BUSINESS DEVELOPMENT PROGRAMS AND THEIR (LIMITED) IMPACT ON ENTREPRENEURS’ PROFIT

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ABSTRACT

Business training is an essential part of business development programs (BDPs). Yet, its impact on entrepreneurs’ profits is limited. We develop a theoretical framework where limited effect of business training is due to mismatch between a BDPs’ narrow focus on business-promoting strategies and a wider context in which microfinance clients operate. We assume households have multiple sources of income, e.g. business and wage incomes, that are correlated with each other. Furthermore, entrepreneurs’ objectives go beyond profit-maximization and are a combination of business- and livelihood-ambitions (Verrest, 2013). We show that when the wider context (multiple income sources, multiple ambitions) is considered, the training impact varies and can result in post-training profit decline. If, however, we narrow the context to either one income source (business) or one ambition (profit-maximization), the post-training profit always goes up. The paper highlights the importance of applying holistic approach to microfinance clients when evaluating efficiency of business programs.
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Abstract

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Keywords: Business development programs, entrepreneurship, business training, household livelihoods, microfinance

JEL Classification Codes: O12, O16

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1 Introduction

Muhammad Yunis, in his “Banker to the Poor”, argued that teaching microentrepreneurs is a waste (Yunis, 1999). One cannot improve loan use since borrowers already use it efficiently. After all, the fact that the poor are alive despite all the adversity they face, is the best proof of their innate ability. Recent research, however, questions the scope of the “poor but rational” view. Karlan and Valdivia (2011) test whether microentrepreneurs maximize their profit given constraints and find that “... [many microentrepreneurs'] activities prove to be generating an economic loss” (p. 510). de Mel, McKenzie and Woodruff (2008) find that real returns on capital vary with borrowers’ entrepreneurial ability indicating that not everyone has the innate ability to do the best with what one has. Finally, there is no a priori reason why the “poor but rational” view would be true, as the poor lack the human capital and the connections that help build successful business (Banerjee, 2013).

One well-recognized way to make loan use more efficient is by using business-training programs to improve microentrepreneurs’ business knowledge (Prediger and Gut, 2014). Yet, the effect of business-training programs is mixed. Meta studies have shown that while entrepreneurship programs do have positive impact on business knowledge and practice, they have no impact on business expansion or income (Cho and Honorati, 2014). To make matters worse, some studies have documented negative effect of business trainings on profits. Karlan and Valdivia (2011) report that training of female entrepreneurs in Peru led to a noticeable improvement in “bad months”, and less noticeable or even a decline in good months. Karlan, Knight and Udry (2012) study the effect of training on a group of tailors in Ghana. Business literacy of tailors in their sample increased, but profits declined. Bruhn and Zia (2011) conduct training of 445 clients in Bosnia and Herzegovina. They find that while basic financial knowledge improves, there is no improvement in the survival rate of business start-ups, and additionally they find that profit declines, though insignificantly. de Mel, Mckenzie and Woodruff (2014) conduct a study using a sample of 1252 women in Sri-Lanka. They find that, for women with existing businesses, the training had no impact on profit. The training, however, had a positive impact on new owners’ profitability. Finally, Drexler et al. (2014) report that only simplistic training — which consists mostly of basic rules of thumb — improves profit while the complex one does not.

An immediate explanation, which is that training programs are too complicated for microentrepreneurs to comprehend, is not supported by the evidence. Most papers report noticeable increase in business literacy after training. Giné and Mansuri (2014) specifically note that “business training did lead to an increase in business knowledge, so lack of understanding is not the issue” (p. 19). Another possible explanation is that the limited impact of training is due to “improved accounting”. Many papers investigate whether this channel is responsible for limited impact and find that the answer is negative. Drexler et al. (2014) find that there is a reduction in mistakes (and more consistency across measures how people calculate profits or sales), yet they do not find that it biases their results. de Mel et al. (2014) compares self-reported profits to revenue and cost figures and control for detailed measures of accounting practices as a further robustness check.
They find no significant evidence that training changed reporting.

One weakness of business development programs (BDPs), and the business-trainings they provide, is that their narrow focus on business-promoting strategies ignores a wide economic context in which microfinance clients operate. First, many microfinance clients are neither interested nor “...particularly good at growing [their] businesses” (Banerjee, 2013, p. 512). In a survey conducted in India 80%, of parents hoped their children would get government job, while 0% hoped their children would build successful business (Banerjee and Duflo, 2011). Verrest (2013) argued that BDPs “are relevant to only a minority of entrepreneurs” due to variations in household vulnerability and entrepreneurial ambition (p. 58). Brooks et al. (2016) compare the effectiveness of commonly generic business-training programs offered to residents of Dandora (urban slum northeast of Nairobi) with mentorship by local successful entrepreneurs and find only the latter to be (temporarily) effective. Second, microentrepreneurs do not view their business activities solely as a way to bring in more money. Instead, they consider it either as a valuable diversification tool in order to deal with irregularity in income sources (Krishna, 2004); or to reduce household’s vulnerability to negative shocks such as job-loss or illness (Ellis, 2000); or for consumption and income smoothing (Bateman and Chang, 2009; Banerjee and Duflo, 2011).

The goal of this paper is to develop a theoretical model that shows how a mismatch between BDPs’ business-promoting goals and a complex reality in which microentrepreneurs run their businesses can be responsible for a negative effect on post-training profits even when the training itself is efficient.

