at its benchmark value, 0.21.
Table 3. Benchmark Elasticity of Total Taxable Income with Respect to Corporate Net-of-Tax Rate

<table>
<thead>
<tr>
<th>Tax Rate (%)</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>1.41</td>
</tr>
<tr>
<td>20.0</td>
<td>1.44</td>
</tr>
<tr>
<td>30.0</td>
<td>1.47</td>
</tr>
<tr>
<td>40.0</td>
<td>1.50</td>
</tr>
<tr>
<td>50.0</td>
<td>1.53</td>
</tr>
<tr>
<td>60.0</td>
<td>1.55</td>
</tr>
</tbody>
</table>

Table 3 shows elasticities of total taxable income resulting from a 1% increase in the corporate tax rate from the value shown in column 1. These are calculated based on parameter settings listed in Appendix A.1 and explained in Appendix B. In Table 3, the individual income tax rate is held constant at 0.21.

There are at least five reasons the simulated elasticities differ from their empirically estimated counterparts. First, the empirical estimates of corporate taxable income omit possible effects on individual taxable income. Estimation focused on corporate taxable income may understate tax responsiveness because a tax-induced decline in capital increases the marginal product of capital, so capital income may not change much. However, wage income is likely to decline with the capital stock. The computer model accounts for change in both types of taxable income. Second, empirical estimates of both individual taxable income and corporate taxable income probably reflect only part of the long-run general equilibrium relationships that the computer model is designed to capture, causing them to understate tax responsiveness. In particular, the processes by which changes in incentives result in changes in the rate of technological innovations, and the embodiment of new ideas in commodities counted in real GDP probably are long and slow. Third, actual data are affected by many types of tax avoidance strategies that are not modeled here, such as taxpayer shifting of income receipts to periods when the tax rate is relatively low. The effects of this are ambiguous. Some types of tax avoidance (for example, shifting income overseas) would increase real tax responsiveness. Other types of tax avoidance (for example, changing the timing of income receipts in anticipation of change in the tax rate) may not affect net tax responsiveness (Slenrod 1996). Fourth, the true endogenous innovation data-generating mechanism is unknown. The model could overstate the effects of endogenous innovation. Fifth, and more generally, strong assumptions are required to solve CGE models. I assume an infinite planning horizon, which tends to overstate the interest rate-induced wealth effect, and tax responsiveness. The inferences I draw in the conclusion attempt to take these qualifications into account.

Elasticities of Total Taxable Income when Innovation Is Exogenous

The elasticities reported in Tables 2 and 3 are based on Romer's (1990) model of endogenous innovation, modified to include diminishing marginal returns in technology production (Jones and Williams 2000). Many previous CGE studies of tax policy assume that
Table 4. Elasticities of Total Taxable Income when Innovation Is Exogenous

<table>
<thead>
<tr>
<th>Tax Rate (%)</th>
<th>Individual</th>
<th>Corporate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncompensated Elasticity</td>
<td>Compensated Elasticity</td>
</tr>
<tr>
<td>10.0</td>
<td>0.25</td>
<td>0.67</td>
</tr>
<tr>
<td>20.0</td>
<td>0.25</td>
<td>0.71</td>
</tr>
<tr>
<td>30.0</td>
<td>0.26</td>
<td>0.76</td>
</tr>
<tr>
<td>40.0</td>
<td>0.27</td>
<td>0.81</td>
</tr>
<tr>
<td>50.0</td>
<td>0.27</td>
<td>0.87</td>
</tr>
<tr>
<td>60.0</td>
<td>0.26</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 4 shows elasticities of total taxable income resulting from a 1% increase in the tax rate from the value shown in column 1. The compensated individual elasticities are calculated by holding household income constant: That is, any change in tax liability resulting from a change in the tax rate is offset by a lump-sum transfer.

innovation is exogenous. To compare the results under the different innovation regimes, all the numerical simulations were rerun, this time imposing the restriction that innovation is exogenous.

The results are shown in Table 4. The elasticities in Table 4 are much smaller than those shown in Tables 2 and 3. Both regimes include the interest rate–induced human wealth effect, so the differences do not result from the wealth effect. Rather, the differences result partly from the non-rival nature of technical knowledge and partly from the complementary relationship between saving and innovation. First, because technical knowledge is non-rival, once created, innovations can be provided to additional users at near-zero marginal cost. In this case, the extent of the market for innovative goods has a large effect on profits from innovation. And the income tax affects profits. Second, endogenous innovation and saving are complements (Lin and Russo 2002). A tax rate–induced decline in saving increases the interest rate and the price of innovative goods, discouraging R&D investment. Less innovation, in turn, decreases the return to capital, discouraging saving. 26

The fact that the elasticities are substantially larger under endogenous innovation indicates that tax sensitivity could increase as innovative value-added grows as a share of GDP. Thus, the excess burden of the income tax could increase as economies develop, and the revenue capacity of the capital income tax could evolve with the economy.

