Taxes, the Speed of Convergence, and Implications for Welfare Effects of Fiscal Policy

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Previous studies suggest that income taxes do not affect the convergence speed in neoclassical and new growth models. Those studies use very simple tax structures. This paper shows that a relation between taxes and convergence speed emerges if tax benefits are included in standard macroeconomic models. A welfare example suggests that the economic impact could be large even if the absolute size of the effect of taxes on convergence speed is small.

1. Introduction

Economists’ views on the relation between taxes and the long-run growth rate of per capita income have varied greatly as macroeconomic models have evolved. Neoclassical growth models leave no room for a relation between taxes and steady-state growth (Ramsey 1928; Solow 1956; Swan 1956; Cass 1965; Koopmans 1965). Tax policy can affect growth in early endogenous innovation models (Romer 1986, 1990; Segerstrom 1991; Grossman and Helpman 1991; Aghion and Howitt 1992). However, these models exhibit scale effects.1 When scale effects are removed, the relation between taxes and steady-state growth tends to disappear (Jones 1995b; Segerstrom 1998; Young 1998). Tax policy can affect the growth rate in some versions of endogenous human capital models (King and Rebelo 1990; Rebelo 1991; Jones, Manuelli, and Rossi 1993). However, Lucas’s (1990) simulations indicate that the tax effects are tiny. Stokey and Rebelo (1995) support Lucas’s conclusion.

The short run may be the only run in which a relation between taxes and per capita income growth might hold. Nevertheless, a short-run relation could have important lasting implications. Whether this is true depends on the economy’s convergence speed. For example, a capital income tax cut may increase the saving rate, increasing growth. Even if diminishing returns dictate that faster growth is merely transitory, the temporary growth spurt shifts upward the long-run trajectories of capital per person and welfare. But the increase in saving requires lower consumption in the short run. If the transition is slow, the short-run sacrifice could outweigh the long-run gain (Bernheim 1981; Judd 1987). The duration of the transition and, therefore, the size of the net welfare gain depend on the economy’s convergence speed.

The speed at which economies converge to their steady states has been analyzed extensively (Mankiw, Romer, and Weil 1992; Ortigueira and Santos 1997; Barro and Sala-i-Martin 1999). But

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1 Scale effects are not supported by empirical studies (Backus, Kehoe, and Kehoe 1992; Jones 1995a; Temple 1999).
For even tin First typical ted an convergence the This convergence classical credits as come ing (Ramsey gross ing where income 2. ing (1995) deduplication for there, therefore, the economy’s convergence speed. The accelerated depreciation allowances permitted by the U.S. tax code will have a larger effect than the example studied here. Tax benefits, such as deductions for charity, debt service, and amortization of goodwill and patents; investment tax credits for equipment and research and development (R&D); and the myriad other deductions in typical tax codes, as well as the personal income tax itself, affect required saving and can affect convergence speed.

The next section of the paper incorporates a deduction for economic depreciation in the neoclassical model with exogenous saving (Solow model, hereafter) and explains the effect on the convergence speed. Section 3 explains the effect in the neoclassical model with endogenous saving (Ramsey model). Appendix C derives the same effect in a new growth model. The relation appears to be a robust feature of standard macroeconomic models. Section 4 compares the net welfare effects of income tax cuts with and without a depreciation deduction, under narrow and broad (human plus physical) capital.\(^2\) The depreciation deduction reduces the size of net welfare effects. This is consistent with the fact that the deduction reduces the convergence speed, and suggests that the deduction reduces the size of the net welfare effects because it lengthens the transition, increasing short-run losses. Section 5 concludes the paper.

### 2. Taxes and Convergence Speed in the Solow Model

Net saving is the difference between gross saving and the quantity of saving required to maintain the capital stock. In the Solow model with Cobb–Douglas production, a proportional income tax, and a depreciation deduction, net saving is

\[
\dot{k} = s(1 - \tau)k^\alpha - [(1 - d\tau)\delta + n + g]k, \tag{1}
\]

where \(k\) is capital in effective labor units, \(\dot{k}\) is \(k\)'s time derivative, \(s\) is the exogenous after-tax saving rate, \(\tau\) is the tax rate, \(\alpha\) is capital’s share of output, \(d\) is the proportion of depreciation deductible for tax purposes, \(\delta\) is the rate of economic depreciation, \(n\) is the population growth rate, and \(g\) is the growth rate of labor effectiveness. The first term on the right-hand side of Equation 1 is gross saving. The absolute value of the second term is required saving. Except for the inclusion of the depreciation deduction, Equation 1 represents net saving in a standard Solow model with an income tax.

\(^2\) Mankiw, Romer, and Weil (1992) distinguish between narrow and broad capital and show the effect on convergence speed.
The speed of convergence is the rate at which actual \( k \) approaches its steady-state value. Let \( k^* \) be the steady-state capital stock. Near the steady state, convergence speed is determined by the negative of the coefficient on \( (k - k^*) \) in the first-order Taylor expansion of Equation 1:

\[
\dot{k} \approx \{\alpha s (1 - \tau) k^{*\alpha - 1} - [(1 - d\tau)\delta + n + g]\}(k - k^*). \tag{2}
\]

The coefficient on \( (k - k^*) \) is the response in net saving to a small change in \( k \). The first term in the coefficient is the change in gross saving. The absolute value of the second term is the change in required saving. The convergence speed is determined by the difference in size of these two changes. If they were equal in size, net saving would not respond to \( k \) and the economy would not converge. Since the production function is concave, near the steady state the change in gross saving always is smaller than the change in required saving, net saving responds negatively to a small change in \( k \), and the economy must converge.