Our model is as follows. There is a microentrepreneur with two sources of income: business activity and non-business activity, e.g. farming or wage employment. Non-business income does not require capital investment, while income from business activities does. The total income depends on the amount of capital invested, labor allocation between the two activities, and a state of nature. States determine relative profitability of non-business and business incomes, with higher states making business activity more profitable. Importantly, having multiple sources of income is quite common among poor households, yet this assumption is rarely used in the theoretical microfinance literature. A common approach is to focus on the business part of entrepreneurial income by, for example, normalizing all other incomes to zero. However, as I show in this paper, assuming away multiple sources of incomes comes with loss of generality. The training outcome differs in the setting with one sources of income versus multiple sources. Only in the latter case the post-training profit can decline.

The microentrepreneur has two different ambitions: business-oriented ambition and livelihoods-oriented ambitions. We model a business-oriented ambition as maximizing expected profit, we model a livelihoods-oriented ambition as maximizing the “rainy day” profit, or more formally the worst-case profit. The microentrepreneur’s utility is assumed to be a weighted average of the two

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1The focus on a business part of the household’s income is a common assumption starting from classical papers such as Besley and Coate (1995) or Ghosh and Ray (2001) to more recent papers including Chowdhury (2005), Ahlin and Waters (2014), de Quidt et al. (2015) and Shapiro (2015).

2Broadly speaking, the livelihoods approach applies a holistic view on the poverty that includes economic, social, infrastructural and environmental factors. For example, Verrest (2013) classifies households with a livelihoods-
ambitions: business-oriented ambition (expected profit) and livelihoods-oriented ambition (rainy day profit).

The microentrepreneur takes a business training course, which introduces a new business practice. The new practice is superior to the old one in that in every state it makes the capital more productive and, consequently, the business activity more profitable. The training does not affect profitability of the non-business activity. The microentrepreneur decides whether to adopt the new practice or not and then takes a loan from a credit provider, such as a microfinance institution, and invests it into her business. When the state of nature is realized, the microentrepreneur chooses how to allocate her labor between employment and business activity, and earns profit.

We show that the training (weakly) improves microentrepreneur’s utility and, therefore, from the welfare point of view business training can be viewed as having a positive effect. Even if the post-training expected profit declines, the borrower is more than compensated by an increased insurance of having more funds in the worst-case scenario.

As for expected profit, the impact of training depends on microentrepreneurs’ circumstances and ambitions. If either the microentrepreneur’s business ambition is sufficiently strong, or if the microentrepreneur has only one source of income — business activity — then the training always results in a higher post-training profit. In the former case, microentrepreneurs’ goals and environment are sufficiently aligned with BDPs’ focus on business promoting strategies. In the latter case, the microentrepreneur does not have other sources of income to cushion herself against negative shocks and has to increase investments into her business activities. The superiority of the new practice then ensures that the post-training profit goes up, even if it was not a microentrepreneur’s objective.

If, however, the microentrepreneur has two sources of income and sufficiently strong livelihoods-ambition then it is possible that post-training profit will decline. In this case BDPs’ training is less relevant to microentrepreneurs’ needs and can have negative impact. The intuition is as follows. Consider a livelihood-oriented entrepreneur who wants to maximize her rainy day profit. After the training, states with higher capital profitability are less of a concern for such an entrepreneur. In these states business activities generate enough funds to cushion against negative shocks to other sources of income. Now, it is states with lower capital profitability that are more likely to be rainy-day-states and, therefore, have a stronger effect on the utility of a livelihoods-oriented entrepreneur. Since states with lower capital profitability require lower capital investment, the entrepreneur optimally chooses to invest less capital. Instead of taking advantage of the improved profitability by investing more, the entrepreneur invests less which then can result in a lower expected profit.

This is where the assumption of multiple income sources plays its role. With multiple sources of incomes, the livelihood-oriented microentrepreneur finds it safer to rely on non-business sources of income to protect herself against negative business shocks, diminishing the impact of business training. When the only source of income is business, however, this option is not available. The motivation as those whose goal is to secure their livelihoods be it by ensuring that they have “an apple for a rainy day”, or to increase consumption, or to have a hobby.
microentrepreneur then finds it optimal to take a larger loan which then results in a higher expected profit.

In the second part of the paper, we develop a behavioral justification for our specification of microentrepreneur’s utility. Our primary focus is on ambiguity aversion, though as a robustness check, in the Appendix we also look at the risk aversion case. The reason we primarily focus on ambiguity aversion is two-fold. First, the role of ambiguity aversion in the adoption of new practices has been well-documented in development literature and it is distinct from the risk-aversion’s effect. Second, it provides an additional insight on how the efficient training can result in the post-training profit decline. Heath and Tversky (1991), and Fox and Tversky (1995) developed a link between ambiguity aversion and an individual’s perceived competence/ignorance, so called the comparative ignorance hypothesis. According to the hypothesis, ambiguity aversion is triggered when an individual is reminded of someone more competent, or otherwise made to perceive oneself as more ignorant.

Arguably, business training can have a similar effect. Exposing microentrepreneurs to more knowledgeable individuals, such as trainers or successful entrepreneurs, has the potential of negatively affecting an individual’s perceived competence, thereby triggering and/or increasing microentrepreneurs’ ambiguity aversion. We show that the effect of training depends on the strength of the perceived ignorance effect, i.e. on microentrepreneurs’ perception of the ambiguity of the new practice. If the effect is small, then the microentrepreneur adopts the new practice and the expected profit goes up; if it is moderate, the microentrepreneur adopts the new practice but the expected profit declines; if it is extremely high, the microentrepreneur does not adopt the new practice.