Elasticities of Total Taxable Income when the Non-corporate Sector Is Larger

Carroll et al. (2000, 2001) and Rosen (2005) argue that small business investments are highly sensitive to the individual income tax, suggesting that tax sensitivity to the individual tax increases with the proportion of valued-added produced by small business. The benchmark simulations assume 20% of economy-wide value-added is produced by non-corporate business and taxed by the individual income tax. Table 5 shows the results when the proportion of value-added created by non-corporate firms is increased to 40%. Note that the elasticities with respect to the individual income tax become larger, and the elasticities with respect to the

26 It is interesting to note that in Fullerton’s (1982) model with exogenous innovation, labor income tax revenue peaks at about 79%. In the current model with exogenous innovation, tax revenue peaks at slightly more than 80%. In this case, the revenue graph is very similar in appearance to Fullerton’s. The graph and numerical results are available on request.
Table 5. Elasticities of Total Taxable Income with Larger Non-Corporate Business Sector

<table>
<thead>
<tr>
<th>Tax Rate (%)</th>
<th>Individual Uncompensated Elasticity</th>
<th>Individual Compensated Elasticity</th>
<th>Corporate Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>1.22</td>
<td>1.70</td>
<td>1.30</td>
</tr>
<tr>
<td>20.0</td>
<td>1.24</td>
<td>1.74</td>
<td>1.33</td>
</tr>
<tr>
<td>30.0</td>
<td>1.25</td>
<td>1.79</td>
<td>1.36</td>
</tr>
<tr>
<td>40.0</td>
<td>1.26</td>
<td>1.83</td>
<td>1.38</td>
</tr>
<tr>
<td>50.0</td>
<td>1.26</td>
<td>1.87</td>
<td>1.40</td>
</tr>
<tr>
<td>60.0</td>
<td>1.26</td>
<td>1.92</td>
<td>1.40</td>
</tr>
</tbody>
</table>

*Table 5 shows elasticities of total taxable income resulting from a 1% increase in the tax rate from the value shown in column 1. The compensated individual elasticities are calculated by holding household income constant. That is, any change in tax liability resulting from a change in the tax rate is offset by a lump-sum transfer.*

corporate net-of-tax rate decline. Slemrod (1992, 1996, 1998), Gordon and Slemrod (2000), and Saez (2004) argue that an increase in the relative taxation of corporate income, such as occurred with TRA86, caused income to shift from the corporate to the non-corporate sector, reducing corporate taxable income. This gave the appearance that corporate income was tax responsive, but the effect was not real because taxable income in the non-corporate sector replaced income lost in the corporate sector. The numerical results in Table 5 indicate a real effect of such a tax-induced shift in income, namely, the decline in the elasticity of total taxable income with respect to the corporate net-of-tax rate and increase in the elasticity with respect to the individual income net-of-tax rate. This indicates that the relative tax change may not have net short-run effects but could still have real effects in the long run.

**Elasticities of Total Taxable Income and the R&D Tax Credit**

The simulations thus far assume that firms do not take an R&D tax credit. However, U.S. businesses are eligible for a federal R&D tax credit at a statutory rate of 20% (the Research and Experimentation Tax Credit [RETC]). This subsection reports taxable income elasticities when the R&D tax credit is included in the model.

Research performed by firms choosing to expense R&D is not eligible for the tax credit, and firms choosing the tax credit cannot expense R&D. To see how the corporate tax rate affects corporate decisions under this restriction, return to the research division's profit function and its first-order condition. These relationships are shown in Equations 10 and 11, respectively, for firms that expense R&D and do not take the credit. The corporate tax rate does not enter the first-order condition because all R&D is expensed. Compare this result with the case of a firm that forgoes expensing and does take the credit. The profit function becomes

\[ \pi_A = (1 - \tau^c) P_A \delta \zeta^l A^k - w\zeta^l (1 - ECR), \]

where \( ECR \) is the effective tax credit rate, which is applied against all wages paid to researchers. The first-order condition for skilled labor, \( \zeta^l \), is now

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27 RETC applies to wages and supplies, including nondepreciable property, the use of computers, and 65% of contract research. RETC is restricted to R&D above a "base amount," which is described in Appendix E.
Table 6. Elasticities of Total Taxable Income when Intermediate Goods Producers Take the R&D Tax Credit

<table>
<thead>
<tr>
<th>Tax Rate (%)</th>
<th>Individual Uncompensated Elasticity</th>
<th>Individual Compensated Elasticity</th>
<th>Corporate Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>1.08</td>
<td>1.55</td>
<td>2.02</td>
</tr>
<tr>
<td>20.0</td>
<td>1.08</td>
<td>1.58</td>
<td>2.07</td>
</tr>
<tr>
<td>30.0</td>
<td>1.09</td>
<td>1.62</td>
<td>2.11</td>
</tr>
<tr>
<td>40.0</td>
<td>1.10</td>
<td>1.66</td>
<td>2.15</td>
</tr>
<tr>
<td>50.0</td>
<td>1.10</td>
<td>1.71</td>
<td>2.18</td>
</tr>
<tr>
<td>60.0</td>
<td>1.10</td>
<td>1.76</td>
<td>2.19</td>
</tr>
</tbody>
</table>