Setting Equation 1 equal to zero and solving for \( k^* \):

\[
k^* = \left[ \frac{s(1 - \tau)}{(1 - d\tau)\delta + n + g} \right]^{\frac{1}{\alpha}}. \tag{3}
\]

To get a standard form expression for the convergence speed, substitute Equation 3 into Equation 2 and take the negative of the coefficient on \( (k - k^*) \):

\[
\beta = (1 - \alpha)\{(1 - d\tau)\delta + n + g\}. \tag{4}
\]

Although the term \( s(1 - \tau) \) appears in Equations 1, 2, and 3, it drops out in the derivation of Equation 4. Barro and Sala-i-Martin (1999, p. 37) explain why the saving rate drops out: \( s \) does not affect convergence speed because it affects \( k^* \) and the change in gross saving for given \( k \) in exactly offsetting ways. Likewise, the income tax per se affects \( k^* \) and the change in gross saving in offsetting ways, so it does not directly affect the convergence speed. This was shown by Barro, Mankiw, and Sala-i-Martin (1995), and by Ortigueira and Santos (1997).

Nevertheless, Equation 4 shows that the depreciation deduction, \( d \), reduces the convergence speed. Equation 2 shows that \( d \) reduces the size of the change in required saving, which tends to reduce the size of the response in net saving and the convergence speed. On the other hand, Equation 3 shows that \( d \) has a positive effect on \( k^* \). This reduces the size of the change in gross saving, which tends to increase the size of the response in net saving and the convergence speed. Since the production function is concave, the latter effect must be smaller, so the convergence speed declines with \( d \).

Equation 4 also shows that the income tax rate enters through its relation with \( d \). Although the income tax does not directly affect convergence speed, it does so indirectly, via the depreciation deduction. The larger the tax, the larger the amount of depreciation that is deductible, so an increase in the tax rate has the same effect as an increase in the deduction. Since the deduction causes the convergence speed to decline, the tax does also.

Table 1 reports numerical values of \( \beta \), under narrow and broad concepts of capital (Mankiw, Romer, and Weil 1992), with and without a depreciation deduction. The narrow concept of capital sets \( \alpha \) to 0.3. The broad concept of capital sets \( \alpha \) to 0.75. Also, \( n = 0.01 \), \( g = 0.02 \), and \( \delta = 0.05 \). These values are used in Barro and Sala-i-Martin (1999). \( d \) is set equal to 0 in the first row. In the second row \( d = 1.0 \) if \( \alpha = 0.3 \), and \( d = 0.4 \) if \( \alpha = 0.75 \). The smaller value of \( d \) is used

\[3\] For example, see Chapter 1 in Romer (2001).
Table 1. Convergence Speed in the Solow Model

<table>
<thead>
<tr>
<th>Depreciation Deduction</th>
<th>$\beta$ ($\alpha = 0.3$)</th>
<th>$\beta$ ($\alpha = 0.75$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>0.056</td>
<td>0.020</td>
</tr>
<tr>
<td>Yes</td>
<td>0.044</td>
<td>0.018</td>
</tr>
</tbody>
</table>

$\alpha$ is capital’s share of output in the Solow model. $\alpha = 0.3$ signifies narrow (physical) capital. $\alpha = 0.75$ signifies broad (physical plus human) capital. $\beta$ is the convergence speed from Equation 4.

under broad capital because this concept includes human capital, whose depreciation (unfortunately) is not deductible. The tax rate is set equal to 35%. This is the statutory federal income tax rate on most corporate income.

3. Taxes and Convergence Speed in the Ramsey Model

This section explains the effects of the depreciation deduction on convergence speed in the Ramsey model. After-tax profit of the representative firm is

$$\pi = (1 - \tau_C)(k^\alpha - w - rk - (1 - d\tau_C)\delta k),$$

where $\tau_C$ is a proportional tax on corporate cash flow, $w$ is the wage rate, $r$ is the rental price of capital, and all other symbols are defined as before. The interpretation of $d$, however, differs slightly from before. Here $d$ represents only the corporate depreciation deduction, which is less than the total depreciation deduction. The total depreciation deduction is larger than $d$ because the household effectively gets to deduct depreciation. This is clear in the household’s budget constraint, Equation 8 below.

The firm’s first-order conditions are

$$r = \alpha(1 - \tau_C)k^{\alpha-1} - (1 - d\tau_C)\delta, \quad (6a)$$

$$w = (1 - \alpha)k^\alpha. \quad (6b)$$

The household’s flow budget constraint determines net saving:

$$k = (1 - \tau_P)(w + rk) - c - (n + g)k. \quad (7)$$

Here $\tau_P$ is the proportional tax on personal income and $c$ is consumption per effective labor unit.

