Microfinance and Dynamic Incentives

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February 3, 2015

Abstract

Dynamic incentives, where incentives to repay are generated by granting access to future loans, is one of the methodologies used by microfinance institutions (MFIs). In this paper, I present a model of dynamic incentives where lenders are uncertain over how much borrowers value future loans. Loan terms are determined endogenously, and loans become more favorable as the probability of default becomes lower. I show that in all equilibria but one all borrowers, including the most patient ones, eventually default. I then consider an extension where borrowers can take loans from several lenders, double-dipping. Qualitatively, properties of equilibria with and without double-dipping are similar. In absolute terms, when borrowers are credit-constrained double-dipping equilibrium loans have to be more favorable to outweigh increased gains from default.

JEL classification: C73, D82, O12, O16
Key Words: Microfinance, unsecured credit, dynamic incentives, strategic default, double-dipping.

1 Introduction

As of December 2010, there were 3,652 microfinance institutions (MFIs) reaching more than 200 million people, most of whom were among the poorest when they took their first loan (Maes and Reed, 2012). This is remarkable given the plethora of obstacles that, for a long time, have kept formal credit institutions away from financing the poor. Adverse selection, moral hazard, lack of collaterizable assets, absence of enforcement mechanisms, and high costs should have made microfinance nothing if non-existent, or at least subsidized. As an example, during pre-Grameen times in Bangladesh, loans targeting poor households by traditional banks had repayment rates as
low as 51.6% in 1980, down to 18.8% by 1988-89, and were heavily subsidized by the government (Khalily and Meyer, 1993).

The microfinance methodologies that are responsible for microcredit success are well-known in the literature. They are group lending (where a small group of neighbors is jointly liable for individual loans), dynamic incentives (using access to future loans as incentives to repay the current one), regular-repayment schedules and using collateral substitutes (Morduch, 1999). Among the four, group lending has received the most attention, as it is an innovative and clever way to alleviate the problems of adverse selection and moral hazard. More recently, however, there has been a shift in focus, away from group lending and towards other aspects of microfinance loans. Fischer and Ghatak (2010) cite several factors responsible for this change, such as a decreased reliance on group lending by several major MFIs, as well as, a growing recognition of costs associated with joint liability (see also Banerjee, 2013, and references therein).\(^1\)

The focus of this paper is dynamic incentives in the environment where no other enforcement mechanism is available. According to a standard repeated-game argument, as long as a borrower is sufficiently patient, the threat of limiting the borrower’s access to future loans can serve as a punishment strong enough to deter the default. The contribution of this paper is that it demonstrates limitations of the dynamic incentives methodology despite the presence of sufficiently patient borrowers and full exclusion of defaulters.

The model is an infinitely repeated game, where a borrower faces an ex-post moral hazard, and default leads to a full exclusion of the defaulter. As in Ghosh and Ray (2001), parties cannot commit to contracts longer than one period. In a given period, loan terms are endogenously determined by the (correctly anticipated) probability of default. A lower probability of default in a given period means larger loans on more favorable terms. I assume that lenders are uncertain as to how much a borrower values future loans. I model it as uncertainty over the borrower’s discount factor, $$\delta$$. However, it can be also modeled as uncertainty about the borrower’s outside option (those with a lower outside option value access to future loans more), or as uncertainty over the borrower’s productivity growth (those with higher growth rates have a higher value of future loans).

The model has multiple equilibria. There is an efficient equilibrium where the risk of default is eventually eliminated: the borrower’s types with sufficiently high patience repay every period, and all less patient types eventually default and leave the game. Notably, such an equilibrium is unique. All other equilibria entail default by all types, including the most patient ones. First, there is a continuum of equilibria where loan terms are unfavorable in the beginning and deteriorate even further with time. When future loans are not attractive, most types quickly default, which, in turn, rationalizes lenders providing unfavorable loans. The second class of inefficient equilibria

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\(^1\)Grameen eliminated explicit joint liability by 2002. Rhyne (2002) documents how due to increased competition, new MFIs in Bolivia, such as Caja los Andes, relied exclusively on individual loans to gain a market advantage over group loan competitors, such as BancoSol or Prodem. Currently BancoSol has switched to individual credit technology (see http://www.mixmarket.org/mfi/bancosol, accessed January 2014) and Prodem has begun to use a mix of individual and solidarity loans (see http://www.mixmarket.org/mfi/prodem-ffp, accessed January 2014). Nonetheless, de Quidt et al. (2012) report that in their sample of 715 MFIs as many as 54% of loans are made under solidarity lending, thus indicating that group loans are still widely used.
allows for a temporary improvement in loan terms. Loan terms improve at first, but eventually begin to deteriorate and all types default. In this scenario, types with intermediate patience delay defaulting until they can gain access to larger loans. In equilibrium, lenders correctly anticipate this. As soon as more types find it optimal to default, loan terms begin to worsen. This, in turn, destroys incentives to repay for the more patient types, which worsens loan terms even further. Eventually, all types will prefer to default regardless of their patience.

The existence of inefficient equilibria seems counterintuitive at first. With time, lenders are bound to learn that the borrower is sufficiently patient which should remove the risk of default. The intuition is misleading, however, since even the most patient types can find it optimal to default when future loans are expected to be small or otherwise unfavorable. Bond and Rai (2009) mention several cases where worries about MFIs’ financial solvency, for example, because of an exogenous hike in default rates, rapidly destroy everyone’s incentives to repay. While financial solvency is not an issue in my model, the key point remains valid. Dynamic incentives are not only about the borrower’s patience, but also about the expected value of future loans.

A rapid expansion of micro-credit has resulted in a recent phenomenon called “double-dipping”, in which borrowers take loans from several MFIs (McIntosh and Wydick, 2005, Guha and Chowdhury, 2013). To study the effect of double-dipping on borrower’s incentives, I develop an extension of the main framework where the borrower can take loans from multiple (two) lenders. I show that equilibrium structure remains the same: all equilibria but one lead to eventual default of all borrowers. I further show that when a borrower is credit-constrained, other things being equal, double-dipping (DD) equilibria lead to more favorable loans and a lower default rate than corresponding single-dipping (SD) equilibria. Intuitively, availability of the second loan has two opposite effects on repayment incentives: it increases gains from default and, at the same time, it increases gains from repayment and the value of future loans. For credit-constrained borrowers the former effects dominates. A DD-equilibrium, therefore, requires more favorable future loans to offset higher incentive to default.

2 Literature Review

That unsecured debt, such as microcredit to the poor or sovereign debts, can be self-enforced in the case of repeated interactions was first formalized by Eaton and Gersovitz (1981) and further developed by Eaton et al. (1986), Grossman and Van Hyuck (1988), and many others. Bulow and Rogoff (1989) demonstrated some limitations of repeated interactions. If borrowers can save, then the rate of loan growth has to be higher than the interest rate, which eventually would become unsustainable. Among more recent papers, Albuquerque and Hopenhayn (2004) develop a model of optimal long-run lending contracts, when the debt repayment cannot be perfectly enforced. Differently from my paper, Albuquerque and Hopenhayn (2004) have no asymmetric information between the investor and the entrepreneur. Default can be avoided entirely by properly structuring the long-term debt contract.

Ghosh and Ray (2001) apply a dynamic incentives argument to the case of unsecured micro-
credit loans where lenders do not communicate and borrowers’ repayment history is not publicly observable. There are two types of borrowers: myopic borrowers who always default, and borrowers with a positive discount factor, who do not default in equilibrium. While the history is not public, an individual lender can distinguish between new and old (returning) borrowers. If the borrower repays an initial (new) loan to a lender, the lender is willing to provide a better (old) loan to the borrower. My paper differs from Ghosh and Ray (2001) in that in my paper the borrower’s history is observable by all lenders. It matters, as it removes the exclusive link between the old borrower and his lender. In Ghosh and Ray (2001), incentives to repay come from the fact that there is exactly one lender who has better information about the returning borrower and is willing to provide a better loan that is unavailable elsewhere. Another aspect where my paper is different is that I characterize all equilibria including non-stationary ones.

In the literature, dynamic incentives are often considered in combination with progressive lending. As empirical research shows, microfinance contracts typically structure the loans in such a way that the starting loans are small but increase with each cycle. Robinson (2001) describes 18 loan programs in different countries and shows that 12 of them used progressive lending with amounts rising up to 200% of the initial loan. Armendáriz and Morduch (2005) show that Grameen Bank provides a continuing and increasing series of loans to its clients. Based on a survey of 424 women in Karnataka, India, Kumar (2012) reports that the sixth consecutive loan could be as large as 684% of the initial loan.

From a theoretical point of view, progressive lending reinforces dynamic incentives, as a borrower who defaults on the current loan gives up the possibility of a larger loan(s) in the future. Ghosh and Ray (2001) can be seen as an example of a progressive lending model as new patient borrowers repay initial small loans to reveal their patience and get access to better loans in the future. Egli (2004) develops a model, where it is divisibility of a project that allows for equilibria where “bad” borrowers repay the first-period loans with non-zero probability. My paper is different in that in my framework the exclusive relationship between a borrower and a lender — either due to long-term contract as in Egli (2004) or due to asymmetry of lenders’ information about the borrower as in the O-phase in Ghosh and Ray (2001) — is impossible. Current loan terms are fully determined by the current probability of default. It has a direct impact on progressive lending, as small and expensive loans in my model invariably imply a higher probability of default. That can be only rationalized by worse (or eventually worse) loans in the future, as otherwise the default rate would have to be lower. As a consequence, the only efficient equilibrium in my model has the largest initial loan among all equilibria.

Finally, many papers study double-dipping and its effect on borrowers’ incentives. Due to an expansion of micro-credit, borrowers often have access to multiple loans from different lenders (McIntosh and Wydick, 2005, McIntosh et al., 2005, Armendáriz and Morduch, 2005). While double-dipping does not necessarily prevent lenders from excluding defaulting borrowers (McIntosh and Wydick, 2005), it does weaken borrowers’ incentives to repay. Guha and Chowdhury (2013) provide a model with double-dipping where borrowers face ex-ante moral hazard, and taking more than one loan is always inefficient and always leads to default. It shows that increased competition
can have opposing effects on the borrower’s well-being and interest rates. My paper differs from Guha and Chowdhury, in that it is a model of ex-post moral hazard and in that taking the second loan can be efficient, when the single-loan borrower is credit-constrained.

3 Model

3.1 Setup

Consider infinitely-repeated interactions between a risk-neutral borrower and risk-neutral lenders. The borrower has access to a project with return $F(K)$ but needs external funds to produce output. Function $F$ satisfies Inada conditions, that is, it is an increasing, concave function such that $F(0) = 0$, $F(\infty) = F'(0) = \infty$ and $F'(\infty) = 0$. The borrower discounts future payoffs with discount factor, $\delta$, which is the borrower’s private information and is unknown to lenders. Lenders have a prior that $\delta$ is distributed in $0 \leq [\delta_{\text{min}}, \delta_{\text{max}}] \leq 1$ with cdf $\Phi(\delta)$. I assume that $\Phi(\delta)$ is differentiable, and has a strictly positive density on $[\delta_{\text{min}}, \delta_{\text{max}}]$.

There is no production uncertainty, however, lenders face risk due to the borrower’s ex-post moral hazard. The borrower, once the output is produced, can choose to default on the received loan. I assume that the borrower has no collaterizable assets and that no enforcement by legal authorities is available. Instead, lenders rely on dynamic incentives as an enforcement mechanism. As long as the borrower does not default, lenders are willing to provide new loans. If the borrower defaults, however, no future loans will be given by any lender.