We can use this result to explain how a simplistic training can have a positive effect on the profit when a more complicated one does not (Drexler et al., 2014). Notably, the explanation does not assume that microentrepreneurs do not fully understand a more complex training. Both simple and complicated trainings are equivalent in terms of their effect on the microentrepreneurs’ profit functions. The only difference between the two is that the complex training is perceived as more ambiguous which then translates into negative effect on post-training profit. It also generates two testable predictions: first, it is microentrepreneurs with an extremely high degree of ambiguity aversion towards the new practice who are less likely to adopt it. Second, among the microentrepreneurs who adopt the new practice, those with higher ambiguity aversion are more likely to experience a decline in post-training profit.

Overall, the contribution of the paper is as follows. First, to the best of our knowledge this is the first theoretical paper providing an explanation to a limited, and possibly negative, effect of training

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3Engle-Warnick et al. (2007) study the lack of adoption of new farming technologies among farmers in Peru and find that farmers’ ambiguity aversion — but not risk aversion — predicts farmer’s technology choice. Braham et al. (2014) study adoption of genetically modified soy and corn seeds. They also show that it is ambiguity and not risk aversion that had a large impact on adoption decisions.

4For instance, when subjects in Fox and Tversky (1995)’s experiments were told that trained professionals participated in the same experiment, they exhibited a higher degree of ambiguity aversion than subjects in the control group.
on the profit. McKenzie and Woodruff (2014) offered an empirical explanation, arguing that such issues as sample size and sample heterogeneity made it harder to detect the effect of training on profitability. In a follow-up paper, however, de Mel, McKenzie and Woodruff (2014) addressed those issues by using a large and homogeneous sample of female entrepreneurs. The authors found little impact on the profitability and concluded that "the lack of impacts in most of the existing literature ... may not be just due to power issues" (p. 200). They also conjectured that business training programs might be less effective than previously thought. Second, differently from earlier theoretical literature, our model provides a holistic view of households by explicitly taking into account multiple sources of incomes, diversification needs and a variation in entrepreneurial ambitions. Third, we show that the holistic modeling of the microentrepreneurial decision is crucial to understanding how efficient training can have mixed to negative impact. Only when diversification and lack of entrepreneurial ambition are introduced, can the training have negative impact on profit. Finally, within our framework we explain how simplistic training can have better effect than complex training as documented in Drexler et al. (2014).

The rest of the paper is organized as follows. Section 2 introduces the basic framework, section 3 studies the effect of training and shows how it can lead to a lower profit. Section 4 introduces ambiguity aversion and studies the complexity of training. All the proofs are given in Appendix A. The framework with risk-averse microentrepreneurs is described in Appendix B.

2 Model

2.1 Microentrepreneur’s Ex-Post Profit. Choice of Labor.

Consider a microentrepreneur whose profit comes from two sources. The first source is non-business income, e.g. wage employment or farming. It requires labor input but does not require capital. The second source is income from business, or entrepreneurial, activities that requires both labor and capital investment. Another difference between the two sources of income is that when we introduce the business-training we will assume that it affects only profits from from business activities.

That households commonly rely on diversified income portfolios with multiple income sources — including subsistence and farming activities, wage employment, small-scale enterprises, and temporary or permanent migration — is well-documented. For example, for rural households in South Asia, 60 percent of household income comes from non-farming sources; in sub-Saharan the number is in a range between 30 to 50 percent; and in southern Africa it may attain 80-90 percent (Ellis, 2000, p. 233).^{5}

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^{5}To provide a more specific numbers: a survey of households in Masaka district, Uganda, showed that for an average household 64% of its income came from farm income, 20% from business profits and 10.6% from wages (Table 3.1, Ellis 2000). Survey of households in Mamone, a poor semi-arid community in South Africa, showed that the primary income source was remittances and other transfers (63.4%), wages accounted for 9.1%, business activities for 6.3% and farming activities for 12.8% (Table 3.2, Ellis, 2001). In Botswana wage employment accounted for 21.5% of household income portfolio, crop and livestock farming for 45.8%, other activities (beer brewing, basket weaving, carpentry) for 18.5% (Valentine, 1993).
In what follows we make a simplifying assumption that non-business income is a linear function of labor. The most natural example would be when non-business activity is a wage employment, which is the interpretation we will employ throughout the paper. This is done primarily for simplifying reasons, as the continuity ensures that having a small degree of either convexity or concavity in non-business income will not affect the results.

The wage income is based on wages in state $s$, $w_s$, and the number of hours worked at a regular job, $L_w$. The business income is determined by the state of the nature and the production function, $F(K, L)$, that depends on the invested capital, $K$, and the labor, $L$. The microentrepreneur has $T$ units of labor to split between the regular employment and her business, $L_w + L = T$. The microentrepreneur does not, however, have access to capital and borrows it at the interest rate $R$.

Consider a microentrepreneur whose profit comes from two sources. The first source is the wage income from regular employment opportunities. It requires labor input but does not require capital. The second source is income from business activities that requires both labor and capital investment. The wage income is based on wages in state $s$, $w_s$, and the number of hours worked at a regular job, $L_w$. The business income is determined by the state of the nature and the production function, $F(K, L)$, that depends on the invested capital, $K$, and the labor, $L$. The microentrepreneur has $T$ units of labor to split between the regular employment and her business, $L_w + L = T$. The microentrepreneur does not, however, have access to capital and borrows it at the interest rate $R$.