Optimal net saving is the difference between optimal gross saving and required saving. Substituting $w$ and $r$ from Equation 6 into Equation 7 gives optimal net saving,

$$k = [(1 - \tau_P)(1 - \alpha\tau_C)k^\alpha - c] - [(1 - \tau_P)(1 - d\tau_C)\delta + (n + g)k]. \quad (8)$$

The first square-bracketed term is optimal gross saving. The absolute value of the second term is required saving in this model. Note that personal as well as corporate taxes reduce the cost of

4 Mankiw, Romer, and Weil’s (1992) empirical estimates are consistent with a human capital share equal to 50% of broad capital’s share. In this case it might seem reasonable to assume the depreciation deduction is 50% of broad capital ($d = 0.5$). However, Barro and Sala-i-Martin (1999) use the value 0.75 for broad capital’s share. If physical capital’s share is 0.3, then physical capital is 40% of broad capital ($d = 0.4$). The smaller value of $d$ is conservative.

5 The production function is $Y = K^\alpha(AL)^{1-\alpha}$, where caps indicate variables in levels. In levels, the profit function is $\Pi = (1 - \tau_C)[K^\alpha(AL)^{1-\alpha} - wAL] - rk - (1 - d\tau_C)\delta K$. Setting derivatives to zero gives Equations 6a and 6b.
depreciation to the household. Depreciation effectively is deducted twice: The firm receives the deduction $d\tau_c\delta$, whereas the household deducts an additional $\tau_\rho \delta$.\footnote{There is an additional term, $-\tau_\rho d\tau_c\delta$, which is small.}

Appendix A derives the following expression for optimal saving:

$$
\hat{c} = \frac{c}{\theta} \left( [\alpha(1 - \tau_p)(1 - \alpha \tau_c)k^{x_{-1}} - (1 - \tau_p)(1 - d\tau_c)\delta + n + g]k^*, \right)
$$

where $\rho$ is the household rate of time preference and $\theta$ is the elasticity of marginal utility. The first square-bracketed expression is the rate of return on capital net of depreciation. The second square-bracketed term, $\rho + \theta g$, is the household’s required rate of return. Note that the expression that multiplies $d$ comes from required saving. It enters through the household’s budget constraint and optimization. The higher the depreciation rate, the lower the required saving, and the higher the net return on capital and optimal consumption growth.

The derivations below show that optimal consumption growth, Equation 9, is crucial in the transmission of tax effects to convergence speed in this model. Equation 8, which defines net saving, may appear to be unimportant. In contrast, in the Solow model net saving (recall Eqn. 2) determines the transmission mechanism. This difference is more apparent than real. The derivations below show that net saving plays the same role as before. The difference is that now the effects are transmitted via the net return on saving, and, therefore, optimal consumption growth, because optimal consumption dictates capital accumulation in the Ramsey model.

Using Equation 8 to solve for $c^*$ and Equation 9 to solve for $k^*$ gives

$$
c^* = (1 - \tau_p)(1 - \alpha \tau_c)k^{x_{-1}} - [(1 - \tau_p)(1 - d\tau_c)\delta + n + g]k^*,
$$

$$
k^* = \left( \frac{\alpha(1 - \tau_p)(1 - \alpha \tau_c)}{(1 - \tau_p)(1 - d\tau_c)\delta + \rho + \theta g} \right)^{\frac{1}{x_{-1}}}
$$

The first-order Taylor expansion of Equations 8 and 9 is

$$
\begin{pmatrix}
\hat{k} \\
\hat{c}
\end{pmatrix} \approx \begin{pmatrix}
\frac{\alpha(1 - \tau_p)(1 - \alpha \tau_c)k^{x_{-1}} - [(1 - \tau_p)(1 - d\tau_c)\delta + n + g]}{\theta} & -1 \\
\frac{\alpha(\alpha - 1)(1 - \tau_p)(1 - \alpha \tau_c)k^{x_{-2}}}{\theta}
\end{pmatrix}
\begin{pmatrix}
k - k^* \\
c - c^*
\end{pmatrix}
$$

The convergence speed depends on the matrix coefficient in Equation 12. The $a_{11}$ element of the matrix coefficient is analogous to Equation 2 in the Solow model. Taxes cannot affect the convergence speed through this term. To see this, use Equation 11 to replace $k^*$: $a_{11}$ reduces to $\rho - n - g(1 - \theta)$.

For taxes to affect the convergence speed in the Ramsey model, they must affect the $a_{21}$ element of the matrix coefficient in Equation 12,

$$
a_{21} = \frac{c^*}{\theta} \frac{\alpha(\alpha - 1)(1 - \tau_p)(1 - \alpha \tau_c)k^{x_{-2}}}{\theta}.
$$

Equation 13 is the response in consumption growth to a small change in $k$. This term depends on the change in the rate of return due to a small change in $k$, in square brackets. Since the production function is concave, the change in the rate of return is negative, so consumption growth responds negatively. The convergence speed depends on the size of the decline. The larger the size of the decline, the faster the convergence because the economy approaches $\hat{c} = 0$ more rapidly.