Time is discrete. The borrower cannot save, has no assets and, therefore, needs to borrow capital every period to finance the project. In period $t$, assuming no prior defaults, the borrower receives a loan $(K_t, R_t)$ where $K_t$ is the loan size, and $R_t$ is the interest rate. Except for Section 4.1, I assume that the borrower can take one loan only. I further assume that the borrower and the period $t$ lender cannot commit to a longer than one period contract. Upon receiving the loan, the borrower produces output $F(K_t)$ and then decides whether to repay the debt or not. In the former case, the borrower pays back $R_t K_t$ and keeps the rest, i.e. $F(K_t) - R_t K_t$, to himself. If the borrower defaults the borrower keeps the entire output, $F(K_t)$, to himself. Thus, the gain from default is $R_t K_t$. If the borrower defaults he cannot receive any loans in the future. If the borrower repays the loan, the game continues into period $t + 1$.

In period $t$, lenders believe, correctly in equilibrium, that the probability of repayment is equal to $q_t$. I assume that loan terms, $(K_t, R_t)$, are determined by $q_t$, that is $K_t = K(q_t)$ and $R_t = R(q_t)$. The functions $K(\cdot)$ and $R(\cdot)$ are determined based on lenders’ mission and the market structure, as will be explained later. I assume that $K(0) = 0$ and

**Loan Term Monotonicity (LTM):** Loan size, $K(q)$, borrower’s gain from repayment, $F(K(q)) - R(q)K(q)$, and borrower’s gain from default, $R(q)K(q)$, are strictly increasing and continuously differentiable functions when $q > 0$.\(^2\)

\(^2\)Banerjee (2001) reports studies indicating that bigger loans are associated with lower interest rates which is consistent with an assumption that $K(q)$ is an increasing function. Evidence supporting the assumption that $KR$ is
When the LTM is satisfied, a higher $q_t$ is beneficial for the borrower, since the borrower receives a larger loan and has more funds left after repayment. Furthermore, the LTM allows dynamic incentives to be combined with progressive lending. Since larger loans are more profitable, allowing future access to larger loans reinforces borrower’s incentives to repay. At the same time, the LTM also assumes that the gains from default on larger loans, which are principal and interest payments, are larger. Thus, under the LTM not only are larger loans more beneficial for a repaying borrower, but also they are more tempting to default upon. To keep notations simple, I will often write the loan size and interest rate in period $t$ as $K_t$ and $R_t$, though it should be remembered that they are functions of $q_t$.

The functions $K(\cdot)$ and $R(\cdot)$ are determined based on the lenders’ mission in the case of non-profits and market structure in the case of for-profits, as shown below.

- **Non-profit borrower’s welfare maximizing MFI:** Assume that an MFI’s opportunity cost of funds is $r$. Given $q_t$ an MFI’s expected payoff net of opportunity cost should be equal to zero:
  \[
  E_t(R_tK_t) = q_tR_tK_t + (1 - q_t)0 - rK_t = 0 \quad \text{so that} \quad R_t = R(q_t) = r/q_t. \tag{1}
  \]
  The loan size then is determined as to maximize the borrower’s payoff conditional on payback, that is $K_t = \arg \max K F(K) - R_tK$. Since $R_t$ is a decreasing function of $q_t$, it follows from the envelope theorem that $F(K_t) - R_tK_t$ is an increasing function of $q_t$. Thus for any $F$, such that $R_tK_t(= F'(K_t)K_t)$ is strictly increasing, the LTM is satisfied. An example of such function is $F(K) = AK^\alpha$.\(^3\)

- **For-profit competitive lender:** This case is similar to the non-profit MFI case above. The loans provided by a competitive lender should yield zero (net of opportunity cost) expected payoff. Thus $R(\cdot) = r/q_t$. The loan size is the same as in the case of non-profit MFI because perfect competition forces lenders to maximize the borrower’s welfare. If not, another lender would be willing to provide $K_t$ that maximizes the borrower’s repayment payoff.

- **Non-profit outreach maximizing MFI:** In my model, all borrowers are served as long as they do not default. In particular, all borrowers receive a loan at $t = 0$. In this sense, assuming that MFIs solely maximize an outreach does not put restrictions on loan terms. We will return to outreach-maximizing MFIs later when comparing equilibrium outcomes.

- **Myopic for-profit monopolistic lender:** Assume that for a given $R_t$ the borrower’s demand for capital is given by maximizing the borrower’s after-repayment profit: $\max K F(K) - R_tK$. The borrower’s inverse demand for capital then is $F'(K_t) = R_t$. Given the monopolist’s belief of repayment, $q_t$, the maximization problem is $\max K q_tF'(K)K - rK$, where $r$ is the

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\(^3\)Formally, that $KF'(K)$ is an increasing function of $K$ is equivalent to $-KF''(K)F(K) < 1$. That means that function $F$ cannot be too concave, at least on interval $[0, K(1)]$ so that its marginal productivity does not decline too quickly.
opportunity cost of funds. When the solution is interior, both $K_t$ and $F'(K_t)K_t(=R_tK_t)$ are increasing functions of $q_t$. The borrower’s repayment payoff, $F(K_t) - F'(K_t)K_t$, is an increasing function of $q_t$ due to concavity of $F(\cdot)$.

- **Credit constraint:** In the examples above, the marginal product is equal to the interest rate. Empirical evidence, however, indicates that microloan recipients have their marginal productivity above the interest suggesting some credit constraints. The LTM allows for credit-constrained loans. For example, when $R$ is constant and $K(q)$ is a strictly increasing function such that $F'(K(1)) \geq R$ the LTM is satisfied.

Let $\varepsilon(q_t)$, or for brevity $\varepsilon_t$, denote $K_tR_t/F(K_t)$, so that $\varepsilon_t$ is equal to the share of the output that the borrower must return to the lender. Then the repayment payoff in period $t$ can be written as

$$F(K_t) - R_tK_t = F(K_t)(1 - \varepsilon_t).$$

When the loan terms are determined so that the borrower’s payoff, conditional on repayment, is maximized, then $\varepsilon_t$ is simply an elasticity of the production function at point $K_t$. In the special case of $F(K) = AK^\alpha$, it is constant, that is $\varepsilon_t = \alpha$.

I assume that $\delta_{\text{max}} > \varepsilon(q_t)$ for any $q_t$ in $[0, 1]$ and that $\varepsilon(1) > \delta_{\text{min}}$. The former assumption guarantees the existence of the equilibrium where some types never default. Otherwise, there is no type patient enough for dynamic incentives to work. The latter assumption guarantees that there are also impatient types that will default. In the case of production function $F(K_t) = AK^\alpha$ and a zero-profit lender (whether a borrower’s welfare maximizing MFI or a competitive for-profit) these assumptions are satisfied as long as $\delta_{\text{min}} < \alpha < \delta_{\text{max}}$.

Given lenders’ beliefs $\{q_t\}$, the borrower’s maximization problem is to determine time $0 \leq T \leq \infty$ to default ($T = \infty$ means that the borrower never defaults) that maximizes

$$\max_T \left\{ \sum_{t=0}^{T-1} (1 - \varepsilon_t)F(K_t)\delta^t + F(K_T)\delta^T \right\}. \tag{2}$$

The optimal default time depends on $\delta$, and I will denote the solution to (2) as $T(\delta)$.

In the equilibrium of the model the following conditions should hold. The borrower correctly anticipates lenders’ beliefs $\{q_t\}$ which determine the sequence of loan terms $\{K_t, R_t\}$. Given $\{K_t, R_t\}$, the borrower chooses the optimal time, $T(\delta)$, to default. Lenders, given the borrower’s strategy $T(\delta)$ and history of the game, correctly estimate the probabilities of payback $\{q_t\}$, which determines loan terms $\{(K_t, R_t)\}$.

**Definition 3.1.** For a given $K(q)$ and $R(q)$ the equilibrium of the model is a sequence of beliefs $\{q_t\}_{t=0}^\infty$ and borrower’s strategy $T(\delta)$ such that

i) Given $\{q_t\}$, the borrower’s optimal default time is given by $T(\delta)$, which is a solution to (2);

ii) Given borrower’s strategy $T(\delta)$ and prior $\Phi(\delta)$ for each period $t$ lenders correctly estimate the probability of payback $q_t$, which determines loan terms $(K_t, R_t)$ in period $t$.

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$^4$See e.g. de Mel et al. (2008) and McKenzie and Woodruff (2008). Karlan et al. (2012) report mixed evidence, nonetheless, their conclusion is that firms in the sample are credit constrained.
3.2 Complete Information Case

As a useful benchmark, I first solve the model where $\delta$ is common knowledge. Equilibrium in the model with complete information is defined similarly to Definition 3.1. A borrower plays his best response given lenders’ beliefs $\{q_t\}$. Lenders’ beliefs are correct, given the borrower’s strategy. I will restrict my attention to pure-strategy equilibria, as they are more relevant to the incomplete-information framework where a measure of indifferent types is equal to zero. The probability of default in a given period then is either 0 or 1. Only two scenarios can occur in equilibrium: a riskless scenario where $q_t = 1$ for every period and no default; and a no-loan scenario.

First, consider sequence $q_t \equiv 1$. Since $\{q_t\}$ is constant, the borrower will either default immediately or will not default at all. The latter is optimal when

$$F(K(1)) \leq \sum_{t=0}^{\infty} \delta^t (1 - \varepsilon(1)) \cdot F(K(1)) = (1 - \varepsilon(1)) \cdot F(K(1)) \frac{1}{1 - \delta} \iff \delta \geq \varepsilon(1).$$

Thus when $\delta \geq \varepsilon(1)$, beliefs $q_t \equiv 1$ are rational. Naturally, in order for dynamic incentives to work, the borrower should be sufficiently patient. The no-loan beliefs $q_t \equiv 0$ is also an equilibrium since the borrower would default on any loan received in period $t$, regardless of $\delta$, and, therefore, expectations $q_t \equiv 0$ are rational.5

3.3 Incomplete Information Case. Uniqueness of Efficient Equilibrium

We first solve for the borrower’s optimal time to default given lenders’ beliefs $\{q_t\}$. Let $K_t = K(q_t)$ and $K_{t+1} = K(q_{t+1})$ be loan levels that correspond to $q_t$ and $q_{t+1}$. Let $\delta_t$ be a discount factor such that the borrower with $\delta_t$ is indifferent between defaulting at moment $t$ and moment $t + 1$:

$$F(K_t) = F(K_t)(1 - \varepsilon(q_t)) + \delta_t F(K_{t+1}),$$

which is equivalent to

$$\delta_t = \varepsilon(q_t) \frac{F(K_t)}{F(K_{t+1})}. \tag{4}$$

Equation (3) can be viewed as an analogue of the Euler equation. The Euler equation states that the agent is indifferent to reallocating an infinitesimal amount of consumption between periods $t$ and $t + 1$. In my model, the choice variable in period $t$ is binary. Equation (3) states that the agent with $\delta = \delta_t$ is indifferent between default at $t$ and $t + 1$.

5No other equilibrium in pure strategies exists. Since even the most favorable stream of loans is not sufficient to deter the default if a borrower’s $\delta < \varepsilon(1)$, equilibrium with positive loans can exist only if $\delta \geq \varepsilon(1)$.