*Table 6 shows elasticities of total taxable income resulting from a 1% increase in the tax rate from the value shown in column 1. The compensated individual elasticities are calculated by holding household income constant: That is, any change in tax liability resulting from a change in the tax rate is offset by a lump-sum transfer.*

\[
(1 - \tau^C)P_A \delta A^\lambda = w(1 - ECR).
\]

In sharp contrast with Equation 11, the corporate tax rate does enter Equation 11' because R&D is not expensed. In particular, an increase in \(\tau^C\) now reduces the after-tax marginal return on R&D, reducing the incentive to innovate. The results below show that this difference has a very large effect.

Appendix E shows how to calculate an effective R&D tax credit rate for an economy growing smoothly along its balanced growth path. This is used in the next set of simulations. Table 6 shows the elasticities. The R&D credit has an insignificant effect on elasticities with respect to the individual tax (compare to Table 2). However, elasticities with respect to the corporate net-of-tax rate are much larger than before (compare to Table 3), as explained above.

How Sensitive are Total Taxable Income Elasticities to Initial Parameter Settings?

This section checks the robustness of the simulations with respect to the benchmark parameter settings listed in Appendix A.1. Table 7 shows the sensitivity results. To simplify presentation, elasticities at tax rates of 20–50% are omitted. Note that the first two rows in the table reproduce the benchmark results.

The intertemporal elasticity of substitution governs the change in the household's willingness to substitute future for current consumption in response to a change in the rate of return on savings. These benchmark numerical solutions assume the reciprocal of the intertemporal elasticity of substitution, \(\sigma\), is equal to 2.0. Altig et al. (2001) set \(\sigma\) equal to 4.0. This led me, initially, to increase \(\sigma\). However, the larger \(\sigma\) is, the larger the taxable income elasticities become. Because the simulated elasticities tend to be relatively large, of more

28 The intuitive explanation for this is that a change in the economic environment that makes a larger capital stock desirable requires more saving. The larger \(\sigma\), the less willing consumers are to make the sacrifice; thus, larger increases in the interest rate are required to elicit the increase in saving. This increases tax responsiveness. This is seen in Equation C3, Appendix C, \(g_c = \frac{1}{(1/\sigma)[(1 - \tau^r) + \rho - \sigma g_a]}\). In balanced growth, \(g_c = 0\) and \(r = (\rho + \sigma g_a)/(1 - \tau^r)\). The larger \(\sigma\) is, the more responsive \(r\) is to changes in \(\tau^r\), and the more tax responsive are variables affected by \(r\), increasing tax responsiveness.
Table 7. Sensitivity of Total Taxable Income Elasticities with Respect to Parameter Settings*  

<table>
<thead>
<tr>
<th></th>
<th>Individual</th>
<th>Corporate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncompensated</td>
<td>Compensated</td>
</tr>
<tr>
<td>Benchmark results, under endogenous innovation</td>
<td>1.05</td>
<td>1.51</td>
</tr>
<tr>
<td>Panel A: Reduce the reciprocal of the intertemporal elasticity of substitution, $\alpha$, from 2.0 to 0.25</td>
<td>0.95</td>
<td>1.42</td>
</tr>
<tr>
<td>Panel B: Reduce household rate of time preference, $\rho$, from 0.034 to 0.017</td>
<td>1.00</td>
<td>1.69</td>
</tr>
<tr>
<td>Panel C: Reduce labor supply parameter, $\eta_1$, from 0.5 to 0.1</td>
<td>1.05</td>
<td>1.18</td>
</tr>
<tr>
<td>Panel D: Reduce elasticity of output with respect to physical capital, $1 - \alpha$, from 0.3 to 0.2</td>
<td>0.92</td>
<td>1.34</td>
</tr>
<tr>
<td>Panel E: Increase rate of capital depreciation, $\Delta$, from 0.06 to 0.1</td>
<td>0.99</td>
<td>1.45</td>
</tr>
<tr>
<td>Panel F: Increase the fraction of capital costs expensed, $f$ (output sector), from 0.33 to 0.43</td>
<td>1.05</td>
<td>1.70</td>
</tr>
<tr>
<td>Panel G: Increase the fraction of capital costs expensed, $h$ (research sector), from 0.2 to 0.3</td>
<td>1.04</td>
<td>1.50</td>
</tr>
</tbody>
</table>

* The first row reproduces the benchmark elasticities of taxable income from Tables 2 and 3. Panels A–G show effects of adjusting a single parameter from its original benchmark value, as described in column I. Only elasticities at marginal tax rates of 10% (first row in each panel) and 60% (second row in each panel) are shown.

interest is a change in parameter values that reduces them. Therefore, Panel A in Table 7 shows the results of reducing $\alpha$ to 0.25. The elasticities become smaller, but the declines are not large.