Use Equations 10 and 11 to replace $c^*$ and $k^*$ in Equation 13. This gives

\footnote{This is the steady-state discount rate on lifetime utility in this model.}
\[ a_{21} = \frac{1 - \alpha}{\theta} ((1 - \tau_p)(1 - d\tau_C)\delta + \rho + \theta g) \times \left[ \frac{1}{\alpha} ((1 - \tau_p)(1 - d\tau_C)\delta + \rho + \theta g) - ((1 - \tau_p)(1 - d\tau_C)\delta + n + g) \right]. \] (14)

Although the term \((1 - \tau_p)(1 - \alpha \tau_C)\) appears in Equations 8–13, it drops out in the derivation of Equation 14. This is true because the personal and corporate income taxes affect \(c^*, k^*\), and the change in consumption growth for given \(c\) and \(k\), in exactly offsetting ways. The term \((1 - \tau_p)(1 - \alpha \tau_C)\) drops out here for the same reason that \(s(1 - \tau)\) drops out of the convergence speed in the Solow model.

However, Equation 14 shows that \(d\) affects the convergence speed. Although the direction of the effect is not obvious from Equation 14, the intuition here is stronger than in the Solow model, and makes clear the direction. Taxpayers receive a deduction for depreciation. From the point of view of private agents, the government bears some of the burden of capital depreciation. This diminishes the size of the decline in the private rate of return that results from a small increase in \(k\) and, therefore, the size of the response in consumption growth (Eqn. 13). This tends to reduce the convergence speed. Of course, the depreciation deduction also increases \(k^*\) (Eqn. 11), which tends to increase the convergence speed. Since the production function is concave, the former effect outweighs the latter, so the convergence speed declines.

The income tax rates affect convergence speed indirectly, via their relation with the depreciation deduction. This is seen in the term \((1 - \tau_p)(1 - d\tau_C)\delta\) in Equation 14. The larger either the personal tax or the corporate tax, the larger the amount of depreciation that is deductible, so an increase in either tax has the same effect as an increase in the deduction. Since the deduction causes the convergence speed to decline, the personal and corporate taxes reduce the convergence speed. However, the personal income tax rate affects convergence speed even if \(d\) is zero in Equation 14. The household receives a depreciation deduction by virtue of the fact that the income the household receives through its ownership of the firm is necessarily net of depreciation. \(d\) does not have to be positive for this to occur.

Table 2 reports values of the convergence coefficient of the linearized Ramsey model, \(\beta_L\), and a global measure of the convergence speed, \(\beta_G\). The first is an approximation to the convergence speed in small neighborhoods of the steady state. More generally, the convergence speed also depends on the distance from the steady state. \(\beta_G\) measures this value. Appendix B derives \(\beta_L\) and \(\beta_G\) for this model. The derivations follow Barro and Sala-i-Martin (1999). Following these authors, \(\beta_G\) is measured at \(k = 0.1(k^*)\).

In Table 2 the ‘base case,’ excluding taxes, serves as a frame of reference. Panels 1, 2, and 3 show convergence speeds for various combinations of corporate and personal income tax rates and depreciation deductions. Except for the value used for the personal tax (19%), all parameter values are as in Table 1.\(^8\)

Comparing the first row in panel 1 with the base case indicates (as expected) that the corporate tax has no effect on the convergence speed when there is no corporate depreciation deduction. Comparing the second row in panel 1 with the first row shows that the convergence speed declines when the corporate depreciation deduction is included in the model.

\(^8\) 19% is Jorgenson’s (1993) estimate of the effective marginal tax rate on corporate-source personal income. See his Tables 1 and 2.
Table 2. Convergence Speed in the Ramsey Model

<table>
<thead>
<tr>
<th>Corporate Depreciation Deduction</th>
<th>Tax Rate</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>0%</td>
<td>0.082</td>
<td>0.015</td>
</tr>
<tr>
<td>Panel 1</td>
<td>$\tau_C = 35%$</td>
<td>0.072</td>
<td>0.014</td>
</tr>
<tr>
<td>Panel 2</td>
<td>$\tau_P = 19%$</td>
<td>0.075</td>
<td>0.014</td>
</tr>
<tr>
<td>Panel 3</td>
<td>$\tau_C = 35%, \tau_P = 19%$</td>
<td>0.075</td>
<td>0.013</td>
</tr>
</tbody>
</table>

$\alpha$ is capital’s share of output in the Ramsey model. $\alpha = 0.3$ signifies narrow (physical) capital. $\alpha = 0.75$ signifies broad (physical plus human) capital. $\beta_L$ is the convergence speed, calculated analytically, from the linearized Ramsey model. $\beta_U$ is a global measure of convergence speed, calculated numerically. The base case excludes taxes from the model.

Comparing the first row of panel 2 with the base case indicates that the individual tax decreases the convergence speed even if there is no corporate depreciation deduction. The household effectively receives a deduction for depreciation even if $d = 0$, because the capital income transferred from the firm to the household is net of depreciation. This tax benefit increases with $\tau_p$, which decreases the change in the rate of return in Equation 13 and the convergence speed. Comparing the second row in panel 2 with the first row shows that the corporate depreciation deduction does not alter the decline in convergence speed resulting from the personal income tax. This holds because the household depreciation deduction is unrelated to the size of the corporate depreciation deduction, $d$. This is important to the interpretation of the welfare gains from tax cuts reported in the next section.