The state of nature is the only source of uncertainty and we label states by integer numbers from 1 to $n$, $s \in \{1, \ldots, n\}$. The probability of state $s$ is $p(s)$. The ex-post profit function given the realized state, $s$, microentrepreneur’s choice of capital and labor is:

$$\pi^{ex-post}(s, L, K) = w_s \cdot (T - L) + sF(K, L) - RK. \quad (2.1)$$

Several technical assumptions are imposed on the production function. We assume that $F$ is an increasing function of its inputs, $F'_K > 0, F'_L > 0$, and it is a strictly concave function: $F''_{KK} < 0, F''_{LL} < 0$ and $F'_K F''_{LL} - (F''_{KL})^2 > 0$. We also assume that $F(\cdot, K)$ is not too concave, as a function of $K$. Specifically we require that $F'_K + K F''_{KK} \geq 0$. Capital and labor are assumed to be complements, $F''_{KL} \geq 0$. Finally: $F'_K(0, L) = +\infty, F'_L(K, 0) = +\infty$ and $F'(+\infty, L) = 0$ for any $L > 0$.

The timing is as follows. First, the microentrepreneur chooses $K$, then the state of nature is realized, then the microentrepreneur allocates labor between the employment and her enterprise. Thus, we assume that the choice of labor is flexible and can be adjusted given the realized state of nature. For example, if the microentrepreneur becomes unemployed, $w_s = 0$, she will adjust by using all of her labor for the business activity. Capital investment, on the other hand, is less flexible, as it requires external financing, and is inherently risky, as it is made before the uncertainty is realized.

Given that labor choice is made after the state realization, it is chosen to maximize the ex-post
profit, \( L^*(s, K) = \arg \max_L \pi^{\text{ex-post}}(s, K, L) \). When the solution is interior, \( L^*(s, K) \) is given by the FOC

\[-w_s + s F'_L(K, L^*) = 0. \quad (2.2)\]

The corner solutions is possible too. By our assumptions on \( F'_L \), \( L^* \) cannot be zero. It can, however, be equal to \( T \). That occurs for states \( s \) such that: \( s \geq w_s / F'_L(K, T) \).

Let \( L^*(s, K) \) denote the optimal labor choice given capital investment and realized state \( s \). For a given \( s \) and given \( K \) the state-profit function, is

\[ \pi(s, K) = w_s \cdot (T - L^*(s, K)) + s F(K, L^*(s, K)) - RK. \]

The next two propositions characterize properties of state-profit functions. All the proofs are given in the Appendix. The first proposition is an immediate corollary of concavity of production function.

**Proposition 1** State-profit functions \( \pi(s, K) \) are differentiable, concave, single-peaked functions of \( K \).

So far we have not impose any requirements on how the two sources of income, non-business and business, are related to each other. Arguably, many risks affecting income sources available to poor households, e.g. own-farm production and agricultural wage labor, exhibit a high correlation (Ellis, 2000, p. 60) with events such as draughts adversely affecting all income streams simultaneously. At the same time, the governmental programs can result in negative correlation among income sources. In Botswana, for example, the government drought relief program during 1985-86 created wage employment opportunities substituting for decreased share of livestock in crops in income portfolios (Valentine, 1993). Furthermore, households themselves try to find income sources that are not correlated with their primary activity in order to reduce their vulnerability. For instance, in many African societies it is customary for households to maintain strong links between rural and urban branches of the family, thereby providing access to income sources that are not correlated with each other (Berry, 1993).

The restriction we will impose on wages will allow for both negative and positive correlation between the two income sources. Specifically, we will assume that \( w_t - \frac{w_s}{t} w_s > 0 \) when \( s > t \). This assumption is satisfied whenever wages decrease with states (negative correlation), when they are constant (independence), or when they increase with states (positive correlation) but at a declining rate. When this assumption is satisfied, then in higher (lower) states business activity is more (less) marginally profitable. In particular, as the next proposition shows, \( L^*(1, K) \leq \cdots \leq L^*(n, K) \) and \( \pi''_{sK} > 0 \). The former means that it is optimal to allocate more labor to a business activity (to employment) for higher (lower) states. The latter implies that \( K^*(1) < \cdots < K^*(n) \) where \( K^*(s) = \arg \max_K \pi(s, K) \). That is, in higher states, it is optimal to invest more capital.

**Proposition 2** Assume that \( \{w_s\} \) is such that \( w_t - \frac{w_s}{t} w_s > 0 \) when \( s > t \). Then \( L^*(1, K) \leq \cdots \leq L^*(n, K) \).
$L^*(n, K)$ and $\pi'_K(s, K) > \pi'_K(t, K)$ whenever $s > t$.

In what follows we will refer to $\pi'_K(s, K)$ being greater than $\pi'_K(t, K)$ when $s > t$ as complementarity; and to its corollary, $K^*(1) < \cdots < K^*(n)$, as weak complementarity.\(^6\) Complementarity means that states are capital are complements in that capital is more profitable in higher states.

### 2.2 Microentrepreneur’s Ex-Ante Utility. Choice of Capital

The timing of our model is such that $K$ is chosen prior to the realization of uncertainty, and the choice depends on microentrepreneur’s ex-ante objective. We assume that the microentrepreneur is not solely focused on maximizing her profit. Instead, her utility is assumed to be a combination of two ambitions: business-oriented ambition and livelihoods-oriented ambition. We model business-oriented ambition as expected-profit maximization. We model livelihoods-oriented ambition as maximizing the “rainy day” profit, where we define the rainy day profit as the profit in the worst-case state, $\min_s \pi(s, K)$.\(^7\) We will use terms “rainy day profit” and “worst-case profit” interchangeably throughout this paper.