This result may appear to be at odds with sensitivity results reported in Altig et al. (2001). These authors study the effects of fundamental tax reforms. Among other things, they replace a U.S.-style federal-state tax system with a consumption tax. Income and the capital stock increase substantially in the long-run. The sensitivity analysis of Altig et al. shows that reducing $\alpha$ increases the size of these responses. In apparent contrast, I find that reducing $\alpha$ reduces the elasticity of taxable income. The difference stems from the very different experiments studied in their paper and mine: they study the effect of fundamental shifts in tax structure. I fix the tax structure and study the effects only of changing a marginal tax rate. To check how $\alpha$ affects the results of tax reform in my model, I replaced the personal and corporate income taxes with a consumption tax. In my model, as in that of Altig et al., as $\alpha$ is reduced, increases in capital and income grow larger.

In the benchmark, the rate of time preference, $\rho$, is set equal to 0.034. A smaller value reduces tax responsiveness for the same reason that a smaller value of $\alpha$ reduces responsiveness. Panel B in Table 7 shows the results of reducing $\rho$ by half. The elasticities become slightly smaller. Again, however, the differences are small and do not affect the main results.

The benchmark numerical solutions assume the parameter controlling the short-run responsiveness of labor supply, $\eta_1$, is 0.5. Panel C shows the effect of reducing the value to 0.1. Elasticities with respect to the corporate income net-of-tax rate are nearly unchanged, as might be expected. In contrast, the compensated elasticity with respect to the individual income tax

29 See the formula for $r$ in the previous footnote.
declines substantially. However, lower $\eta$ also decreases the income effect, and the uncompensated elasticities are nearly unchanged. This is an interesting result and seems to indicate the possibility that a less elastic labor supply may not reduce the size of total taxable income elasticities, even though it does reduce the inefficiency effects of taxes.

The benchmark numerical solutions assume that the elasticity of final output with respect to capital, $1 - \alpha$, is 0.3. Mankiw, Romer, and Weil (1992) show that increasing the value of the latter elasticity in the Solow growth model matches many stylized facts. They suggest the correct value could be as high as 0.7. However, the larger the value of the elasticity of output with respect to capital, the less concave the production function, the lower the economy’s speed of convergence, and the more responsive output tends to be to exogenous changes in the environment. Of more interest is a change in $1 - \alpha$ that reduces responsiveness. I reduced the value to 0.2, much lower than the economics literature suggests. The results are shown in Panel D. All the elasticities become smaller, but the declines are not large.

Following Stokey and Rebelo (1995), the benchmark numerical simulations assume that the rate of depreciation in physical capital, $\Delta$, is 6%. Jones, Manueli, and Rossi (1993) argue the correct value is closer to 0.1. Panel E shows the results in this case. All the elasticities decline somewhat, but the declines are not large. Following the literature, the benchmark simulations assume the fraction of capital asset costs expensed by output firms, $f$, is 0.33. I increased this value to 0.43. The results are shown in Panel F. The elasticities with respect to the corporate net-of-tax rate decline somewhat. This occurs because debt service is expensed and escapes the corporate tax. However, the main results are preserved. The benchmark simulations assume that the fraction of capital asset costs expensed by capital goods manufacturers, $h$, is 0.2. I increased this to 0.3. The elasticities decline, as shown in Panel G, but again, the main results are preserved.

5. Conclusion

The elasticity of taxable income determines the way tax revenue responds to marginal income tax rates and influences the welfare effects of income taxes. Long-run general equilibrium relationships are likely to affect the elasticity and may take a long time before they are revealed in available data. Empirical estimation of the elasticity is difficult and may not completely reflect the economy’s tax responsiveness. This paper constructs a CGE model of long-run growth and studies what this tool reveals about taxable income elasticities. Because privately financed technical innovation in modern economies earns monopoly profits, the model includes endogenous innovation.

The model is used to simulate total taxable income elasticities with respect to the individual net-of-tax rate and with respect to the corporate net-of-tax rate. Compensated and uncompensated elasticities are reported for the individual income tax. The elasticities reported are larger than many, though not all, of the elasticities reported in the empirical literature. The sensitivity analysis indicates that this is not an artifact of the initial benchmark parameter settings.