Sequence $q_0 = \cdots = q_t = 1$ and $q_s = 0$ when $s > t$ is not an equilibrium. The borrower will default at $t$ making $q_t = 1$ an incorrect belief. Thus, there exists $t$ such that $q_t = 0$ and $q_{t+1} = 1$. This is not an equilibrium either. A lender in period $t$ could offer a riskless loan based on $q_t = 1$, which would be repaid by the borrower. This is because it is strictly more profitable to default at $t+1$ than at $t$ if $\delta > \varepsilon(1)$, and is just as profitable if $\delta = \varepsilon(1)$. This means $q_t = 0$ is an incorrect belief. Thus, no other pure-strategy equilibrium exists.
From (3), all types with \( \delta < \delta_t \) prefer default at \( t \) to default at \( t + 1 \); all types with \( \delta > \delta_t \) will prefer default at \( t + 1 \) to default at \( t \).\(^6\) Proposition 3.2 below shows that in equilibrium, \( \{\delta_t\} \) has to be a strictly increasing sequence and that for a borrower with \( \delta_t < \delta < \delta_{t+1} \) it is optimal to default at \( t + 1 \).

**Proposition 3.2.** In any equilibrium \( \{\delta_t\} \) is a strictly increasing sequence. The borrower’s optimal strategy \( T(\delta) \) is then defined as follows: for types with \( \delta \in (\delta_t, \delta_{t+1}) \) it is optimal to default at period \( t + 1 \).

The intuition is straightforward. Let \( \pi_t(\delta) \) denote the utility of the borrower with discount factor \( \delta \) from default at \( t \). If \( \delta \)-sequence is strictly increasing then

\[
\pi_0(\delta) < \pi_1(\delta) \cdots < \pi_t(\delta) < \pi_{t+1}(\delta) > \cdots
\]  

(5)

Indeed, since \( \{\delta_t\} \) is increasing, \( \delta > \delta_t \) implies \( \delta > \delta_{\tau} \) for any \( \tau \leq t \). Therefore, \( \pi_{\tau}(\delta) < \pi_{\tau+1}(\delta) \) for any \( \tau \leq t \). Similarly, \( \delta < \delta_{t+1} \) implies \( \delta < \delta_{\tau} \) for any \( \tau \geq t + 1 \) and, therefore, \( \pi_{\tau}(\delta) > \pi_{\tau+1}(\delta) \) for any \( \tau \geq t + 1 \).

The reason why \( \delta \)-sequence has to be increasing is as follows. The value of \( \delta_t \) reflects the intertemporal comparison between loans in periods \( t \) and \( t + 1 \). Higher (lower) values of \( \delta_t \) mean that the loan in period \( t \) is more (less) favorable than the loan in period \( t + 1 \). When \( \delta_t > \delta_{t+1} \) it means, roughly speaking, that loans in periods \( t \) and \( t + 2 \) are better than the loan at \( t + 1 \). But then no one will default at \( t + 1 \). Less patient borrowers default at \( t \). Those who were patient enough to wait until \( t + 1 \), will prefer to wait until a better loan at \( t + 2 \). This leads to a contradiction. If no one defaults at \( t + 1 \), then \( q_{t+1} = 1 \) and the loan at \( t + 1 \) is actually the most favorable one.

Next, we derive lenders’ beliefs given the borrower’s strategy. In the beginning of period \( t \), it is known that \( \delta > \delta_{t-1} \) and that borrowers with \( \delta \in (\delta_{t-1}, \delta_t) \) will default. Therefore, the probability of payback is equal to the probability of \( \delta > \delta_t \) conditional on \( \delta > \delta_{t-1} \):

\[
q_t = \text{Prob}\{\delta \geq \delta_t|\delta \geq \delta_{t-1}\} = \frac{1 - \Phi(\delta_t)}{1 - \Phi(\delta_{t-1})},
\]

where \( \delta_{t-1} \) is defined as \( \delta_{\text{min}} \).

Combining the two, we have that the equilibrium dynamic should satisfy the system:

\[
\begin{cases}
\delta_t = c(q_t) \frac{F(K_t)}{F(K_{t+1})}, & t \geq 0 \\
q_{t+1} = \frac{1 - \Phi(\delta_{t+1})}{1 - \Phi(\delta_t)}, & t \geq -1
\end{cases}
\]

(6)

Given \( q_0 \), one can unravel the equilibrium dynamic as follows.\(^7\) From the second equation, which when \( t = 0 \) becomes \( q_0 = 1 - \Phi(\delta_0) \), we determine \( \delta_0 \). Using the first equation, \( q_0 \) and \( \delta_0 \) determine

\(^6\)Note that (3) does not mean that for \( \delta < \delta_t \) (\( \delta > \delta_t \)) it is optimal to default at \( t \) (at \( t + 1 \)). It does, however, mean that types with \( \delta < \delta_t \) (\( \delta > \delta_t \)) will not default at \( t + 1 \) (at \( t \)), since default at \( t \) (at \( t + 1 \)) gives a strictly higher profit.

\(^7\)It is worth mentioning that even though the value of \( q_0 \) pinpoints the equilibrium dynamic it is not an exogenous parameter. It represents lenders’ beliefs in period 0, which are determined in equilibrium by the rational expectation condition, that is beliefs have to be correct given the borrower’s strategy.
Given \( q_1 \), one can use the second equation of (6) to determine \( \delta_1 \) and so on. Having \( K(q) > 0 \) ensures that \( q_{t+1} > 0 \); however, it is possible to get \( q_{t+1} > 1 \) when solving (6). It happens if \( q_t \) is so high that even most favorable (riskless) loan in \( t + 1 \) is not good enough to guarantee the repayment rate of \( q_t \). For \( \delta \)-sequence, as long as \( 0 \leq q_t \leq 1 \) for all \( t \), the second equation of (6) ensures that \( \delta_t \in [\delta_{-1}, \delta_{\text{max}}] \). Thus, a solution to (6) is an equilibrium iff \( q_t \leq 1 \) for every \( t \). In Proposition 3.4, I will prove that equilibrium exists.

The next proposition compares two equilibria with initial beliefs such that \( q_0 > Q_0 \). It shows that an equilibrium with higher initial beliefs has better loan terms and lower default rates in every period.\(^\text{9}\) Combined with the LTM, it means that for every period, loans along the \( q \)-equilibrium are higher, \( K(q_t) > K(Q_t) \), and are more favorable in terms of the period-\( t \) profit of the repaying borrower: \( F(K(q_t)) - R(q_t)K(q_t) > F(K(Q_t)) - R(Q_t)K(Q_t) \).\(^\text{10}\)

**Proposition 3.3.** Let \( \{(q_t, \delta_t)\} \) and \( \{(Q_t, \Delta_t)\} \) be two equilibria. If \( q_0 > Q_0 \) then \( q_t > Q_t \) and \( \delta_t < \Delta_t \) for every \( t \).

To understand why Proposition 3.3 holds note that \( q_t > Q_t \) means that the \( q \)-loan is more favorable and, therefore, more tempting to default upon. In order for \( q_t \) to be rational, the \( t + 1 \)-loan has to be more favorable as well to offset larger gains from default. Thus, \( q_{t+1} > Q_{t+1} \), which, in turn, implies that \( \delta_{t+1} < \Delta_{t+1} \). Similarly, lower \( \delta_t \) means that the \( t + 1 \)-loan must be more favorable because more impatient borrowers are willing to wait until \( t + 1 \). Again, this means that \( q_{t+1} > Q_{t+1} \) and thus \( \delta_{t+1} < \Delta_{t+1} \).

System (6) has two limit points, \( (0, \delta_{\text{max}}) \) and \( (1, \varepsilon(1)) \), and they correspond to the two equilibria of the complete information model. The former is the no-loan equilibrium, and the latter is an efficient equilibrium where patient types receive riskless loans and never default. Proposition 3.4 shows that equilibria exist and, as expected, any equilibrium trajectory must converge to one of these two limit points. Thus, the two equilibria of the complete information case are the only possible long-run outcomes.

**Proposition 3.4.** Equilibrium trajectories exist and converge to either the efficient steady state \((1, \varepsilon(1))\) or the no-loan steady state \((0, \delta_{\text{max}})\).

The next proposition is one of the main results in the paper, and it highlights the limitations of dynamic incentives. It shows that, while there exists an equilibrium that converges to \((1, \varepsilon(1))\), such an equilibrium is unique. The result seems to be counter-intuitive at first. Proposition 3.2

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\(^8\) For example, consider \( q_0 = 1 \). Then from the second equation of (6) \( \delta_0 = \delta_{\text{min}} \), that is nobody defaults at \( t = 0 \). Since we assumed that \( \varepsilon(1) > \delta_{\text{min}} \), it would imply that \( K_1 > K_0 = K(1) \) and therefore \( q_1 \) has to be greater than 1.

\(^9\) Note that in my model an equilibrium is an infinite sequence that specifies lenders’ beliefs and borrower’s decision for every period \( \{(q_t, \delta_t)\} \). Thus, multiplicity of equilibria means that there exist many equilibrium sequences. However, within a given equilibrium in a given period there is no multiplicity. That is, if equilibrium is \( \{(q^*_t, \delta^*_t)\} \), then the only thing that can happen at period \( t \) is that types with \( \delta^*_{t-1} < \delta^*_t < \Delta^*_t \) default and lenders’ beliefs are \( q^*_t \).

\(^10\) Intuitively, one would expect \( R(q) \) to be a decreasing function of \( q \). However, it is not required for the LTM to be satisfied (see the example of constrained loans on p. 7), which is why I did not assume \( R(\cdot) \) to be decreasing. Nonetheless, in relevant economic settings, such as those considered on pp. 6 and 7, \( R(q) \) is a decreasing function of \( q \), which means that along the equilibrium with higher initial beliefs, the interest rates are lower.
established that in any equilibrium the $\delta$-sequence must be increasing and, therefore, eventually lenders learn that the borrower is sufficiently patient. Dealing with a patient borrower should imply that there is a smaller risk of default and, therefore, lenders should be willing to provide more favorable loans which would give incentives to repay them.

The reason this intuition fails is that the knowledge of the borrower’s high patience does not necessarily remove the risk of default. If the future loans are not favorable, then even the most patient types will prefer to confiscate the loaned amount which, in turn, would justify low confidence and unfavorable loan terms thereby justifying defaults by patient types in the first place. This logic is similar to Bond and Rai (2009) who documented several cases where concerns about MFI’s financial solvency quickly destroyed incentives to repay. In my model, there is no question about lenders’ insolvency. However, the result has the same spirit. When future loans are less valuable, it undermines the power of dynamic incentives.

**Proposition 3.5.** There exists an equilibrium that converges to $(1, \varepsilon(1))$. This equilibrium is unique.

To understand the reason why the efficient equilibrium is unique assume that there are two equilibria, $(q_t, \delta_t)$ and $(q'_t, \delta'_t)$, that converge to $(1, \varepsilon(1))$ and assume that $q_0 > q'_0$. From Proposition 3.3, it follows that $q_t > q'_t$ and $\delta_t < \delta'_t$ for all $t$. Since $\{\delta_t\}$ and $\{\delta'_t\}$ converge to $\varepsilon(1)$ and $\delta_t < \delta'_t \leq \varepsilon(1)$, the mass of types that will default after period $t$ is always larger along the first equilibrium. It has to be the case then that at some point $t$, the probability of default along the second equilibrium is less than or equal to the probability of default along the first one ($q_t \leq q'_t$). This would contradict Proposition 3.3 and, therefore, $(q'_t, \delta'_t)$ cannot converge to $(1, \varepsilon(1))$.