Comparing panel 3 with the other panels indicates that the size of the decline in convergence speed is largest when capital income is taxed twice. Since state governments also tax personal income and permit depreciation deductions for business income, the effects would be larger than indicated here if state income taxes were included in the model.

Ortigueira and Santos (1997) include an income tax in their endogenous growth model. The tax does not affect the convergence speed. However, Appendix C shows that a depreciation deduction does affect convergence speed in their model. Since the depreciation deduction depends on the tax rate, the convergence speed indirectly depends on the tax rate. These relations appear to be robust features of standard macroeconomic models. By induction, any tax parameter that affects required saving in macroeconomic models affects convergence speed. Any tax benefit that varies with the tax rate will induce an indirect relation between the tax rate and convergence speed.

4. Implications for Welfare Effects of Fiscal Policy

**Measured Convergence Speeds May Not be Indicative of Economic Impact**

The changes in convergence speed in Table 2 may appear to be small. But there are a number of reasons that the absolute size of the changes may be a misleading indicator of their economic importance. First, income taxes distort behavior. For example, the tax on capital income tends to cause a substitution of consumption for saving. Since households choose consumption bundles that would not otherwise have been chosen, the social cost of the tax exceeds the revenue raised. A hallmark of tax analysis is that this excess burden increases more than proportionately with tax
rates. Therefore, even small changes in tax rates can produce large changes in welfare. This characteristic of tax burdens could easily transfer over to convergence speed. In that case, small changes in convergence speed could cause large changes in welfare.

Second, the convergence speed determines the length of the transition period. Therefore, changes in convergence speed affect net welfare gains. Suppose, for example, that the capital income tax is cut. To keep things as simple as possible, also suppose the tax cut is financed by a decrease in lump-sum taxes. Initially saving rises and consumption declines, so welfare declines in the short run. The net welfare gain from the tax cut includes the short-run losses as well as the long-run gains (Bernheim 1981; Judd 1987). The smaller the convergence speed, the longer the losses persist, and the smaller the net welfare gain.

A relatively small reduction in convergence speed could have a disproportionately large effect. This possibility receives support from the fact that after the tax cut (continuing the example), the initial transitional losses receive the most weight in lifetime utility. This is true because compound discount rates rapidly reduce the contribution to total welfare of changes in utility that occur later in the transition. For example, in the 24 welfare experiments I performed, the discount factor one period after a tax cut averaged 0.92, and ranged from 0.89 to 0.95. The discount factor after 34 periods averaged 0.11, and ranged from 0.04 to 0.18. Therefore, a relatively small increase in the number of periods of initial sacrifice could have a disproportionate effect on welfare.

This is particularly important in cases where the short-run and the long-run welfare responses have opposite signs. In that case, the change in the convergence speed can change the sign of the net welfare gain: A policy with an apparent welfare gain could, in fact, produce a welfare loss.

An Example: Net Welfare Effects of Tax Cuts

This section reports simulations of the effects on welfare of capital income tax cuts financed by lower lump-sum transfers. Tax rate cuts financed by lump-sum transfers have no income effects. Therefore, after a tax cut, saving initially increases, and consumption and welfare initially decline. Higher saving increases the capital stock, so consumption and welfare increase in the long run.

The values reported are net (short-run plus long-run) welfare changes, based on the economy’s full dynamic transition path, after a tax cut. To derive these values, the differential equation system in Equations 8 and 9 is solved numerically. Then the net welfare values are calculated numerically, on the basis of the global solution of the model. The net welfare changes are measured in terms of the equivalent variation (Auerbach 1985; Creedy 1998). Equivalent variation is the amount that pre-tax reform consumption must be reduced to deliver the post-tax reform level of utility. In all experiments reported here, the tax cuts increase net welfare, so the equivalent variations are negative. Since there are no income effects, equivalent variation is identical to excess burden. Therefore, negative values represent net reductions in excess burden. Appendix D describes the solution method and welfare calculations.

The tax cuts are unanticipated and permanent. The cut in the corporate income tax reduces it from 35% to 0. The cut in the personal income tax reduces it from 19% to 0. All parameter values are the same as in section 3.

Each panel in Table 3 shows net welfare gains from tax cuts before and after the corporate depreciation deduction is included in the model. It is important to keep in mind that the sole difference between the two rows within each panel is the corporate depreciation deduction. Panel 2

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Footnote: That is, initial consumption would have to increase to deliver the larger post-tax cut level of welfare.
Table 3. Net Welfare Effects of Capital Tax Cuts in the Ramsey Model

<table>
<thead>
<tr>
<th>Corporate Depreciation Deduction</th>
<th>Tax Cut in</th>
<th>Net Change in Excess Burden ($\alpha = 0.3$)</th>
<th>Net Change in Excess Burden ($\alpha = 0.75$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel 1</td>
<td>No</td>
<td>$\tau_C$</td>
<td>$-6.9$</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>$\tau_C$</td>
<td>$-0.1$</td>
</tr>
<tr>
<td>Panel 2</td>
<td>No</td>
<td>$\tau_P$</td>
<td>$-7.0$</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>$\tau_P$</td>
<td>$-7.0$</td>
</tr>
<tr>
<td>Panel 3</td>
<td>No</td>
<td>$\tau_C$ and $\tau_P$</td>
<td>$-14.2$</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>$\tau_C$ and $\tau_P$</td>
<td>$-6.5$</td>
</tr>
</tbody>
</table>