Estimated elasticities may underestimate tax responsiveness because they are reported either for individual income or for corporate income, rather than for total income, and because the economy converges slowly over time (Barro and Sala-i-Martin 2004), capital evolves slowly.
In response to exogenous shocks. Thus, empirical estimates may miss some of the economy's long-run general equilibrium tax responsiveness. However, the simulation results also must be qualified. The model I use tends to overstate tax responsiveness somewhat because it assumes infinite planning horizons. More generally, strong assumptions are required to solve CGE models. Only steady states are studied here. The analysis of transition paths when innovation is endogenous is difficult and is left for future work.

With these caveats in mind, the simulation results indicate the following important points:

(i) Long-run elasticities can be relatively large, even if initial impacts of taxes on labor supply and saving are small.
(ii) Long-run elasticities with respect to the corporate tax can exceed uncompensated elasticities with respect to the individual tax by large margins.
(iii) Elasticities are much larger under endogenous innovation than under exogenous innovation.

The simulations also indicate that the R&D tax credit appears to increase tax responsiveness with respect to the corporate net-of-tax rate by large amounts and that a form of tax avoidance often characterized as having no real short-run effects appears capable of real long-run effects. The numerical simulations indicate that a shift from corporate to non-corporate form, in response to higher corporate taxes, could reduce elasticities with respect to the corporate tax, while elasticities with respect to the individual tax increase.

If these results reflect the true underlying long-run relationship between tax rates and tax responsiveness, they have important policy implications. The second numbered point above indicates that if policy makers' choices are motivated by tax efficiency, they should look toward improving the corporate income tax. But the policy debate over income taxes in the United States appears heavily weighted toward concern for the individual income tax: The majority of changes in the federal tax code implemented during the past 25 years affect the individual tax. The second and third numbered points appear to have implications for policy makers concerned with tax equity. In a world where innovation becomes an ever larger share of economic activity, substantial shifting of tax incidence away from the corporate sector may occur in the long run. Capital income could become harder to tax in economies evolving toward higher levels of innovative activity, and true progressivity in the income tax may become harder to sustain, in spite of apparent progressivity of statutory taxes.
Appendix A

A.1 Symbols of Set Parameters and Their Benchmark Settings

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Denotation</th>
<th>Benchmark Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>% of debt-financed capital in final output sector</td>
<td>0.33</td>
</tr>
<tr>
<td>( h )</td>
<td>% of debt-financed capital in intermediate goods sector</td>
<td>0.2</td>
</tr>
<tr>
<td>( g_L )</td>
<td>Growth rate of population</td>
<td>0.0144</td>
</tr>
<tr>
<td>( g_A )</td>
<td>Growth rate of technology</td>
<td>0.018</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Elasticity of final output with respect to labor</td>
<td>0.7</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>Depreciation rate, physical capital</td>
<td>0.06</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Parameter determining the elasticity of labor supply</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Household rate of time preference</td>
<td>0.034</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Reciprocal of the intertemporal elasticity of substitution</td>
<td>2.0</td>
</tr>
<tr>
<td>( \tau^P )</td>
<td>Proportional tax rate on individual income</td>
<td>0.21</td>
</tr>
<tr>
<td>( \tau^C )</td>
<td>Proportional tax rate on corporate income</td>
<td>0.35</td>
</tr>
</tbody>
</table>

A.2 Other Symbols*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Denotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>An index of the level of technical knowledge</td>
</tr>
<tr>
<td>( AGR )</td>
<td>Average gross receipts used to compute ECR (used only in Appendix E)</td>
</tr>
<tr>
<td>( b )</td>
<td>Short-term corporate bonds, per person</td>
</tr>
<tr>
<td>( BA )</td>
<td>A firm's base amount of R&amp;D used to compute ECR (used only in Appendix E)</td>
</tr>
<tr>
<td>( c )</td>
<td>Consumption, per person</td>
</tr>
<tr>
<td>( d )</td>
<td>Dividend yield</td>
</tr>
<tr>
<td>( ECR )</td>
<td>Effective R&amp;D tax credit rate</td>
</tr>
<tr>
<td>( e )</td>
<td>Equity, per person</td>
</tr>
<tr>
<td>( FBP )</td>
<td>Fixed base percentage use to compute ECR (used only in Appendix E)</td>
</tr>
<tr>
<td>( G )</td>
<td>Government consumption per person</td>
</tr>
<tr>
<td>( GR )</td>
<td>A firm's gross receipts used to compute ECR (used only in Appendix E)</td>
</tr>
<tr>
<td>( g_C )</td>
<td>Growth rate of consumption per person</td>
</tr>
<tr>
<td>( g_k )</td>
<td>Growth rate of capital per person</td>
</tr>
<tr>
<td>( g_t )</td>
<td>Growth rate of time spent working</td>
</tr>
<tr>
<td>( g_Y )</td>
<td>Growth rate of real GDP</td>
</tr>
<tr>
<td>( k )</td>
<td>Physical capital, per person</td>
</tr>
<tr>
<td>( l )</td>
<td>Fraction of one unit time endowment supplied as labor</td>
</tr>
<tr>
<td>( L )</td>
<td>Labor stock</td>
</tr>
<tr>
<td>( LST )</td>
<td>Lump sum transfer, per person</td>
</tr>
<tr>
<td>( p_e )</td>
<td>Equity share price</td>
</tr>
<tr>
<td>( p(i) )</td>
<td>Rental price, innovative capital good ( i )</td>
</tr>
<tr>
<td>( P_A )</td>
<td>Price of a patent</td>
</tr>
<tr>
<td>( r )</td>
<td>Interest rate</td>
</tr>
<tr>
<td>( R )</td>
<td>Tax revenue, per person</td>
</tr>
<tr>
<td>( SCR )</td>
<td>The statutory R&amp;D credit rate used to compute ECR (used only in Appendix E)</td>
</tr>
<tr>
<td>( TI )</td>
<td>Taxable income, per person</td>
</tr>
<tr>
<td>( ti )</td>
<td>Taxable income, per effective labor unit</td>
</tr>
<tr>
<td>( w )</td>
<td>Wage rate per worker</td>
</tr>
<tr>
<td>( x(i) )</td>
<td>( i ) th innovative capital good, per person</td>
</tr>
<tr>
<td>( y )</td>
<td>Final output, per person</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Productivity parameter in research</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Scale parameter in the production of new technical knowledge</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Fraction of skilled labor employed in the intermediate goods sector</td>
</tr>
<tr>
<td>( (1 - \zeta) )</td>
<td>Fraction of skilled labor employed in the final output sector</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Parameter that determines the extent of congestion in research</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Elasticity of technology with respect to level of technical knowledge</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Profit</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Shadow price of assets</td>
</tr>
</tbody>
</table>