### 3.4 Incomplete Information Case. Types of Equilibria.

The next theorem characterizes dynamic properties of the equilibria. Figures 1, 2 and 3 provide examples of possible equilibrium dynamics. It turns out that, in addition to the unique efficient equilibrium, there are two types of inefficient equilibria. First, there are equilibria with no loan term improvement, where $\{q_t\}$ is a decreasing sequence converging to zero. Loan terms become less and less favorable with every period. Even the most patient borrowers eventually prefer to default, $\delta_t \rightarrow \delta_{\text{max}}$, which, in turn, justifies unfavorable loan terms. Second, there are equilibria with temporary loan term improvement. For any such equilibrium, $\{q_t\}$ increases at first but then becomes declining and converges to zero. Loan terms improve but only temporarily. In such an equilibrium, borrowers with medium patience simply wait until they get access to better loans so that benefits from default are higher. Lenders correctly anticipate this: as soon as more types find it optimal to default, the loan terms become less attractive to reflect a higher risk of default. That, in turn, negatively affects incentives of more patient borrowers. Eventually, all borrowers, including the most patient ones, default, $\delta_t \rightarrow \delta_{\text{max}}$.

**Theorem 3.6.** Let $q_0^{eff}$ be the initial belief that corresponds to the efficient equilibrium.

i) There is no equilibrium with $q_0 > q_0^{eff}$. For any initial belief, $q_0 \leq q_0^{eff}$, there exist an equilibrium.
ii) In any equilibrium, either \( \{q_t\} \) is increasing, or \( \{q_t\} \) is decreasing, or \( \{q_t\} \) is increasing at first but then becomes decreasing.

iii) Equilibria with sufficiently low \( q_0 \) exhibit no loan term improvement, and equilibria with sufficiently high \( q_0 \) exhibit temporary loan term improvement.

Part ii) shows that there are at most three types of equilibrium dynamics. It follows from the fact that if \( q_t \geq q_{t+1} \), then \( q_{t+1} > q_{t+2} \), and, therefore, \( q_{s+s} > q_{s+s+1} \) for any \( s > 0 \). Intuitively, if \( q_t > q_{t+1} \) then default is more likely in period \( t+1 \). However, \( \delta_t < \delta_{t+1} \). That is, the borrowers who default in period \( t+1 \) are more patient than those who default in period \( t \). Thus, the loan terms in period \( t+2 \) should be worse than in period \( t+1 \). An equilibrium with increasing \( \{q_t\} \) is the unique efficient equilibrium. As for inefficient equilibria, it follows from part ii) that inefficient equilibria with \( q_1 > q_0 \) are equilibria with temporary loan term improvement; equilibria with \( q_1 \leq q_0 \) are equilibria with no loan term improvement. Part iii) shows that both types always exist.\(^{11}\)

Consider the following example. Let \( F(K) = \sqrt{K} \), and \( \delta \sim U[0,1] \). Assume that lenders are non-profit borrower’s welfare maximizing MFIs. That is, \( R_t = R(q_t) = r/q_t \), and the loan size, \( K_t \), maximizes borrower’s payoff conditional on payback. By solving \( \max_K \sqrt{K} - (r/q_t)K \), we get that \( K_t = \left( \frac{q_t}{2r} \right)^2 \), \( F(K_t) = \frac{q_t}{2r} \), and borrower’s profit after repayment is \( \frac{q_t}{4r} \). In this example, the borrower always has to pay back 50\% of the produced output, \( \varepsilon(K_t) = 1/2 \).

![Figure 1: The unique efficient equilibrium. The solid line is a \( q \)-sequence. It converges to 1 and the risk of default is eventually eliminated. The dashed line is a \( \delta \)-sequence. At the beginning of moment \( t+1 \) types in \([0, \delta]\) have already defaulted and types in \([\delta, 1]\) are still in the game. The \( \delta \)-sequence converges to 1/2 which is \( \varepsilon(1) \) in this example. The figure is plotted for \( \delta \sim U[0, 1] \), \( F(K) = \sqrt{K} \) and \( q_0^{\varepsilon_f} \approx 0.632 \).](image)

The efficient equilibrium dynamic is shown in Figure 1. Numerically, one can estimate that \( q_0^{\varepsilon_f} \approx 0.632 \). The \( q \)-line converges to 1 and the risk of default eventually disappears. The \( \delta \)-line converges to \( \varepsilon(1)(=1/2) \) meaning that all types with \( \delta < \varepsilon(1) \) eventually default, and types with \( \delta \geq \varepsilon(1) \) stay in the game permanently. The efficient equilibrium is an example of equilibrium with

\(^{11}\)Generally speaking, one cannot say that there is a threshold \( \tilde{q}_0 \) such that all equilibria with \( q_0 < \tilde{q}_0 \) exhibit no loan term improvement and all equilibria with \( q_0^{\varepsilon_f} > q_0 > \tilde{q}_0 \) exhibit temporary loan term improvement. From (6), \( F(K_1) = F(K_0) \) iff \( \varepsilon(K(q_0)) = \delta_0 \), where \( \delta_0 \) satisfies \( q_0 = 1 - \Phi(\delta_0) \). Depending on loan terms and \( F(\cdot) \), it is possible to have multiple \( q_0 \)'s such that \( K_1 = K_0 \). This is why, part iii) is only stated in terms of equilibria with sufficiently low and sufficiently high \( q_0 \)'s. However, in the example used for Figures 1, 2 and 3, \( \varepsilon(K(q)) = 1/2 \) for every \( q \). There exists a unique \( \tilde{q}_0 (=1/2) \) such that \( K_1 = K_0 \).
progressive lending, where the initial loan terms are small and, as long as the borrower repays, loan terms improve until the risk of default is fully eliminated. In contrast to other papers, progressive lending only works in the efficient equilibrium and it cannot start with loans that are too small. In fact, as Theorem 3.6 shows, \( q_{0}^{eff} \) and the corresponding initial loan are the highest among all equilibria. The difference is due to the fact that, in my model, any exclusive borrower-lender relationship, such as long-term contracts, is impossible. Loan terms in period \( t \) are determined solely based on a likelihood of default in period \( t \). And as shown in Proposition 3.3, a lower initial loan must imply higher default rates which are then rationalized by even worse future loans.

![Figure 2](image)

Figure 2: An equilibrium with no loan term improvement. The solid line is a \( q \)-sequence. The dashed line is a \( \delta \)-sequence. At the beginning of moment \( t + 1 \) types in \([0, \delta_t]\) have already defaulted and types in \([\delta_t, 1]\) are still in the game. The figure is plotted for \( \delta \sim U[0, 1], F(K) = \sqrt{K} \) and \( q_0 = 0.48 \).

An example of the equilibrium with no loan term improvement can be seen in Figure 2. It is plotted for the case of \( q_0 = 0.48 \), though any \( q_0 \in [0, 1/2] \) would result in no loan term improvement (see footnote 11). Along this equilibrium, repayment rates, \( q_t \), are quickly decreasing. The value of future loans is low to begin with and becomes even lower with every new period. Eventually, even the most patient types default. Some evidence from studies comparing the repayment rates of new borrowers with those of returning borrowers suggest that the repayment rates among the latter tend to be lower (Vogelgesang, 2003, Pollio and Obuobie, 2010), which corresponds to equilibria with declining \( \{q_t\} \).

An example of equilibrium dynamics with temporary loan term improvement can be seen in Figure 3. It is plotted for \( q_0 = 0.62 \), though any \( q_0 \in (0.5, q_0^{eff}) \) would result in temporary loan improvement. At first, all but the most impatient types prefer to pay back the loans and get access to more favorable loans. However, the funds inflow is not large enough to make these incentives permanent. The types with medium patience are paying back simply to get access to more favorable loans.

In addition to the papers cited in the literature review, Thomas and Worrall (1994) study how to structure a contract between a host country and a multinational corporation that is interested in investing in the country given that any such contract has to be self-enforced. As with progressive lending, investment is initially underprovided but increases over time and, depending on parameters, can reach the efficient level. Watson (1999) and (2002) develop a model of a dynamic partnership game with two-sided incomplete information. Both papers show that it is possible for players to eventually achieve a cooperative equilibrium by starting small, no matter how pessimistic they are at the beginning of the partnership.
loans, which would be more profitable to default upon. Lenders correctly anticipate this, so as soon as more types find it optimal to default, loan terms become less favorable, destroying incentives for the more patient types. Eventually all types default.

![Figure 3: An equilibrium with temporary loan term improvement. The solid line is a q-sequence. The dashed line is a δ-sequence. At the beginning of moment \( t + 1 \) types in \([0, \delta]\) have already defaulted and types in \([\delta, 1]\) are still in the game. The figure is plotted for \( \delta \sim U[0, 1] \), \( F(K) = \sqrt{K} \) and \( q_0 = 0.62 \).](image)

I conclude this section by a remark on the equilibrium selection. Regardless of \( \delta \), the borrower prefers equilibria with higher \( q_0 \). For lenders the result is similar, though the justification depends on the mission and market structure. By the logic above, the non-profit MFI maximizing borrower’s welfare prefers equilibria with higher \( q \). Competitive lenders are indifferent across equilibria as their expected profit is zero. A for-profit monopolistic lender prefers equilibria with a higher \( q \) (follows from the envelope theorem). Finally, as borrowers default later in equilibria with higher \( q_0 \), an outreach maximizing non-profit MFIs also prefers equilibria with higher \( q_0 \).

Since, except for the case of competitive lenders, both sides strictly prefer the efficient equilibrium — the one with the highest \( q_0 \) — and the competitive lenders are indifferent, one might expect the efficient equilibrium to become the focal equilibrium that players could coordinate on. However, as argued in Harsanyi and Selten (1988), in coordination games with multiple Pareto-ranked equilibria, it is risk-dominant and not payoff-dominant equilibrium that is likely to be selected. In my model, the efficient equilibrium is also the riskiest as it requires the largest loans. If a lender mis-coordinates and provides a larger than an equilibrium loan, the expected loss is higher than in the case of providing a smaller than an equilibrium loan. Thus, if lenders are worried about mis-coordinating, it is possible that an efficient equilibrium is selected.

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13 Consider the two equilibria \( \{(q_t, \delta_t)\} \) and \( \{(Q_t, \Delta_t)\} \), where \( q_0 > Q_0 \). Let \( \pi_t(\delta) \) denote the borrower’s profit from default at \( t \) along the \( q \)-equilibrium, and \( \Pi_t(\delta) \) profit along the \( Q \)-equilibrium. Let \( t^* \) denote the optimal time to default along the \( q \)-equilibrium and \( T^* \) denote the optimal time to default along the \( Q \)-equilibrium. From Proposition 3.3 follows that \( \pi_t(\delta) > \Pi_t(\delta) \) for every \( t \). Thus, \( \pi_t(\delta) \geq \pi_{T^*}(\delta) > \Pi_{T^*}(\delta) \) and the borrower is better off in the \( q \)-equilibrium.
4 Extensions of the Model

4.1 Double Dipping

One of the consequences of a rapid expansion of micro-credit is that borrowers have access to multiple lenders. Guha and Chowdhury (2013) cite the Wall Street Journal’s article from November 27, 2011: “Surveys have estimated that 23% to 43% of families borrowing from micro-lenders in Tangail borrow from more than one.” This phenomenon is called double-dipping and, in this section, I study how double-dipping affects dynamic incentives.

Assume that there are two lenders who are willing to lend to the borrower and neither lender can contract on the borrower’s interactions with the other lender. I also assume that the loan terms provided by each lender are the same and determined by a probability of default. Each period, the borrower with no prior defaults has an option of borrowing money from one lender or two lenders. Upon getting the loan(s), the borrower use the funds to produce output and then decides whether or not to default. As in the main model, defaults are assumed to be publicly observable, despite a borrower taking multiple loans, and will prevent the borrower from getting future loans. The assumption is consistent with McIntosh et al. (2005)’s observation that “ Particularly in rural areas, clients are extremely unlikely to be able to default in one joint-liability network without acquiring a reputation that would preclude their joining another.”