$\alpha$ is capital’s share of output in the Ramsey model. $\alpha = 0.3$ signifies narrow (physical) capital. $\alpha = 0.75$ signifies broad (physical plus human) capital. $\tau_C$ is the tax rate on corporate income; $\tau_P$ is the tax rate on personal income.

shows that the net welfare gain from cutting the personal income tax rate is unaffected by including the corporate depreciation deduction in the model. Panels 1 and 3 show that when the corporate income tax rate is cut, including the corporate depreciation deduction causes the size of the welfare gains to decline dramatically. Both of these results are consistent with the argument that the decline in convergence speed causes the decline in net welfare gains because the short-run losses persist longer.

As emphasis for this point, note that the welfare gains decline if and only if convergence speed declines. Recall that in panels 1 and 3 of Table 2, the convergence speed declines when the corporate depreciation deduction is included in the model. Likewise in panels 1 and 3 of Table 3 the tax cuts produce much smaller welfare gains when the depreciation deduction is included. In contrast, in panel 2 of Table 2, including the corporate depreciation deduction does not affect the convergence speed. Likewise, in panel 2 of Table 3, including the corporate depreciation deduction does not affect the size of the welfare gains. This suggests that the declines in convergence speed cause the declines in welfare gains.

5. Conclusion

As macroeconomic models have evolved, economists have come to an apparent consensus that fiscal policy does not affect long-run growth. Recently, economists have argued that fiscal policy cannot affect transitional growth. For example, one argument is that near the steady state of macroeconomic models taxes on output, firms’ revenue, or household income affect gross saving for given levels of capital and consumption and the steady-state capital stock in equal but offsetting ways. Therefore, these taxes do not affect the economy’s convergence speed.

However, the simple tax structure that led to this result does not reflect many features of actual tax systems that affect behavioral responses to fiscal policy. In the United States, the federal government, most state governments, and some local governments provide tax benefits, such as amortization, depreciation, and interest deductions. This paper uses the depreciation deduction to provide a simple example showing that these features of tax systems do affect convergence speed. In the Solow, Ramsey, and a new growth model, the depreciation deduction has a direct effect on the convergence speed because it reduces required saving. This deduction also induces an indirect relation between the tax rate and the convergence speed: The size of the benefit from the tax deduction increases with the tax rate; therefore, an increase in the tax rate has the same effect as an increase in the deduction.
The depreciation deduction causes convergence speed to decline. The size of the declines in these examples may appear to be small. But the absolute size may not be indicative of economic impact. A tax’s excess burden increases more than in proportion with the tax rate. This property may carry over to convergence speed. Also, a small change in the convergence speed can have a relatively large effect on the welfare effects of fiscal policy because welfare changes occurring early in the transition get more weight than later welfare changes. This is particularly important if the short-run and long-run welfare responses have opposite signs, because it can affect the sign of the net welfare gain, in that case.

In an attempt to gauge the economic importance of tax benefit-induced changes in convergence speed, the paper numerically calculates the global solution to the Ramsey model, and uses the results to measure welfare changes from capital income tax cuts financed by cuts in lump-sum transfers. In these experiments tax cuts cause net welfare to increase. However, the net welfare gains decline substantially when depreciation deductions are included in the model. This suggests that the reduction in convergence speed causes the reduction in net welfare gains because the short-run losses last longer.

For understandable reasons, macroeconomists have tended to include simplified tax structures in their models. The example of the depreciation deduction, however, suggests that the simplifications are not always innocuous. Overly simplified tax structures can lead to measurement error in transitional growth rates, and could affect predictions about the welfare effects of fiscal policy.10

Appendix A

Using Equation 8, the Hamiltonian for the household’s choice problem is

\[
H = e^{-\rho t} \psi(\tau - \theta) \left[ \frac{c^\star - 0}{1 - \theta} + \mu[(1 - \tau_p)(1 - \alpha \tau_c)k^\star - ((1 - \tau_p)(1 - d\tau_c)\delta - (n + g))k - c] \right],
\]

where \(\rho\) is the rate of time preference, \(\theta\) is the elasticity of marginal utility, and \(\mu\) is the costate variable (the value at time zero of an additional unit of capital at time \(t\)). Equation A1 assumes \(L(t) = L(0)e^{\eta t}\) and \(L(0) = 1\). The Maximum Principle and the costate equation imply

\[
\dot{c} = \frac{c}{\theta} \left[ (\alpha(1 - \tau_p)(1 - \alpha \tau_c)k^{\star - 1} - (1 - \tau_p)(1 - d\tau_c)\delta) - [\rho + \theta g],
\]

which is Equation 9.