* Symbols in Appendix C include a tilde, which indicates that a variable is normalized by \( A \), except in the case \( \tilde{P}_A = P_A/L \).
Appendix B

Thirteen parameter values must be set in the calibration of the model. The parameters in the benchmark numerical simulations are given the following values: \( \sigma = 2.0, \rho = 0.034, \eta = 0.5, \alpha = 0.7, \Delta = 0.06, g_A = 0.018, g_L = 0.0144, f = 0.33, k = 0.2, \tau^A = 0.35, \) and \( \tau^L = 0.21. \) The fraction of R&D financed privately is set at 50%, and the fraction of total output produced by corporate firms is set at 80%. These benchmark parameter values were chosen in the following way. The reciprocal of the intertemporal elasticity of substitution, \( \sigma, \) the household's rate of time preference, \( \rho, \) and the parameter, \( \eta, \) are taken from Lucas (1990). Labor's share of final output, \( \alpha, \) is set equal to the historically observed value of GDP earned by labor, 0.7. Stokey and Rebelo (1995) suggest that a realistic value for the depreciation rate, \( \Delta, \) is 0.06. U.S. growth in output per person, \( g_A, \) averaged about 1.8% per year between 1870 and 2000 (Barro and Sala-i-Martin 2004). The long-run trend growth in the U.S. labor force, \( g_L, \) is 1.44% (Jones and Williams 2000). Cordes, Hauger, and Watson (1987) suggest that established firms finance 33% of capital from debt, while new firms finance much less from debt. Gravelle (1994) says that 33% is a rule of thumb for the fraction of capital financed by debt. And tables 20-1 and 10-2 in Fullerton and Karayannis (1993) indicate that the U.S. industry financed slightly more than 33% of capital from debt in 1990, but that U.S. manufacturers financed about 20% of capital from debt in 1990. Therefore, I set the fraction of debt-financed assets in the final output sector, \( f, \) equal to 33%. For the capital good manufacturers I set this fraction, denoted \( h, \) equal to 20%. Historically, more than 50% of R&D has been privately financed in the United States (OECD 1989; Stern, Porter, and Furman 2000). In the past two decades the figure has trended upward. The numerical simulations assume that the relatively conservative value of 50% of innovation is endogenous. Table 2 in Gravelle and Kotlikoff (1989) indicates that 76–86% of final output was produced by the corporate sector in the United States between 1975 and 1982. The computer model assumes that 80% of final output is subject to the corporate income tax, and the remainder is subject to the individual income tax.