In order to make single-dipping (SD) and double-dipping (DD) frameworks comparable I assume that, for a given \( q \), single loan terms are the same in SD and DD frameworks, \((K(q), R(q))\). To what extent this assumption is realistic depends on the MFI’s mission and market structure. For instance, whether DD is available or not, a welfare-maximizing non-profit MFI will provide a loan at the level where the marginal productivity of a single loan is equal to the loan’s interest rate. In this case, the repaying borrower will take a single loan and the effect of double-dipping will manifest itself in how the probability of default is determined. On the other hand, in the case of a for-profit lender with some market power, the assumption is less appropriate as increased competition will affect the loan terms.

The analysis of Section 3 is applicable to the DD-framework if the LTM holds for loans with double-dipping. The borrower planning to repay in period \( t \) will borrow at level \( K^*_t \) that maximizes his payoff after the repayment: \( \max_{K \in [K(q_t), 2K(q_t)]} F(K) - R(q_t)K. \) The borrower planning to default in period \( t \), will take loans from both lenders at the maximum amount, \( 2K(q_t) \). The borrower’s gain from default, therefore, is \( F(2K(q_t)) - (F(K^*_t) - R(q_t)K^*_t) \). The next proposition

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14 This assumption is similar to how Attar et al. (2011) define nonexclusive trade: “Trading is nonexclusive in the sense that no buyer [lender in my model - DS] can contract on the trades that the seller [borrower in my model - DS] makes with other buyers” (p. 1877). In this sense, lending contracts in the framework with double-dipping are nonexclusive.

15 An implicit assumption in Section 3 was that the borrower always takes the full amount of loan, i.e. \( F'(K(q)) \geq R(q) \). In the case of double-dipping, I maintain the assumption that for a repaying borrower it is optimal to take at least one full loan. With the second loan I allow for two possibilities: either the borrower remains credit-constrained with two loans (then \( F''(2K(q)) \geq R(q) \) and \( K^* = 2K(q) \)), or the second loan removes credit constraints (\( K^* < 2K(q) \) and \( F''(K^*) = R(q) \)).
shows that both the repayment payoff and the gain from default are increasing functions of $q$; therefore, the LTM holds in the DD-framework.

**Proposition 4.1.** Let $K(q)$ and $R(q)$ satisfy the LTM. Assume also that $F'(K)K$ is an increasing function of $K$ and $R(q)$ a decreasing function of $q$. Then $\max_{K \in [K(q), 2K(q)]} F(K) - R(q)K$ and $F(2K(q)) - \max_{K \in [K(q), 2K(q)]} F(K) - R(q)K$ are increasing functions of $q$.

Since the LTM holds, the results from Section 3 can be immediately extended to the case of double-dipping. For the DD-borrower to be indifferent between default in periods $t$ and $t + 1$ his $\delta$ should satisfy

$$\delta_t = \frac{F(2K_t) - (F(K_t^*) - R_tK_t^*)}{F(2K_{t+1})}.$$  \hspace{1cm} (7)

The Bayesian update condition is the same as before,

$$q_{t+1} = \frac{1 - \Phi(\delta_{t+1})}{1 - \Phi(\delta_t)}. \hspace{1cm} (8)$$

Double-dipping equilibria should satisfy equations (7) and (8). As was established in Section 3, there is a multiplicity of equilibria and the efficient equilibrium always exists and is unique.

The next two Propositions compare borrowers’ incentives along SD and DD equilibrium dynamics. Intuitively, double-dipping has two opposite effects on borrower’s incentives. On one hand, the borrower’s gains from defaults are higher. On the other hand, for a credit-constrained borrower gains from a repayment are higher as well. When assumptions of Proposition 4.2 are satisfied, a DD-equilibrium requires more favorable future loans in order to generate the same repayment rate as in a SD-equilibrium. That means that the former effect dominates. With double-dipping available, the borrower has stronger incentives to default which have to be offset by better future loans. Proposition 4.3 is an immediate extension of Proposition 4.2. It compares the efficient equilibria with and without double-dipping and shows that when a DD-borrower has higher incentives to default, the initial DD-loan has to be smaller than under single-dipping.

**Proposition 4.2.** Assume that loan terms, $(K(q), R(q))$, satisfy assumptions of Proposition 4.1. Assume further that either (i) a double-dipping borrower is credit-constrained, i.e. $F'(2K(q)) \geq R(q)$ for every $q$; or (ii) $F'(K)K/F(K)$ is a weakly decreasing function of $K$. Then for any SD and DD-equilibria with the same $q_0$: $q_{t}^{dd} > q_{t}^{sd}$ and $\delta_{t}^{dd} < \delta_{t}^{sd}$ for every $t > 0$.

**Proposition 4.3.** If assumptions of Proposition 4.2 are satisfied then the double-dipping efficient equilibrium requires a lower initial loan than the single-dipping efficient equilibrium, $q_{0}^{sd,eff} > q_{0}^{dd,eff}$.

As mentioned above, double-dipping has two opposite effects on the borrower’s incentives since gains from both default and repayment become higher. It turns out that, when a double-dipping borrower is credit-constrained, the former effect always dominates. Intuitively, for the same loan terms in period $t$, the credit-constrained borrower’s gain from default, $F(2K_t) - (F(2K_t) - 2R_tK_t) = 2R_tK_t$, is twice as high under DD than under SD. However, the output from the future loan less than doubles, $F(2K_{t+1}) < 2F(K_{t+1})$, due to concavity of $F$. Therefore, to maintain repaying
incentives, loan terms for future single loans under DD should become more favorable. The logic is similar in spirit to de Quidt et al. (2012), where borrowers are better off under the monopolistic lender. The monopolistic lender in de Quidt et al. (2012) has to give more rent to borrowers as their incentive constraints are tightened. Similarly, in my model, DD-loan terms should become more favorable in order to offset a higher incentive to default.

If the double-dipping borrower is not credit-constrained then the borrower’s gain from default becomes a concave function of $K_t$: $F(2K_t) - (F(K^*) - R_tK^*)$. This makes a comparison between SD and DD ambiguous. The sufficient condition that guarantees that DD loans remain higher is that $KF'(K)/F(K)$, which is elasticity of $F$, is a weakly decreasing function of $K$. To interpret this condition, note that a weakly decreasing elasticity implies $(\ln F(2K))' \leq (\ln F(K))'$. Thus, an increase in single-loan terms, i.e. $K$, leads to a higher percentage in output’s increase under SD than under DD. Therefore, to maintain the incentives along the DD-equilibrium, DD single loan terms should be more favorable.

Empirical evidence on double-dipping suggests that double-dipping can be an unanticipated shock for lenders. For instance, McIntosh et al. (2005) report a decline in repayment performance, which indicates that lenders got caught by surprise and have not adjusted to a presence of double-dipping borrowers. In my framework, if unbeknown to lenders the borrower in period $t$ gets access to multiple loans the short-run incentives to default are higher. Consider the borrower who in period $t$ of the SD-equilibrium gets access to two loans but lenders do not adjust the loan terms to account for double-dipping. If the borrower expects future single loan terms to be determined according to the original SD-equilibrium, then the borrower’s intertemporal trade-off changes from $F(K^{sd}_t) - R_tK^{sd}_t + F(K^{sd}_{t+1})\delta^{sd}_{t} = F(K^{sd}_t)$ to $F(K^{*}_t) - R_tK^{*}_t + F(2K^{sd}_{t+1})\delta^{dd}_{t} = F(2K^{sd}_t)$. One can show that $\delta^{dd}_t > \delta^{sd}_t$ (the proof is identical to the proof of Proposition 4.3), meaning that default in period $t$ becomes more attractive and higher patience is required to postpone it.

4.2 Types Separation

In the main framework, the only way patient types can separate themselves is by consistently repaying the loans. In this section, I consider two extensions to allow for a possibility of types’ separation. First, I study whether it is possible to have an equilibrium with different loan contracts, thereby allowing types to self-select according to their patience. This is different from double-dipping in that, even though two loans are offered, the borrower can take only one loan at a given period. Second, I introduce possibility of costly signaling.

Assume that there are two sequences of loans, one specified by beliefs $\{q_t\}$ and another by beliefs $\{Q_t\}$. As before, loan terms are fully determined by the probability of default. Given $\{q_t\}$ and $\{Q_t\}$, the borrower’s strategy is to decide when to default, as well as which loan to use in period $t$. Note that I allow switching between $q$-loans and $Q$-loans. That is, a borrower can choose a $q$-loan
in period \( t - 1 \), and then upon repayment, a \( Q \)-loan in period \( t \); however, this is not essential for the argument below. In equilibrium, lenders’ beliefs should be rational given the borrower’s strategy.

**Proposition 4.4.** In any equilibrium \( q_t = Q_t \) for every \( t \).

**Proof.** Assume not. Let \( t \geq 0 \) be the first period, when \( q_t \neq Q_t \) and without loss of generality assume that \( q_t > Q_t \). Two cases are possible. First, no type defaults at \( t \), in which case \( q_t = Q_t = 1 \) which is a contradiction to \( q_t > Q_t \). Second, some types default. All defaulting types should choose a \( q \)-loan, as it offers a larger loan. But then \( Q_t = 1 \), which is a contradiction with \( q_t > Q_t \).\(^{16}\)

The reason why the separation is not possible in my model is the failure of the single-crossing condition, which is the necessary condition for separation. On one hand, as the single-crossing condition requires, a marginal improvement in the future loan terms leads to a higher marginal utility of more patient borrowers. On the other hand, a marginal improvement of the current loan terms leads to the same increase in marginal utility, regardless of \( \delta \). Therefore, the single-crossing condition fails.

The failure of the single-crossing condition is due to an inherent feature of dynamic incentives in which the reward or punishment of current behavior does not come until the next period. This is why exclusive reliance on dynamic incentives cannot separate borrowers with different discount factors. However, the separation can be possible if there are other instruments that have an impact during the same period when the borrower’s default decision is made.

As an example, consider an extension of the original model where a borrower can send a costly signal. In general, we would expect the poor to lack the ability to send costly signals, both due to a lack of funds and a lack of options. Nonetheless, there are documented examples of costly signaling in development literature such as obtaining land titles in Indonesia (Dower and Potamites, 2007). In Indonesia, acquiring land titles is a costly process that is both lengthy and very bureaucratic. Even though these titles could be used as collateral, they seem to be used instead as an ex-ante signal of borrower’s creditworthiness. Only 40% of the first-time borrowers use land titles as collateral, and doing so does not improve loan terms beyond the effect of having one.

Formally, signaling is modeled as follows. Assume that in period \( t - 1 \), a borrower can experience a signaling shock that enables a borrower to send a signal to lenders. I assume that the timing of the signaling shock is unrelated to \( \delta \), and importantly, whether the borrower sends a signal in a given period or not does not reveal any new information about his patience. This assumption substantially simplifies the analysis since, until the signal is sent, equilibria are defined by (6). For example, in the case of land titles in Indonesia, the timing of getting the title could be beyond the control of the borrower due to the unpredictability of bureaucratic procedures. Alternatively, the borrower might lack resources to send an informative signal to lenders unless an income shock occurs which is unrelated to the borrower’s interactions with lenders.