We assume that the microentrepreneur’s ex-ante utility is the weighed average of business-oriented and livelihoods-oriented ambitions:

$$U(K) = (1 - \eta) \sum_s p(s)\pi(s, K) + \eta \min_s \pi(s, K). \quad (2.3)$$

Parameter $\eta$ is the weight that the microentrepreneur puts on her livelihood ambition, as captured by the rainy-day profit. When $\eta = 0$, the microentrepreneur has only business-oriented ambition, and her objective is expected profit. When $\eta = 1$ the microentrepreneur has only livelihood-oriented ambition and her objective is the rainy-day profit.

In what follows, we will denote the worst-case profit, $\min_s \pi(s, K)$, as $\pi_w(K)$. We will denote the capital that maximizes the expected profit as $K^*$; the capital that maximizes the microentrepreneur’s utility, $U(K)$, as $K^*_\eta$; and the capital that maximizes the worst-case profit as $K_w$. By definition, $K^*_0 = K^*$ and $K^*_1 = K_w$.

The next two Propositions characterize properties of the rainy-day profit and the microentrepreneur’s utility, (Proposition 3), as well as their corresponding optima (Proposition 4). Both propositions are proved in the Appendix A. Proposition 3 and the first part of Proposition 4 are straightforward corollaries of state-profit functions being concave. The second part of Proposition 4 follows from the weak complementarity.

**Proposition 3** The rainy-day profit, $\pi_w(K)$, and the microentrepreneur’s utility, $U(K)$, are concave and single-peaked functions. Both functions are continuous and admit left and right derivatives.

\(^6\)The complementarity condition implies weak complementarity but not vise versa. For some results in this paper the weak complementarity which, as the title suggests is a weaker condition, will be sufficient.

\(^7\)Rainy day profit is not the only possible motivation of microentrepreneurs with livelihoods-ambition. Other motivations can also include increased consumption and to have a hobby, see Verrest (2013, p. 63).

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that are non-increasing.

**Proposition 4** For a given \( \eta \), the optimal capital level, \( K^*_\eta \), satisfies

\[
U'(K^*_\eta -) \geq 0 \geq U'(K^*_\eta +). \tag{2.4}
\]

For the optimal worst-case capital, \( K_w \), there are two alternatives. Either, \( K_w = K^*(\sigma) \), where \( \sigma \in \arg\min_s \pi(s, K^*(s)) \), or there exist two states, \( s < s' \), such that \( \pi(s, K_w) = \pi(s', K_w) \leq \pi(t, K_w) \) and \( \pi'_K(s, K_w) < 0 < \pi'_K(s', K_w) \).

### 2.3 Example

We conclude this section with the following example. Let the income from business activity be given by \( sF(K, L) = s \ln(K \cdot L) \), and hourly wages be \( w_s = w \) so that capital shocks are independent of labor shocks. Earlier we discussed a complicated nature of correlation among different income sources, which is why we chose to use constant wage for our example so that nothing depends on whether income sources are positively or negatively correlated. When \( w_s = w \) for every state \( (2.1) \) can be re-written as

\[
\pi^{ex-post}(s, L, K) = w(T - L) + s \ln(KL) - RK, \tag{2.5}
\]

and the optimal labor choice is

\[
L^*(s, K) = \begin{cases} 
\frac{s}{w} & \text{if } \frac{s}{w} \leq T \\
T & \text{o/w}
\end{cases}
\]

That \( L^*(s, K) \) does not depend on \( K \) is a convenient corollary of having logarithm as the production function and, generally, will not be the case. Plugging \( L^*(s, K) \) into \( (2.5) \) we get

\[
\pi(s, K) = \begin{cases} 
wT - s + s \ln \frac{sK}{w} - RK & \text{if } \frac{s}{w} \leq T \\
s \ln(TK) - RK & \text{o/w}
\end{cases} \tag{2.6}
\]

It is immediate to verify that state-profit functions are single-peaked, concave and satisfy the complementarity condition, since \( \pi''_{sK} > 0 \). For a given state, \( s \), the optimal capital level is \( K^*(s) = s/R \) and is an increasing function of state.

Figure 1 plots state-profit functions \( (2.5) \) for values \( w = 11, T = 10, R = 0.2 \) and states 1 through 3. It also plots the worst case profit and expected profits. Point \( K_w \approx 7.5 \) shows the investment level that maximizes the rainy day profit, point \( K^* = 13.5 \) maximizes the expected profit. In state \( s = 1 \) business activities are the least productive. At \( K^*(1) = 5 \) most of the income comes from wages and, due to low business profitability, \( K^*(1) \) is the lowest among the three states.
In state $s = 3$ the capital is the most productive. The optimal investment for state 3 is $K^*(3) = 15$ and it is the highest among the three states. For the microentrepreneur whose objective is to maximize the rainy day profit, the optimal investment level is $K_w \approx 7.5$. Lower levels of capital are suboptimal since the microentrepreneur does not fully utilize her business. Higher levels of capital make an entrepreneur exposed to negative capital-productivity shocks. Capital investment of $K_w$ balances labor and business incomes and maximizes the rainy day profit. Regardless of which state is realized, the microentrepreneur guarantees to herself profit of at least $\approx 107.54$.