Appendix C

The variables in the model described in the text are measured per person. To derive numerical solutions, variables must be transformed so that their growth rates are 0 on the balanced growth path. Variables measured in units of effective labor have this property. This appendix shows how the model is transformed into units of effective labor. I use the definitions \( \tilde{c} = c/A, \tilde{k} = k/A, \tilde{w} = w/A, \tilde{R} = R/A, \) and \( \tilde{P}_x = P/L. \) Also, \( k = \int_0^1 \bar{c}(l) dl = Ax, \) as in Rivera-Batiz and Romer (1991); therefore, \( \tilde{k} = x. \) I solve the model for nine endogenous variables, \( \tilde{c}, \tilde{k}, \tilde{p}, \tilde{P}, \tilde{r}, \tilde{w}, \tilde{R}, \) and \( \tilde{z}. \)

Because there are no adjustment costs on balanced growth paths, Tobin's \( q \) is equal to one, and \( (b+c)(k+AP/L) = 1. \) There is no uncertainty in the model, so the return on bonds and equity must be equal. Thus, Equation 2 can be rewritten as

\[
k + A\tilde{P}_A + A\tilde{P}_A = (1 - \tau^A)[\tilde{w} + r(k + A\tilde{P}_A)] - c = (\tilde{z} + A\tilde{P}_A)g_L.
\]

Divide this equality by \( A\tilde{k} \) and use the facts that \( k/A = \tilde{k} + \tilde{g}_A \) and \( g_{P_A} = g_L = 0 \) on the balanced path to get

\[
0 = g_L = \frac{\tilde{w} + r(\tilde{k} + \tilde{P}_A)}{\tilde{k}} - c = (\tilde{g}_L + g_A)\tilde{k} + \tilde{P}_A.
\]

Combine Equations 3a and 3b and divide by \( A \) to get

\[
\frac{\tilde{c}}{1 - \tilde{I}} = (1 - \tau^A)\tilde{w}.
\]

On the balanced path \( g_r = g_c = 0. \) Transforming Equation 4, the growth rate of consumption in effective labor units is

\[\text{Equation C5 in Appendix C is } (1 - \tau^A)(1 - g)(1 - \tilde{z})(1 - \tilde{I})\tilde{k} = p/(1 - \tau^L). \] To account for the 80% of business income taxed as corporate income versus the 20% taxed as individual income, Equation C5 would have to be written as \( 0.8 \) * \( (1 - \tau^A)(1 - g)(1 - \tilde{z})(1 - \tilde{I})\tilde{k} = 0.2 \) * \( (1 - \tau^A)(1 - g)(1 - \tilde{z})(1 - \tilde{I})\tilde{k} = 0.8 \) * \( p/(1 - \tau^L) + 0.2 \) * \( p/(1 - \tau^L) \), which is much more complicated.\]
0 = g_L = \frac{1}{C}(1 - \tau^p) r - \rho - \sigma g_A. \quad (C3)

Dividing Equation 7a by \( A \) gives

\[ \alpha[(1 - \zeta)l^{1-\gamma}]^{k^{1-\gamma}} = \tilde{w}. \quad (C4) \]

Replacing \( x \) by \( \tilde{k} \) in Equation 7b gives

\[ (1 - \tau^C)(1 - \alpha)[(1 - \zeta)l]^{\tilde{k}^{1-\gamma}} = p(1 - f^C). \quad (C5) \]

Dividing Equation 9 by \( A \), the growth rate of technology is \( g_A = \delta \zeta/LA^{\lambda-1} \). For intermediate goods producers who expense R&D, divide Equation 11 by \( A \) and use the definition of \( g_A \) to get

\[ \tilde{P}_A g_A(\xi) = \tilde{w}. \quad (C6) \]

Divide Equation 5 and Equation 16 by \( \xi \), solve Equation 5 for the balanced growth path value of \( \tilde{y} \), and substitute for \( \tilde{y} \) in Equation 16. Also, divide Equation 11 by \( A \) and use the definition of \( g_A \) to get

\[ \tilde{P}_A g_A(\xi) = \tilde{w}. \quad (C6) \]

Equations C1–C7, along with Equations 13 and 15 in the text, are used to solve for \( \tilde{e}, \tilde{k}, \tilde{l}, p, \tilde{P}_A, r, \tilde{w}, \tilde{R}, \) and \( \zeta \).

### Appendix D

Appendix C transforms the model into units of effective labor in order to derive numerical solutions. The computer calculations, therefore, generate taxable income in units of effective labor. Let \( TI \) be taxable income per person and let \( ti = TI/A \) be taxable income per effective labor unit. If innovation is endogenous, income tax rates affect the level of technical knowledge, \( A \); thus, \( ti \) and \( TI \) are affected differently by a change in tax rates. The computer output in units of effective labor must be transformed back into units per person before the elasticity of taxable income can be measured correctly. This appendix explains the transformation from units of effective labor to units per person.