\(^{16}\)In the proof, I implicitly assumed that each loan is used by a positive mass of types. If no one takes the \( Q \)-loan, then it could be possible that \( q_t > Q_t \). However, the main insight remains valid. There is no separation of the types in the equilibrium. In every period, either they all use the same loan, or they use different loans but with equivalent terms. Also, note that the proof could be easily extended to the case of three or more sequences of loans. For any two loans that are used by a positive measure of borrowers, the terms have to be equivalent.
The next proposition provides an example of an equilibrium where sending a signal credibly reveals that the borrower is patient and leads to an infinite sequence of riskless loans.

**Proposition 4.5.** There is an equilibrium with signaling where signal $c_{t-1} = (F(K(1)) - F(K_t))\epsilon(1)$, sent in period $t - 1$, credibly reveals that the borrower is patient.

Given the LTM, $K_t$ is an increasing function of $q_t$. Thus, more favorable beliefs imply that the signal cost is lower. This observation has an interesting implication. For equilibria with temporary loan term improvement the best period for a borrower to receive a signaling shock is around the peak of a $q_t$-trajectory. First, when $q_t$ is high, the signal itself is less costly. Second, the borrower receives more capital making the signal more affordable. Thus, signaling even in this simple extension can reduce the inefficiency of equilibria of the original model. Whether the equilibrium is efficient or not when the signaling shock occurs, patient types can credibly signal their patience in exchange for an access to riskless loans.

### 4.3 Alternatives to the Uncertainty over $\delta$ Assumption

Lenders’ uncertainty over $\delta$ can be broadly interpreted as lenders’ uncertainty over how much the borrower values future loans and, therefore, how responsive he is to dynamic incentives. As the value of future loans can depend on factors other than borrower’s discount coefficient, in this section I describe three alternatives to the uncertainty over the $\delta$ assumption, under which results of Section 3 will remain intact.$^{17}$

**Uncertainty over the rate of productivity growth:** Assume that the production function grows with rate $A$ so that $F_{t+1}(K) = AF_t(K)$, where $A$ is the borrower’s private information. $A$ is distributed on interval $[A_{\text{min}}, A_{\text{max}}]$, where $\delta A_{\text{max}} < 1$ so that the borrower’s utility is well-defined. If we define $\hat{\delta}$ as $\delta A$, then it becomes identical to the setting in Section 3 where the borrower’s discount coefficient is $\hat{\delta}$, and it is distributed on interval $[\delta A_{\text{min}}, \delta A_{\text{max}}]$.

An important underlying assumption is that lenders cannot learn $A$ in any way other than as described in Section 3. This assumption could be reasonable, for example, if the only information provided by credit bureaus is that the borrower has no prior defaults. However, if the loan size is determined so as to maximize $\max_K F_T(K_t) - R_t K_t$, and the sizes of prior loans are observed, then the lenders could use this additional information to deduce $A$.

**Uncertainty over proportional collateral:** Assume that in the case of default, the borrower looses some fixed portion, $\theta$, of the loan as post-collateral. While lenders might know the monetary value of the collateral, they are uncertain how much the borrower values it.$^{18}$ Thus, the borrower’s profit from payback is, as before, $(1 - \epsilon_t)F(K_t)$, but the payoff from default is $(1 - \theta)F(K_t)$. If $\theta > \epsilon_t$, then no default will occur. Therefore high collateral is enough to stop strategic defaults.

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$^{17}$The formal model that generalizes Section 3 to include these alternatives is available on the author’s website.

$^{18}$An example where lenders could be uncertain about value of collateral is the so-called nontraditional collateral introduced by Bank Rakyat Indonesia. Nontraditional collateral is something that might not have a resale value, but is, nonetheless, valuable to the borrower, such as a domestic animal or land that is not secured by title.
The combination of small collateral and dynamic incentives, however, is plagued with the same problem as just using dynamic incentives.

**Uncertainty over an outside option:** The borrower has an outside option, with a payoff of $\theta$. The option is available if lenders restrict the borrower’s access to the credit market. The value of the outside option is unobserved by lenders. The payback payoff is $(1 - \varepsilon_t)F(K_t)$, and the default payoff is $F(K_t) + \theta$.

### 5 Conclusion

Dynamic incentives is a relatively well-understood microfinance methodology. A simple game-theoretical model can readily show that for a patient borrower, the long-term gain from accessing future loans can outweigh the short-term gain from default. Thus, as long as a borrower is sufficiently patient, and lenders can punish defaulters by full (or partial) exclusion from the credit market, the borrower will prefer to repay current loans.

In this paper, I develop a new model to study dynamic incentives. I assume that lenders face uncertainty with regards to how much the borrower values future loans and, therefore, how responsive the borrower is to dynamic incentives. Another feature of my model is the impossibility of exclusive contracts between lenders and the borrower. In particular, the parties cannot commit to a contract which is longer than one period, and lenders’ information with regards to a given borrower is symmetric.

As I show in Section 3, only one equilibrium eliminates the risk of default in the long-run. For the rest of equilibria, all borrowers including the most patient ones default. This highlights the limitations of dynamic incentives. The environment with sufficiently patient types and the full exclusion of defaulters is not enough to prevent patient types from default. Another result of the benchmark model is the drawback to starting small when an exclusive relationship between the borrower and lenders is impossible, and the current loan terms are determined by the anticipated default rate. In my model, starting with smaller loans can be only rationalized by the higher default rate and, as I show in the paper, the eventual default of even the most patient types. The only efficient equilibrium is the one with the highest initial loan among all equilibria.

Next, I study dynamic incentives in the presence of double-dipping. In general, the effect of double-dipping can be ambiguous. Having two loans can not only increase default gains, but also can increase repayment gains. I show that when a double-dipping borrower is credit-constrained the former effects dominates. In a double-dipping equilibrium, future loans have to be more favorable to generate the same repayment rate as in a single-dipping equilibrium. That, in particular, implies that the default rates along the double-dipping equilibrium are lower. It is important to note, however, that if access to multiple loans occurs unexpectedly for lenders, in the short-run, this negatively affects borrower’s incentives. This is consistent with an increase in default rates reported in the empirical literature.
6 Appendix A: Proofs

Proof of Proposition 3.2: Proof by contradiction. Let \( \tau \) be a moment when \( \delta_\tau > \delta_{\tau+1} \). Then \( q_{\tau+1} = 1 \). Indeed, by definition of \( \delta_\tau \), types with \( \delta < \delta_\tau \) prefer default at \( \tau \) to default at \( \tau + 1 \). Since \( \delta > \delta_\tau \) implies \( \delta > \delta_{\tau+1} \) types with \( \delta > \delta_\tau \) will prefer default at \( \tau + 2 \) to default at \( \tau + 1 \). Type with \( \delta = \delta_\tau \) has measure zero.

From (4)
\[
\delta_\tau = \varepsilon(q_\tau) \frac{F(K(q_\tau))}{F(K(1))} = \frac{R(q_\tau)K(q_\tau)}{F(K(1))}
\]
\[
\delta_{\tau+1} = \varepsilon(1) \cdot \frac{F(K(1))}{F(K(q_{\tau+2}))} = \frac{R(1) \cdot K(1)}{F(K(q_{\tau+2}))}.
\]

Since \( K(q) \) and \( R(q)K(q) \) are increasing functions of \( q \) and since \( q_\tau \) and \( q_{\tau+2} \) are less than or equal to one, \( \delta_\tau \leq \delta_{\tau+1} \) which is a contradiction. Furthermore, notice that \( \delta_\tau = \delta_{\tau+1} \) only if \( q_\tau = q_{\tau+1} = q_{\tau+2} = 1 \), which in turn would imply \( \delta_\tau = \delta_{\tau+1} = \varepsilon(1) \).

Lemma 6.1. Assume that \( \{\delta_t\} \) is a weakly increasing sequence and assume that \( \delta_t < \delta_{t+1} \) for some \( t \). Then for any borrower with \( \delta \) such that \( \delta_t < \delta < \delta_{t+1} \) it is optimal to default at \( t + 1 \).

Proof. Let \( \pi_t(\delta) \) denote the utility of the borrower with discount factor \( \delta \) from default at \( t \). Then
\[
\pi_0(\delta) < \pi_1(\delta) < \cdots < \pi_t(\delta) < \pi_{t+1}(\delta) > \pi_{t+2}(\delta) > \cdots.
\] (9)

Indeed, since \( \{\delta_t\} \) is weakly increasing \( \delta > \delta_t \) implies \( \delta > \delta_\tau \) for any \( \tau \leq t \) and, therefore, \( \pi_\tau(\delta) < \pi_{\tau+1}(\delta) \) for any \( \tau \leq t \). Similarly, \( \delta < \delta_{t+1} \) implies \( \delta < \delta_\tau \) for any \( \tau \geq t + 1 \) and, therefore, \( \pi_\tau(\delta) > \pi_{\tau+1}(\delta) \) for any \( \tau \geq t + 1 \).

Finally, I will show that \( \{\delta_t\} \) is a strictly increasing sequence. Assume not. Let \( \tau \) be the first moment, such that \( \delta_\tau = \delta_{\tau+1} \). If \( \tau = 0 \) then \( q_0 = 1 \), \( \delta_0 = \delta_1 = \varepsilon(1) \) and from weak-monotonicity \( \delta_t \geq \varepsilon(1) \) for every \( t \). Since \( \varepsilon(1) > \delta_{\min} \), we can use the same logic as in Lemma 3.1 to show that types with \( \delta < \varepsilon(1) \) will default at 0, which contradicts \( q_0 = 1 \). Similarly, if \( \tau > 0 \) then \( \delta_{\tau-1} < \delta_\tau \) and by Lemma 3.1 types with \( \delta \in [\delta_{\tau-1}, \delta_\tau] \) default at \( \tau \) which contradicts \( q_\tau = 1 \). The fact that \( \{\delta_t\} \) is a strictly increasing sequence together with Lemma 3.1 complete the proof of the Proposition.

Proof of Proposition 3.3: Let \( q_{\tau+1}(q, \delta) \) and \( \delta_{\tau+1}(q, \delta) \) denote the result of one iteration of (6) given \( (q, \delta) \). Also I will say that pair \( (q, \delta) \) is feasible if \( 0 \leq q \leq 1 \) and \( \delta_{\min} \leq \delta \leq \delta_{\max} \). The following Lemma does not require \( (q, \delta) \) to be a part of an equilibrium trajectory. The only requirement is that one iteration of (6) based on \( (q, \delta) \) will result in a feasible \( (q_{\tau+1}, \delta_{\tau+1}) \).

Lemma 6.2. If \( (q, \delta) \) and \( (q_{\tau+1}, \delta_{\tau+1}) \) are feasible, then \( q_{\tau+1} \) is an increasing function of \( q \) and a decreasing function of \( \delta \); \( \delta_{\tau+1} \) is a decreasing function of \( q \) and an increasing function of \( \delta \).