3 Effect of Training

3.1 Basic Setup

We will apply the framework developed in the previous section to study the impact of a microentrepreneurs’ training offered by business development programs (BDPs). The scope and level of training vary between different BDPs. In Karlan, Knight and Udry (2012), the training was on a small scale and involved targeted lessons such as keeping time and transaction records, separating business and personal money, etc. de Mel, McKenzie and Woodruff (2014), on the other hand, used the global Start-and-Improve Your Business (SIYB) training program. The SIYB is a program with an outreach of more than 4.5 million people in more than 95 countries. It involves 3 to 5 day training courses and covers topics such as organization of staff, record keeping and stock control, marketing and financial planning.
We introduce the training into the framework of Section 2 as follows. We assume that the microentrepreneur takes a training to learn about a new business practice, or technology. There is no cost associated with taking the training and no cost associated with implementing the new practice. Thus it is always (weakly) optimal for the microentrepreneur to undertake the training. Whether it is optimal to adopt the new practice or not depends on which practice will result in higher utility. In what follows, we will use superscript \( \text{new} \) to refer to variables and functions related to the new practice. For example, \( \pi_{\text{new}}(s, K) \) is the state-profit function under the new practice; \( K^*_{\text{new}} \) is the capital level that maximizes the new utility, i.e. the utility if the new practice is adopted.

The microentrepreneur adopts the new practice if and only if she finds it optimal, that is:

\[
U_{\text{new}}(K^*_{\text{new}}) = (1 - \eta)E_s\pi_{\text{new}}(s, K^*_{\text{new}}) + \eta\pi_{\text{w}}(K^*_{\text{new}}) > (1 - \eta)E_s\pi(s, K^*_\eta) + \eta\pi_{\text{w}}(K^*_\eta) = U(K^*_\eta).
\]

(3.1)

The expression on the left is the utility given the new state-profit functions and new optimal capital, \( K^*_{\text{new}} \). The expression on the right is pre-adoption utility given the old state-profit functions and old optimal capital \( K^*_\eta \). We can re-write (3.1) as:

\[
(1 - \eta)\left(E_s\pi_{\text{new}}(s, K^*_{\text{new}}) - E_s\pi(s, K^*_\eta)\right) + \eta\left(\pi_{\text{new}}(K^*_{\text{new}}) - \pi_{\text{w}}(K^*_\eta)\right) > 0 \quad (3.2)
\]

Thus, the microentrepreneur switches to the new practice if, and only if, a change in the expected profit plus the change in the worst-case profit is positive. The strength of livelihoods-ambition, \( \eta \), measures a relative importance of the two terms. The microentrepreneur can switch to the new practice even if the post-training expected profit will decline. That would occur if the improvement in the rainy-day profit is sufficiently high to outweigh the decline in the expected profit. Similarly, the microentrepreneur might choose not to adopt the new practice even if a change in the expected profit is positive. That would occur if there is a sufficiently large decrease in the rainy-day profit to outweigh the expected profit increase.

Given the focus of BDPs on microentrepreneur’s business activities, we assume that the training impacts only the business-activity income, and not the employment income. Specifically, the effect of the training is that it makes capital more productive:

\[
\pi_{\text{ex-post, new}}(s, L, K) = w_s \cdot (T - L) + sF(\lambda_s K, L) - RK,
\]

where \( \lambda_s > 1 \) for all \( s \). Thus the training is beneficial as it improves profit in every state \( s \) and for every \( K \): \( \pi_{\text{ex-post, new}}(s, L, K) \geq \pi_{\text{ex-post}}(s, L, K) \). The assumption that training is profit-improving for every \( s \) and \( K \) is intentionally generous. It assumes away the profit decline due to training inefficiency, and allows us to focus on the post-training profit decline due to mismatch between microentrepreneur’s and BDP’s goals.

To impose further structure we assume that \( \lambda_1 < \cdots < \lambda_n \) so that the training effect is stronger in higher states that are more business-favorable. For example, if trainees learn how to
find cheaper suppliers or become more efficient at inventory management that will have stronger effect during good states when the demand is high.\footnote{Brooks et al. (2016) mention Prudence who was a participant of one the treatment (the mentor treatment) and who used to purchase inventory from suppliers at the entrance of a market area. After training she started to purchase at stalls deep into the market and only after comparing prices. Her cost dropped from 250 Ksh to 100 Ksh as a result, while she kept her sale price exactly the same. Clearly, a reduction in marginal cost has stronger effect during states that are favorable to the business activity.} Mathematically, the assumption \( \lambda_1 < \cdots < \lambda_n \) has an advantage that Proposition 2 will hold for post-training state-profit functions. However, this particular assumption is not essential for a possibility of post-training profit decline. We will briefly describe the case \( \lambda_1 > \cdots > \lambda_n \) in Section 3.3.

It is straightforward to verify that post-training state-profit functions remain to be concave and single-peaked functions of \( K \). Furthermore, as mentioned earlier, one can easily to adopt the proof of Proposition 2 to the case of new production function and show that Proposition 2 and weak complementarity hold for \( \pi_{\text{new}}(s,K) \). Thus new state-profit functions satisfy all the properties derived in Section 2.

### 3.2 Effect of Training. Two Benchmarks.

We will consider two benchmarks. The first benchmark is when the microentrepreneur is business-oriented, \( \eta = 0 \), and the second benchmark is when the microenterprise is the only source of income. It turns out that in both benchmarks the training always results in expected-profit improvement. The intuition is straightforward. BDPs are designed to promote business-oriented strategies such as business growth or production strengthening (Verrest, 2013). The training improves profitability of business activities ignoring their diversifying role with other sources of income as well as a possible lack of entrepreneurial ambition. When \( \eta = 0 \), the microentrepreneur’s only objective is a business-oriented ambition. The training’s focus is perfectly aligned with microentrepreneur’s objectives and profit goes up. When the only source of income is business activity then the diversification component is absent in the decision-making. The microentrepreneur does not have other sources of income to diversify against negative business shocks. The safest strategy available is to increase the capital investment. The superiority of the new practice then ensures that the post-training profit goes up, even though, it was not a microentrepreneur’s objective.