Divide Equation 9 by \( A \) to get the growth rate of technology, \( g_A = \delta \zeta/LA^{\lambda-1} \). Use the definition \( \delta = \delta(\xi/L)^{-(1-\theta)} \) to eliminate \( \delta \), giving \( g_A = \delta(\xi/L)^{\theta}A^{\lambda-1} \). Because \( g_A \) must itself be constant on the balanced growth path, I can solve for \( g_A \) in terms of parameters: Taking logarithmic time derivatives and solving gives \( g_A = \theta g_L/(1 - \lambda) \) (this follows Jones 1995a). Given \( g_A \) and \( g_L \) on the balanced growth path the ratio of \( \theta \) to \( (1 - \lambda) \) must be

\[ \frac{g_A}{g_L} = \frac{\theta}{1 - \lambda}. \quad (D1) \]

Next note that only two variables in \( g_A = \delta(\xi/L)^{\theta}A^{\lambda-1} \) are affected by a change in tax rates, namely \( \zeta \) and \( A \). Using Equation D1, the percent change in \( A \) from the initial balanced growth path to the new balanced growth path after a tax rate change is

\[ \%\Delta A = \frac{g_A}{g_L} \%\Delta \zeta. \quad (D2) \]

Recall \( TI = (ti)A \). Therefore, using Equation D2 I get
Innovation and the Elasticity of Income

\[ \%\Delta TI = \%\Delta (u_l) + \frac{g_A}{g_L} \%\Delta u_r. \]  

Next, let subscripts 0 and 1 denote balanced growth path values before and after a change in a tax rate, respectively, so that \( T_1 = (t_1)A_1 = (t_0)e^{g_L\Delta t}A_0e^{g_A\Delta \lambda}. \) Writing this in logs,

\[ \log[T_1] = \log[t_0] + \%\Delta (u_l) + \log[A_0] + \%\Delta A. \]  

In Equation D4, \( \%\Delta (u_l) \) is approximately equal to \( \log[t_1] - \log[t_0]. \) Also, \( A_0 \) can be set to 1, without loss of generality because the percentage change in \( A, \) not its level, matters. Using this and Equation D2 in Equation D4 gives

\[ \log[T_1] = \log[t_1] + \frac{g_A}{g_L} \%\Delta u_r. \]  

Exponentiating Equation D5 gives the new balanced growth path value of taxable income in units per person. This is then used to approximate the percent change in taxable income per person resulting from a change in a tax rate and to calculate the elasticities of taxable income.

**Appendix E**

U.S. firms can expense R&D, or they can take an R&D tax credit. The Statutory R&D Credit Rate (SCR) is 20%. However, the United States employs an incremental R&D credit; that is, a firm’s R&D spending must exceed a base amount in order to be eligible for the credit. Therefore, the Effective Credit Rate (ECR) is a fraction of SCR. ECR in an economy growing smoothly along a balanced growth path is derived as follows.

First, the R&D credit statute defines a Fixed Base Percentage (FBP) in year \( t. \) FBP is the ratio of R&D to Gross Receipts (GR) in the prior year:

\[ FBP_t = \frac{R&D_{t-1}}{GR_{t-1}}. \]  

Along the economy’s balanced growth path, R&D and GR must grow at constant and equal rates, so FBP must be constant. Using this in Equation E1 implies

\[ GR_{t-1} = \frac{R&D_{t-1}}{FBP}. \]  

Now use Equation E2 to write an expression for Average Gross Receipts (AGR) in the four years prior to the year a firm intends to take the credit:

\[ AGR_t = \frac{1}{4} \sum_{i=1}^{4} GR_{t-i} = \frac{1}{4FDP} \sum_{i=1}^{4} R&D_{t-i}. \]  

The Base Amount (BA) of R&D, used to calculate ECR, is the product of FBP and AGR,

\[ BA_t = FBP_t AGR_t = FBP_t \left( \frac{1}{4} \sum_{i=1}^{4} R&D_{t-i} \right) = \frac{1}{4} \sum_{i=1}^{4} R&D_{t-i}. \]  

Along the economy’s balanced growth path, \( R&D_{t-i} = R&D_0(1 + g_y)^{t-i}, \) where \( g_y \) is the economy’s constant long-run growth rate. Use this to replace \( R&D_{t-i} \) in Equation E4 to get the base amount along the balanced growth path:

\[ BA_t = \frac{R&D_0}{4} \sum_{i=1}^{4} (1 + g_y)^{t-i}. \]
Only R&D in excess of BA is eligible for the credit. Using Equation E5, the amount of R&D eligible for the credit in year t is $R&D_t - BA$:

$$[\text{Eligible } R&D_t] = R&D_t \left(1 - \frac{1}{4} \sum_{i=1}^{4} (1 + g_Y)^{-t} \right).$$  \hspace{1cm} (E6)

The expression in parentheses is the fraction of total R&D, that is eligible for the credit. $ECR$ is the product of $SCR$, the fraction of $R&D$ eligible for the credit,

$$ECR = SCR \left(1 - \frac{1}{4} \sum_{i=1}^{4} (1 + g_Y)^{-t} \right).$$  \hspace{1cm} (E7)

Assuming $g_Y$ is 3.25% per year, the fraction of total R&D eligible for the credit is 7.6% along the balanced growth path. With a statutory credit rate of 20%, $ECR$ is 1.53%, so this value is used in the experiment that studies the effect of the R&D tax credit (Table 6 in the text).

References


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