Proof. Pair \( (q, \delta) \) defines \( q_{\tau+1} \) by the first equation of (6) which given the definition of \( \varepsilon(q) \) can be re-written as
\[
F(K(q_{\tau+1})) = \frac{\varepsilon(q) \cdot F(K(q))}{\delta} = \frac{R(q)K(q)}{\delta}.
\]
Thus $\frac{\partial q_{t+1}}{\partial \delta} < 0$, and since $R(q)K(q)$ is an increasing function $\frac{\partial q_{t+1}}{\partial q} > 0$. Given that $q_{t+1} \leq 1$ from the second equation of (6) it immediately follows that $\delta_{t+1}$ is a decreasing function of $q_{t+1}$ and an increasing function of $\delta$. Together with the established monotonicity of $q_{t+1}$, this implies that $\delta_{t+1}$ is a decreasing function of $q$ and an increasing function of $\delta$. ☐

Now we can prove the proposition. Given that along the equilibrium $q_0 = 1 - \Phi(\delta_0)$, having $q_0 > Q_0$ implies $\delta_0 < \Delta_0$. Iteratively applying Lemma 6.2 we get that $q_t > Q_t$ and $\delta_t < \Delta_t$ for all $t$. The requirement of Lemma 6.2 is satisfied because $\{(q_t, \delta_t)\}$ and $\{(Q_t, \Delta_t)\}$ are equilibria and, therefore, are feasible. ☐

**Proof of Proposition 3.4:** Similarly to the proof of Proposition 3.3 I will denote $q_{t+1}$ and $\delta_{t+1}$ as the $t^{th}$ iteration of (6) based on initial values $(q, \delta)$. Also I will say that pair $(q, \delta)$ is feasible if $0 \leq q \leq 1$ and $\delta_{\text{min}} \leq \delta \leq \delta_{\text{max}}$.

First, we prove that equilibrium trajectories exist. Let $(Q, \Delta) = (1, \varepsilon(1))$ and $(q_0, \delta_0) = (1 - \Phi(\varepsilon(1)), \varepsilon(1))$ be two initial conditions. Note that while both are feasible, $(1, \varepsilon(1))$ cannot be an equilibrium pair of initial values. This is because we assumed that $\varepsilon(1) > \delta_{\text{min}}$ and, therefore, $Q \neq 1 - \Phi(\Delta)$ when $Q = 1$ and $\Delta = \varepsilon(1)$.

The pair $(Q, \Delta) = (1, \varepsilon(1))$ is a steady state of (6) and, therefore, $(Q_{t+1}, \Delta_{t+1})$ is equal to $(1, \varepsilon(1))$ for every $t$ and is feasible. Given that $Q > q_0$ and $\Delta = \delta_0$ from Lemma 6.2 it follows that $q_1 < Q_{t+1} = 1$. Having $q_1 < 1$ implies that $\delta_1$ is well-defined, and so the pair $(q_1, \delta_1)$ is feasible. Again, by Lemma 6.2, $\delta_1 > \Delta_{t+1} = \varepsilon(1)$. Iteratively applying the same argument, we get the feasibility of the trajectory starting with $(q_0, \delta_0)$ which, therefore, is an equilibrium.

Next, I show that in equilibrium the $q$-sequence is either monotone or eventually becomes monotone. That is, if $q_t > q_{t+1}$ then $q_s > q_{s+1}$ for any $s > t$. According to Proposition 3.3, $q_{t+1}$ is an increasing function of $q_t$ and a decreasing function of $\delta_t$ and, according to Proposition 3.2, $\delta_{t+1} > \delta_t$. Therefore, if $q_t > q_{t+1}$ then $q_{t+2} < q_{t+1}$. By the same argument, one can show that $q_s > q_{s+1}$ for any $s > t$. Thus, the $q$-sequence is either increasing, or eventually becomes decreasing.

Since both the $q$-sequence and $\delta$-sequences are monotone (or eventually monotone) and bounded, they converge. Assume that $\delta_t \rightarrow \delta(\neq \delta_{\text{max}})$. Then from the second equation of (6) it follows that $q_t$ should converge to 1, and then from the first equation $\delta_t \rightarrow \varepsilon(1)$. That corresponds to the limit point $(1, \varepsilon(1))$. Assume that $\delta_t \rightarrow \delta_{\text{max}}$. If the $q$-sequence converges to a positive limit, $\hat{q}$ then the first equation converges to $\delta_{\text{max}} = \varepsilon(\hat{q})$, which is impossible given our assumption $\delta_{\text{max}} > \varepsilon(q)$. Thus, $q_t$ has to converge to zero which corresponds to the second limit point $(0, \delta_{\text{max}})$. ☐

**Proof of Proposition 3.5:** As before in the proof of this Proposition I will say that pair $(q, \delta)$ is feasible if $0 \leq q \leq 1$ and $\delta_{\text{min}} \leq \delta \leq \delta_{\text{max}}$.

i) Existence: Let $\tilde{A}$ be the set of all $q_0 \in [0, 1]$ that generate feasible trajectories, and let $\bar{A}$ be the set of all $q_0 \in [0, 1]$ that generate infeasible trajectories. Both sets are non-empty. $\tilde{A}$ is non-empty by Proposition 3.4; $\bar{A}$ is non-empty because $\varepsilon(1) > \delta_{\text{min}}$ and, therefore, the trajectory
that starts at $q_0 = 1$ is not feasible. From Proposition 3.3, it follows that if $q_0 \in A$ then all $Q_0 < q_0$ are in $A$ and, similarly, if $q_0 \in \bar{A}$ then all $Q_0 > q_0$ belong to $\bar{A}$. $\bar{A}$ is an open set. Indeed, take an infeasible trajectory generated by $q_0' \in \bar{A}$ and let $T$ be the first moment such that $q_T' > 1$. By (6), function $q_T(q_0)$ is continuous in a neighborhood of $q_0'$ and therefore $q_T(q_0'', T) > 1$ when $q''_0$ is sufficiently close to $q_0'$. Let \( \bar{q}_0 = \inf \bar{A} \) and, since $\bar{A}$ is an open set, $\bar{q}_0 \in A$. The $q$-sequence of the dynamic generated by $\bar{q}_0$ converges to 1. Assume not, in which case it converges to zero. Consider a period $T \geq 0$ such that $T$ is the maximum of $\{ \bar{q}_t \}$ and $\bar{q}_{T+1} < \bar{q}_T$. The feasibility implies that $\bar{q}_T$ is strictly less than 1.\(^{20}\) For feasible trajectories and fixed $T$, functions $q_{T+1}(\cdot)$ and $q_T(\cdot)$ are continuous functions of the initial value. Thus, by continuity, there exists $q_0 > \bar{q}$ such that $q_t(q_0) < 1$ for all $t \leq T$ and such that $q_T(q_0) > q_{T+1}(q_0)$. The last part implies that the $q$-sequence becomes decreasing after $T$, which, in turn, implies that a trajectory that starts with $q_0$ is also a feasible trajectory since $q_t(q_0) \leq q_T(q_0) < 1$ for all $t$. This is a contradiction since $q_0 > \bar{q}$ and thus should belong to $\bar{A}$.

ii) Uniqueness: Assume not. Then there are two equilibrium trajectories $(q_t, \delta_t)$ and $(Q_t, \Delta_t)$ that converge to $(1, \varepsilon(1))$. Assume that $q_0 > Q_0$. Let $\Psi$ be a function that maps a feasible pair $(q, \delta)$ into $(q_{t+1}, \delta_{t+1})$. In other words, $\Psi$ is an outcome of one iteration of system (6) so that $(q_{t+1}, \delta_{t+1}) = \Psi(q_t, \delta_t)$. Using Taylor decomposition of $\Psi$ we get:

\[
\begin{pmatrix}
Q_{t+1} - q_{t+1} \\
\Delta_{t+1} - \delta_{t+1}
\end{pmatrix}
\approx
\begin{pmatrix}
Q_t - q_t \\
\Delta_t - \delta_t
\end{pmatrix}
\begin{pmatrix}
\frac{\partial q_{t+1}}{\partial q_t} & \frac{\partial q_{t+1}}{\partial \delta_t} \\
\frac{\partial \delta_{t+1}}{\partial q_t} & \frac{\partial \delta_{t+1}}{\partial \delta_t}
\end{pmatrix}
\begin{pmatrix}
Q_t - q_t \\
\Delta_t - \delta_t
\end{pmatrix}.
\]

(10)

In Proposition 3.3 it was already established that $\frac{\partial q_{t+1}}{\partial q_t} > 0$ and $\frac{\partial q_{t+1}}{\partial \delta_t} < 0$. In fact, we can establish that the latter is bounded away from zero in the neighborhood of $(1, \varepsilon(1))$. Indeed, re-writing the first equation of (6) as $\delta_t F(K_{t+1}) - \varepsilon(q_t) F(K_t) = 0$ and using the implicit function theorem we get that

\[
\frac{\partial q_{t+1}}{\partial \delta_t} = -\frac{F'(K_{t+1})}{\delta_t F'(K_{t+1})} \frac{\partial K(q_{t+1})}{\partial q_{t+1}} < 0,
\]

for some small negative $\nu$. Such $\nu$ exists because $K(\cdot)$ is assumed to be continuously differentiable and, therefore, $K'(\cdot)$ is bounded from infinity.

Re-writing the second equation of (6) as $\Phi(\delta_{t+1}) - \varepsilon(q_t) (1 - \Phi(\delta_t)) = 0$ and using the implicit function theorem we get that

\[
\frac{\partial \delta_{t+1}}{\partial q_t} = \frac{- (1 - \Phi(\delta_t)) \frac{\partial q_{t+1}}{\partial \delta_t}}{\Phi(\delta_{t+1})},
\]

and

\[
\frac{\partial \delta_{t+1}}{\partial \delta_t} = \frac{q_{t+1} \phi(\delta_t)}{\phi(\delta_{t+1})} - \frac{(1 - \Phi(\delta_t)) \frac{\partial q_{t+1}}{\partial \delta_t}}{\phi(\delta_{t+1})}.
\]

The first expression is always non-positive because $\frac{\partial q_{t+1}}{\partial \delta_t} > 0$. The second expression consists of two terms. Given that $\varepsilon(1) < \delta_{\text{max}}$, the first term converges to 1 as $(q_t, \delta_t)$ converges to $(1, \varepsilon(1))$.\(^{20}\) Assume there exists feasible trajectories such that $q_t = 1$ for some $t$. This would imply then that $\delta_t = \delta_{t-1}$, which violates Proposition 3.2.
Given that \( \partial q_{t+1} / \partial \delta_1 \) is bounded away from 0 the second term is negative and also bounded away from zero. Therefore, in the neighborhood of \((1, \varepsilon(1))\), we have that \( \frac{\partial \delta_{t+1}}{\partial \delta_1} > 1 + \mu \) for some \( \mu > 0 \).

Given established inequalities, together with the fact that \( Q_t < q_t \) and \( \Delta_t > \delta_t \), we have from (10) that

\[
\Delta_{t+1} - \delta_{t+1} = \frac{\partial \delta_{t+1}}{\partial q_t} \cdot (Q_t - q_t) + \frac{\partial \delta_{t+1}}{\partial \delta_t} \cdot (\Delta_t - \delta_t) > \Delta_t - \delta_t
\]

when both trajectories are close to \((1, \varepsilon(1))\). Therefore, there cannot be two trajectories converging to the efficient outcome. ■

**Proof of Theorem 3.6:** i) There is no equilibrium with \( q_0 > q_0^{eff} \). Otherwise, by Proposition 3.3, the \( q \)-sequence of such equilibrium would have to converge to 1, which would contradict the uniqueness result of Proposition 3.5. It also follows from Proposition 3.3 that any initial belief, such that \( q_0 < q_0^{eff} \), will result in an equilibrium trajectory, as \( q_t < q_t^{eff} \leq 1 \) for any \( t \).

ii) There are three possible types of equilibrium loan-term dynamics: \( \{q_t\} \) is increasing; \( \{q_t\} \) is decreasing; and \( \{q_t\} \) is increasing at first but then it becomes decreasing. This is because if \( q_t \geq q_{t+1} \), then \( q_{t+1} > q_{t+2} \), and, therefore, \( q_{t+s} > q_{t+s+1} \) for any \( s > 0 \). Indeed, if \( q_t \leq q_{t+1} \) then from (6)

\[
F(K_{t+2}) = \frac{\varepsilon_{t+1} F(K_{t+1})}{\delta_{t+1}} \leq \frac{\varepsilon_t F(K_t)}{\delta_{t+1}} < \frac{\varepsilon_t F(K_t)}{\delta_t} = F(K_{t+1}),
\]

and, therefore \( q_{t+1} > q_{t+2} \). Here, the first inequality follows from the LTM, and the second from Proposition 3.2.

iii) Consider \( q_0^{low} < q_0^{eff} \) that is sufficiently close to 0 so that the corresponding \( d_0^{low} \) is sufficiently close to \( \delta_{max} \). By assumption, \( \delta_{max} > \varepsilon(q) \) for any \( q \). Therefore, \( d_0^{low} > \varepsilon(q_0^{low}) \) and from the first equation of (6) then it follows that \( d_1^{low} < q_0^{low} \) which means that the \( \{q_t^{low}\} \) sequence is decreasing. Equilibria with a temporary increase of \( q \) also exist. Along the efficient equilibrium \( q_t^{eff} > q_0^{eff} \).