Appendix B

To get the convergence speed from the linearized Ramsey model, \(\beta_L\) in the text, expand Equations 8 and 9 around \(k^\star\). This gives

\[
\begin{pmatrix}
\dot{k} \\
\dot{c}
\end{pmatrix} \approx \begin{pmatrix}
\frac{\alpha(1 - \tau_p)(1 - \alpha \tau_c)k^{\star - 1} - ((1 - \tau_p)(1 - d\tau_c)\delta + n + g)}{c_e} & -1 \\
\frac{\alpha(\alpha - 1)(1 - \tau_p)(1 - \alpha \tau_c)k^{\star - 2}}{c_e} & 0
\end{pmatrix} \begin{pmatrix}
k - k^\star \\
c - c^\star
\end{pmatrix},
\]

To get the eigenvalues, use Equations 10 and 11 to eliminate \(c^\star\) and \(k^\star\) from Equation B1, then use the condition

\[10\] The referee points out another simplification necessary for convergence speed to be independent of income taxes, namely Cobb-Douglas production. In this case, the marginal product of capital is proportional to its average product. In models with less restrictive technologies (e.g., Constant Elasticity of Substitution), income taxes affect convergence. One might argue that the restriction is justified because it greatly simplifies solving the model. However, the same cannot be said for taxes: The analysis in this paper shows it is easy to solve models including realistic tax structures; the simulations indicate the importance this has for the welfare effects of taxes.
to solve for \( k \). The roots alternate in sign. The negative root is the stable root. \( \beta_z \) is the absolute value of the stable root. To get a global expression for the convergence speed, substitute in Equation 8 by \( k \), set the resulting value of \( \beta zk \) to zero, and solve this for the steady-state value

\[
(1 - \tau p)(1 - \alpha \tau c) = \left(1 - \tau p(1 - dtc)\delta + n + g + \frac{c^*}{k^*}\right) \frac{1}{k^{2s-1}}.
\]

Substitute this back in \( \dot{k}/k \) and write the result in terms of \( \log[k/k^*] \). This gives

\[
\dot{k}/k = \left(1 - \tau p(1 - dtc)\delta + n + g + \frac{c^*}{k^*}\right)e^{1/3 \log[k/k^*]} - (1 - \tau p)(1 - dtc)\delta - n - g - e^{log[k/k^*]}.
\]

Therefore,

\[
\beta z = \frac{d(\dot{k}/k)}{d(\log[k])} = (1 - \alpha)\left(1 - \tau p(1 - dtc)\delta + n + g + \frac{c^*}{k^*}\right)(k/k^*)^{2s-1} - \frac{c^*}{k^*} + c'(k).
\]

To get numerical values for \( \beta z \), the differential equations for optimal growth in \( k \) and \( c \) must be solved numerically. The time paths for these are then substituted in Equation B4 to get numerical solutions.

**Appendix C**

Ortigueira and Santos (1997) examine convergence speed in a model with endogenous human capital. They do not include tax benefits, such as depreciation, so taxes do not affect convergence speed in their model. However, if a depreciation deduction is included in the Ortigueira/Santos model, taxes do affect the model’s convergence speed. To see this, add a deduction for depreciation of non-human physical capital to their model. Growth in physical and human capital, respectively, is

\[
\dot{k} = [k^s(ah)^{\gamma-3} - \delta k](1 - dt) - nk - c,
\]

\[
\dot{h} = \Delta(1 - u - l)h - \Phi h,
\]

where \( u \) is the proportion of time skills used in final goods production, \( h \) is human capital, \( l \) is the proportion of time spent in leisure, \( \Delta \) is an educational productivity parameter, and \( \Phi \) is depreciation in human capital.

Since human capital grows endogenously, variables in this model are measured in labor units, so the discount rate is \( \rho - n \). The Hamiltonian is

\[
H = e^{(\rho - n)\theta} \left\{\frac{c^*(lh)^{\gamma-3} - \delta k}{1 - \theta} + \eta_1[(k^s(ah)^{\gamma-3} - \delta k)(1 - dt) - nk - c] + \eta_2(\Delta(1 - u - l)h - \Phi h)\right\}.
\]

The parameter \( \gamma \) governs substitution between consumption and leisure, and \( \eta_1 \) and \( \eta_2 \) are the costate variables for \( k \) and \( h \). The first-order conditions for \( c, lh, \) and \( uh \), and the costate equations for \( k \) and \( h \) are

\[
[c^*(lh)^{\gamma-1}]^{\gamma-1}c^*(lh)^{\gamma-\gamma} = \eta_1, \quad \text{(C3a)}
\]

\[
[c^*(lh)^{\gamma-1}]^{\gamma}(1 - \gamma)c^*(lh)^{\gamma} = \eta_2\Delta, \quad \text{(C3b)}
\]

\[
\eta_1(1 - \alpha)k^s(ah)^{\gamma-3}(1 - dt) = \eta_2\Delta, \quad \text{(C3c)}
\]

\[
\dot{\eta}_1 = -\eta_1[k^s(ah)^{\gamma-1} - \delta](1 - dt) + \eta_1\rho, \quad \text{(C3d)}
\]

\[
\dot{\eta}_2 = -[c^*(lh)^{\gamma-1}]^{\gamma}(1 - \gamma)c^*(lh)^{\gamma}l - \eta_1(1 - \alpha)k^s(ah)^{\gamma-3}(1 - dt)u + \eta_2[\Delta(1 - u - l) - \Phi] + \eta_2(\rho - n). \quad \text{(C3e)}
\]

Using Equations C3b and C3c in Equation C3e gives

\[
\dot{\eta}_2 = \eta_2[\Phi + \rho - n - \Delta]. \quad \text{(C4)}
\]