The next Proposition shows that for the first benchmark, when a microentrepreneur has only one source of income, the training effect on profit is always positive.

**Proposition 5** If business activity is the only source of income then post-training expected profit will increase.

Now consider the benchmark, where the microentrepreneur has two sources of incomes, but her only ambition is business-oriented, \( \eta = 0 \). Under the old practice, the optimal capital investment is \( K^* \) and the expected profit is \( E_s \pi(s,K^*) \). If the new practice is adopted, the optimal capital level, \( K_{\text{new}}^* \), satisfies \( E_s \pi_{\text{new}}(s,K_{\text{new}}^*) = 0 \). The business-oriented microentrepreneur will always
prefer the new practice and will have a higher expected profit after the training:

\[ E_s\pi(s, K^*) < E_s\pi^{new}(s, K^*) < E_s\pi^{new}(s, K^{*new}). \] (3.3)

Here, the first inequality is due to the fact that \( \pi^{new}(s, K) > \pi(s, K) \), and the second inequality is due to the fact that \( K^{*new} \) is the optimal capital level for the new expected profit.

The two inequalities in (3.3) represent the two effects that an adoption of the new practice has on the expected profit. The first effect is the profit improvement effect, which is \( E_s\pi^{new}(s, K^*) - E_s\pi(s, K^*) \). Based on our assumptions, it is always positive. Even if the microentrepreneur does not change capital investment, the expected profit goes up. The second effect is the capital adjustment effect, which is \( E_s\pi^{new}(s, K^{*new}) - E_s\pi^{new}(s, K^*) \). Since \( K^* \) is no longer optimal, the microentrepreneur will invest \( K^{*new} \). In the case of the business-oriented microentrepreneur, the second effect is also always positive: \( E_s\pi^{new}(s, K^{*new}) - E_s\pi^{new}(s, K^*) > 0 \). This proves that for business-oriented microentrepreneur the post-training profit will increase. This result is summarized in Proposition 6.

**Proposition 6** If the microentrepreneur has only business-oriented ambitions, \( \eta = 0 \), then post-training expected profit will always increase.

By continuity, if the microentrepreneur has both business-oriented and livelihood-oriented ambitions but the former dominates then the post-training expected profit will increase.

**Corollary 1** If \( \eta > 0 \) but sufficiently close to zero then post-training expected profit will always increase.

### 3.3 Post-training profit decline.

It turns out that if the microentrepreneur has two sources of income and the livelihood-ambition then it is possible for training to have a negative effect on the expected profit. Consider a microentrepreneur with \( \eta > 0 \). The microentrepreneur’s utility under the old practice is \( U(K_{\eta}^*) \), and under the new practice is \( U^{new}(K_{\eta}^{*new}) \), where \( U(K) = (1 - \eta)E_s\pi(s, K) + \eta\pi_w(K) \), and \( U^{new}(K) = (1 - \eta)E_s\pi^{new}(s, K) + \eta\pi_w^{new}(K) \). Similarly, to the business-oriented microentrepreneur,

\[ U(K_{\eta}^*) < U^{new}(K_{\eta}^*) < U^{new}(K_{\eta}^{*new}). \]

Thus, regardless of the strength of the livelihood-oriented ambition, the training leads to a higher utility and a microentrepreneur will always prefer the new practice and will always adopt it. From welfare perspective it indicates that the business training can be viewed as having a positive effect on microentrepreneurs’ well-being regardless of its effect on expected profit. Even if the post-
training expected profit declines the borrower is more than compensated by an increased insurance of having more funds in the worst-case scenario.

That microentrepreneurs tend to follow, at least in the short-run, the practices they learn during the training course is well-documented in the literature. In Karlan, Knight and Udry (2012) the authors document that “the consultants’ recommendations were adopted for a time” (p. 5). Similarly, in Karlan and Valdiva (2011) many people responded in a follow-up survey that they switched to the new practice. Table 8 in McKenzie and Woodruff (2014)’s survey summarizes the effect of training on business practice adoption with the conclusion that “almost all studies find a positive effect of business training on business practices” (p. 67).

However, the fact that the microentrepreneur is willing to adopt the new practice does not mean that the post-adoption profit will increase. As in the case of business-oriented microentrepreneur, the adoption of the new practice has two effects on the expected profit:

\[
E_s \pi(s, K^*_{new}) - E_s \pi(s, K^*_n) = \left[ E_s \pi_{new}(s, K^*_n) - E_s \pi(s, K^*_n) \right] + \left[ E_s \pi_{new}(s, K^*_{new}) - E_s \pi_{new}(s, K^*_n) \right]
\]

(3.4)

The first term is the profit improvement effect. It is equal to a change in the expected profit if the microentrepreneur does not adjust the capital. Similarly to the case of business-oriented microentrepreneur, the profit improvement effect is always positive given that \( \lambda \)'s are greater than 1. The second term is the capital adjustment effect. The capital level that maximizes the microentrepreneur’s post-training utility changes which has an effect on the expected profit. Differently from the case of business-oriented microentrepreneur, the capital adjustment effect can be negative. The reason is the mismatch between the microentrepreneur’s focus on her livelihood and the BDP’s focus on expected profit. For example, in the case of \( \eta = 1 \) the microentrepreneur will adjust the capital in order to maximize the rainy day profit, disregarding a potentially negative effect on the expected profit. As we will show, not only can the capital adjustment effect be negative but it can outweigh the profit improvement effect so that the total effect is negative.