By continuity, this inequality will also hold for an equilibrium with \( q_0 \) sufficiently close to \( q_0^{eff} \). ■

**Proof of Proposition 4.1:** First consider the case when a double-dipping borrower is credit-constrained, that is \( F'(2K(q)) \geq R(q) \). Then the gains from default are \( 2R(q)K(q) \), which is an increasing function of \( q \) as \( K(q) \) and \( R(q) \) satisfy the LTM. The repayment payoff is \( F(2K(q)) - 2R(q)K(q) \). Its derivative with respect to \( q \) is equal to \( 2F'(2K(q))K'(q) - 2R'(q)K(q) - 2R(q)K'(q) \).

It is positive because \( F'(2K(q)) \geq R(q) \) and \( R'(q) < 0 \).

Next, consider the case when a double-dipping borrower is not credit-constrained and the optimal loan \( K^* < 2K(q) \). Then \( F'(K^*) = R(q) \) and the repayment payoff is \( F(K^*) - R(q)K^* \). By the envelope theorem, its derivative with respect to \( q \) is \( -R'(q)K^* > 0 \). Thus the repayment payoff satisfies the LTM. The gain from default is

\[
F(2K(q)) - \max_{K \in [K(q), 2K(q)]} \{F(K) - R(q)K\} = F'(K^*) K^* + F(2K(q)) - F(K^*).
\]

Taking the derivative with respect to \( q \) we have

\[
F''(K^*) K^* (K^*)' + F'(2K)(2K)' - F'(K^*) (K^*)' = F''(K^*) K^* (K^*)' + F'(2K)(2K)',
\]

(11)
Applying the implicit function theorem to the FOC, \( F'(K^*) - R(q) = 0 \), we get that \( (K^*)'_q = R'(q)/F''(K^*) \) and so (11) can be re-written as
\[
R'(q)K^* + F'(2K)(2K)'_q.
\]
By the LTM \( (RK)'_q > 0 \) and, therefore, \( R'(q)K(q) + R(q)K'(q) > 0 \). Then,
\[
R'(q)K^* + F'(2K)(2K)'_q = K'(q) \left( \frac{R'(q)}{K'(q)} K^* + F'(2K) \cdot 2 \right) > K'(q) \left( -\frac{R(q)}{K(q)} K^* + F'(2K) \cdot 2 \right) = \frac{K'(q)}{K(q)} (-R(q)K^* + F'(2K) \cdot 2K(q)) > 0.
\]
Here, I used that \( F'(K^*) = R(q) \), \( K^* < 2K(q) \) and that \( F'(K)K \) is an increasing function of \( K \).

**Proof of Proposition 4.2:** i) Consider SD- and DD-equilibria that start with the same \( q_0 \) and, therefore, the same \( \delta_0 \). I will show that \( q_{1dd} > q_{1sd} \) and then a similar argument can be extended to any \( t \). Let \( K_0 = K(q_0) \) and \( R_0 = R(q_0) \) and let \( k_{sd} \) be a solution to
\[
\delta F(k_{sd}) = R_0K_0.
\]
Equation (12) implicitly defines \( k_{sd} \) as a function of \( \delta \). Note that \( k_{sd}(\delta_0) = K_{1sd}^* \). Similarly, for \( \alpha \geq 1 \) let \( k_{dd} \) be a solution to
\[
\delta F(\alpha k_{dd}) = F(\alpha K_0) - (F(K^*_\alpha) - R_0K^*_\alpha),
\]
where \( K^*_\alpha \) is the optimal loan size given \( R_0 \) on the interval \([K_0, \alpha K_0] \). (13) implicitly defines \( k_{dd} \) as a function of \( \alpha \) and \( \delta \). Note that \( k_{dd}(1, \delta_0) = (R_0K_0)/\delta_0 = K_{1sd}^* \) and \( k_{dd}(2, \delta_0) = K_{1dd}^* \).

As long as \( \alpha \) is such that \( K_{1\alpha}^* = \alpha K_0 \), i.e. the borrower is credit-constrained, then \( k_{dd} > k_{sd} \). Indeed,
\[
\delta F(\alpha k_{dd}) = F(\alpha K_0) - (F(K^*_\alpha) - R_0K^*_\alpha) = \alpha R_0K_0 = \alpha \delta F(k_{sd}) > \delta F(\alpha k_{sd}),
\]
where the last inequality follows from concavity of \( F(\cdot) \) and \( F(0) = 0 \). Thus, if a double-dipping borrower remains credit-constrained, i.e. \( K_{1\alpha}^* = 2K_0 \), then the single DD-loan in period 1 will be higher than the SD-loan. From \( K_{1dd}^* > K_{1sd}^* \) it follows that \( q_{1dd} > q_{1sd} \) and therefore, \( \delta_{1dd} > \delta_{1sd} \). By iteratively applying the logic above one can show that \( q_{tdd} > q_{tsd} \) and \( \delta_{tdd} > \delta_{tsd} \) for every \( t \).

ii) When a double-dipping borrower is no longer credit-constrained, the outcome depends on \( F(\cdot) \). Let \( \hat{\alpha} \) be such that \( F'(\hat{\alpha}K_0) = R_0 \) and \( 1 \leq \hat{\alpha} < 2 \). From the argument above \( k_{dd}(\hat{\alpha}, \delta_0) > k_{sd}(\delta_0) \). Thus, if \( k_{dd} \) is an increasing function of \( \alpha \), then \( K_{1dd}^* = k_{dd}(2, \delta_0) > k_{sd}(\delta_0) = K_{1sd}^* \). Applying the implicit function theorem to (13) and taking into account that \( K_{1\alpha}^* \) is a constant when \( \alpha > \hat{\alpha} \) we get
\[
\frac{\partial k_{dd}}{\partial \alpha} = \frac{\delta k_{dd} F'(\alpha k_{dd}) - K_0 F'(\alpha K_0)}{\delta \alpha F'(\alpha k_{dd})}.
\]

\(^{21}\) In what follows, there is nothing special about 2. If the borrower has access to \( N \) lenders, what matters is whether the borrower with \( N \) loans is credit-constrained or not.
The denominator is positive. From (13), we have $k^{dd} < K_0$ when $\delta = 1$. Thus, when $\delta = 1$ and $KF'(K)$ is an increasing function, then the numerator is negative and the entire derivative is positive. The term $\delta k^{dd} F'(\alpha k^{dd})$ is a weakly increasing function of $\delta$ as long as $KF'(K)/F(K)$ is a weakly decreasing function of $K$.

Therefore, the numerator of (14) is negative for every $\delta \leq 1$, and $k^{dd}$ is an increasing function of $\alpha$ for every $\delta \leq 1$. This proves that $K^{dd}_1 > K^{dd}_0$.

Just like in the proof of i) from $K^{dd}_1 > K^{sd}_1$, it follows that $q^{dd}_1 > q^{sd}_1$ and, therefore, $\delta^{dd}_1 < \delta^{sd}_1$. By iteratively applying the logic above one can show that $q^{dd}_t > q^{sd}_t$ and $\delta^{dd}_t < \delta^{sd}_t$ for every $t$.

**Proof of Proposition 4.3:** The efficient double-dipping equilibrium converges to $(1, \delta^{dd, eff})$, and the efficient single-dipping equilibrium converges to $(1, \delta^{sd, eff})$, where

$$
\delta^{dd, eff} = \frac{F(2K(1)) - (F(K^*(1)) - rK^*(1))}{F(2K(1))} \quad \text{and} \quad \delta^{sd, eff} = \frac{rK(1)}{F(K(1))} = \varepsilon(1).
$$

As before, $r = R(1)$ and $K^*(1)$ is the the DD-optimal loan size given the riskless interest rate.

When assumptions of Proposition 4.2 are satisfied, then $q^{dd}_0 = q^{sd}_0$ implies that $q^{dd}_1 > q^{sd}_1$ and $\delta^{dd}_1 < \delta^{sd}_1$ for every $t$. Thus, to show that $q^{dd, eff}_t > q^{sd, eff}_t$ it is sufficient to prove that $\delta^{dd, eff} > \delta^{sd, eff}$.

The function $F(\cdot)$ is concave, $F(0) = 0$ and, therefore, $K/F(K)$ is increasing. Since $2K(1) \geq K^*(1) \geq K(1)$, we have that

$$
\delta^{dd, eff} F(2K(1)) = F(2K(1)) - (F(K^*(1)) - rK^*(1))
\geq F(2K(1)) - F(K^*(1)) + rK^*(1)
\geq F(2K(1)) \frac{rK^*(1)}{F(K^*(1))} = \delta^{sd, eff} F(2K(1)).
$$

The first inequality is strict unless $K^*(1) = 2K(1)$. The second inequality is strict unless $K^*(1) = K(1)$. Thus, $\delta^{dd, eff} > \delta^{sd, eff}$.

**Proof of Proposition 4.5:** The equilibrium has the following structure. Until the signal is received, the dynamic unravels according to (6). Upon receiving the signal at period $t - 1$, lenders believe that $q_T = 1$ for every $T \geq t$ and offer the loan $(q, \delta) = (1, \varepsilon(1))$. To ensure that lenders’ beliefs are correct, it has to be the case that types with $\delta < \varepsilon(1)$ will prefer not to send a signal.

Consider a type with $\delta < \varepsilon(1)$. Suppose that without signaling the best time to default is $T \geq t$. Let $\pi_{t,T}(\delta)$ be the profit from default at $T$ from the period $t$ point of view. Since $T \geq t$, $\pi_{t,T}(\delta) \geq \pi_{t,T}(\delta) = F(K_t)$.

If type $\delta$ signals at $t - 1$, then upon getting access to an infinite sequence of loans $(1, \varepsilon(1))$, this type will default immediately at period $t$. Its payoff then is $-c_{t-1} + \delta F(K(1))$. If type $\delta$ does not
signal and instead defaults at \( T \), its payoff (from period \( t - 1 \) point of view) is \( \delta \pi_{t,T}(\delta) \). It is easy to establish that

\[
-c_{t-1} + \delta F(K(1)) = -(F(K(1)) - F(K_t))\varepsilon(1) + \delta F(K(1)) < \delta F(K_t) = \delta \pi_{t,t}(\delta) \leq \delta \pi_{t,T}(\delta),
\]

or in other words, payoff of type \( \delta \) from signaling is lower than from defaulting at \( T \). This means that beliefs are correct. That is, if the signal is observed then there will be no default given loan sequence \((1, \varepsilon(1))\).

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