Use the definition \( x = h/k \) in Equation C3c and take log time derivatives

\[
\dot{u} = \frac{u}{x} \left[\frac{\dot{\eta}_1}{\eta_1} - \frac{\dot{\eta}_2}{\eta_2} - \frac{\alpha}{x}\right].
\]

Using Equations C3d and C4 in Equation C5 gives
log_{10}\left(\frac{\Delta - n}{\Phi} - (1 - \tau)(\tau x^{1-\theta}u^{1-\theta} - d\delta) + n \cdot \frac{x^2}{\delta}\right). \tag{C6}

Equation C6 shows that the larger the net productivity of human capital in the production of human capital ($\Delta - \Phi$), the larger the growth of skill used in final goods production. The middle term in square brackets is the after-tax return on physical capital net of depreciation. The larger this return is, the lower is growth in skill used in final goods production. An increase in the net after-tax return on physical capital leads to an increase in the stock of physical capital, which increases the productivity of human capital. This tends to increase the growth of skills used in final goods production. At the same time, the opportunity cost of investment in human capital increases, which leads to a decrease in the human capital investment. The latter effect dominates, so growth in skill used in final goods production declines. This is important to the interpretation of the convergence speed in this model.

Log linearizing (Eqn. C6) around the steady state gives

$$\dot{u} \equiv \frac{u^*}{a} \left[-\sigma(1 - dw)(1 - \alpha)x^{1-\theta}u^{1-\theta} - \sigma(1 - dw)(1 - \alpha)x^{1-\theta}u^{1-\theta} \frac{dx}{du} - \alpha \frac{d(x)}{du}\right] (u - u^*). \tag{C7}
$$

The first term in square brackets is the after-tax marginal product of capital. The convergence speed depends on this term. To see this, use Ortega and Santos’s Lemma A1. After multiplication through by $u/a$, the coefficient on $(u - u^*)$ in Equation C7 reduces to (this is their Eqn. A14)

$$\lambda = -(1 - dw)(1 - \alpha)x^{1-\theta}u^{1-\theta}. \tag{C8}
$$

This is the decrease in growth of skill used in final goods production due to a small increase in time at work, near the steady state. If the absolute value of Equation C8 increases, the economy converges faster because a larger decrease in the growth in skill used in final goods hastens achievement of the steady-state condition, $u = 0$. The absolute value of Equation C8 depends on the return on physical capital because a larger return ceteris paribus tends to decrease the acquisition of skill, as described earlier.

Now use the steady-state value of Equation C3d in Equation C8. This gives

$$\lambda = -\frac{1}{a} \left[\rho + \delta(1 - dw)\right]. \tag{C9}
$$

Use the steady-state value of Equation C4 in Equation C9. The negative of this is the convergence speed

$$\beta_L = \left(\frac{1}{a}\right)(\delta(1 - dw) + n + \Delta - \Phi]. \tag{C10}
$$

Equation C10 shows that including tax benefits in a standard endogenous growth model causes taxes to affect the convergence speed in the vicinity of the steady state, as does in the Solow and Ramsey models.

Appendix D

$U_0$ is the utility of lifetime consumption in the original steady state:

$$U_0 = \frac{c_0^{1-\theta}}{[\rho - n - g(1 - \theta)][1 - \theta]}, \tag{D1}
$$

where $c_0$ is the original, pretax cut value of consumption. $U_1$ is utility after the tax cut:

$$U_1 = \int_0^\infty e^{-k(t)}c(k(t))^{1-\theta} \frac{1}{1 - \theta} dt. \tag{D2}
$$

In (D2), $k(t)$ and $c(k(t))$ are the optimal time paths of capital and consumption, respectively. $R[k(s)]$ is the time path of the after-tax return received by the household, given by

$$R[k(s)] = \sigma(1 - \tau_c)(1 - \alpha x) c(k(s))^{1-\theta} - (1 - \tau_c)(1 - dx) \delta. \tag{D3}
$$

The time paths are derived from the differential equations $k(t) = f_1[k(t), c(t)]$ (see Eqn. 8) and $\dot{c}(t) = f_2[k(t), c(t)]$ (see Eqn. 9) in the following way. After the tax rate is cut, use these two differential equations to solve $c'(k) = c(t)/k(t)$ for the policy function $c = c(k)$. Then substitute the policy function into $k(t) = f_1[k(t), c(t)]$ and solve for the time path $k(t)$ (the initial condition is $k(0) = k_0^*$, where $k_0^*$ is the steady-state value of $k$ before the tax cut). Substitute the time path $k(t)$ into $R[k(s)]$. Also use $k(t)$ to solve $c(t) = f_2[k(t), c(t)]$, where the policy function is used to find the initial condition $c(0) =$

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11 Mulligan and Sala-i-Martin (1993) describe the derivation of the policy function and the time paths.
\( c(k_0) \). The solution to the last differential equation is the optimal time path \( c(k(t)) \). Use this and \( R[k(s)] \) to compute \( U_l \). Finally, the equivalent variation is the percentage decrease in \( c_0 \) required to equate \( U_0 \) with \( U_l \). Since taxes are decreased in these experiments, \( U_l \) is larger, and the equivalent variations are negative.

References