Recall our example from Section 2.3, where the state profit functions are given by (2.6):

\[
\pi(s, K) = \begin{cases} 
 wT - s + s \ln \frac{sK}{w} - RK & \text{if } \frac{w}{s} \leq T \\
 s \ln(TK) - RK & \text{otherwise}
\end{cases}
\]

and the parameter values are \( w = 11, T = 10 \) and \( R = 0.2 \). Assume there are three states. Probability of states 1 and 2 is 1/10, probability of state 3 is 8/10. We further assume that the microentrepreneur’s only objective is livelihood-oriented ambition, i.e. \( \eta = 1 \).
Figure 2: $\pi(s, K)$ and $\pi_w(K)$ before and after the training. Profit in state 1 did not change; while profit in states 2 and 3 went up. The capital adjustment effect is negative $K^\text{new}_w < K_w < K^*$. 

The training makes capital more productive so that after training the business income becomes $s F(\lambda_s K, T - L) - RK = s \ln(\lambda_s K(T - L)) - RK$, where $\lambda_1 = 1$ and $\lambda_2 = \lambda_3 = 1.2$. Figure 2 shows new and old state-profit and rainy-day profit functions. The thick solid lines correspond to old and new rainy-day profits. The thin solid line is $\pi(1, s)$ and the training does not affect state 1. Dashed lines are $\pi(2, K)$ and $\pi^\text{new}(2, K)$. Dotted lines are $\pi(3, K)$ and $\pi^\text{new}(3, K)$. It follows from Figure 2 that for the microentrepreneur with livelihood-oriented ambition, it is optimal to invest less as the result of training: $K^\text{new}_w < K_w$. This is because of the mismatch between the BDP’s goal (business improvement) and the microentrepreneur’s goal (livelihoods-oriented ambition). From the microentrepreneur point of view, after the training the worst-case scenario is more likely to happen when lower states are realized. Specifically, for values of $K \in [K^\text{new}_w, K_w]$ the pre-training worst-case state is $s = 2$, and post-training worst-case state is $s = 1$. Thus, training makes lower states more likely to be the worst-case states, and since lower states require lower capital investment, it is optimal for the livelihoods-oriented microentrepreneur to invest less.

Figure 3 plots the rainy-day profit and expected profit before and after the training. One can see that for a microentrepreneur with the livelihoods-ambition, the post-training expected profit declines: $107.48 \approx E\pi^\text{new}(s, K^\text{new}_w) < E\pi(s, K_w) \approx 107.54$. The profit improvement effect is $E\pi^\text{new}(s, K_w) - E\pi(s, K_w) \approx 0.47$. The capital adjustment effect is $E^\text{new}\pi^\text{new}(s, K^\text{new}_w) - E\pi^\text{new}(s, K_w) \approx -0.53$. It dominates the profit improvement effect and the expected profit declines. This is despite the uniform improvement of all state-profit functions.
The example above considered the case of \( \eta = 1 \) and \( \lambda_1 \leq \ldots \leq \lambda_n \). Neither is essential. By continuity, one can construct a similar example for \( \eta \) less than but sufficiently close to 1. As for the latter, figures 4 and 5 provide an example where it is low states that experience post-training improvement. It is plotted for values of \( w = 30, T = 10 \) and \( R = 0.2 \). Probability of states 2 and 3 is 1/10; probability of state 1 is 0.8. Finally, \( \lambda_1 = \lambda_2 = 1.15 \) and \( \lambda_3 = 1 \). Improvements in state 1 and 2 make state 3 more likely to be the worst-case. The microentrepreneur then find it optimal to invest more so that \( K_w^{\text{new}} > K_w \). However, since \( K_w > K^* \) the expected profit declines.

Figure 4: \( \pi(s, K) \) and \( \pi_w(K) \) before and after the training. Profit in state 3 did not change; while profit in states 1 and 2 went up. The capital adjustment effect is negative \( K_w^{\text{new}} > K_w > K^* \).

Figure 5: Rainy-day profit and expected profit before and after the training: \( E\pi^{\text{new}}(s, K_w^{\text{new}}) \approx 295.30 \) is less than \( E\pi(s, K_w) \approx 295.57 \).

The next proposition provides sufficient conditions for the capital adjustment effect to be negative. We consider a logarithmic production function due to its convenience property that the training does not affect either \( K^*(s) \) or \( K^* \). In this case the capital adjustment effect is determined solely by changes in \( K_w \). When \( K_w < K^* \) and \( \lambda_1 < \ldots < \lambda_n \) then lower states are more likely to be worst-case states after the training, thereby making the entrepreneur invest less, \( K_w^{\text{new}} < K_w \). Thus \( K_w^{\text{new}} < K_w < K^* = K^*^{\text{new}} \) and the capital adjustment effect is negative. The entrepreneur’s investment moves further away from the expected-profit optimum. If \( K_w > K^* \) and \( \lambda_1 > \ldots > \lambda_n \) then higher states are more likely to be worst-case states after the training. Higher states require more capital so that \( K_w < K_w^{\text{new}} \). Thus \( K^* = K^*^{\text{new}} < K_w < K_w^{\text{new}} \) and the capital adjustment effect is negative.\(^9\)

\(^9\)In the paper, we assumed that \( F'_K + K F''_{KK} \geq 0 \), which holds as equality when \( F(K, L) = \ln(LK) \). When \( F'_K + K F''_{KK} > 0 \) then \( K^{\text{new}}(s) > K^*(s) \) for any \( s \), and \( K^{\text{new}} > K^* \). So the capital adjustment is determined by a change in \( K_w \) and a change in \( K^* \) with the latter being always positive, i.e. \( K^{\text{new}} > K^* \). It changes Proposition 7 as follows. For the case of \( \lambda_1 < \ldots < \lambda_n \) and \( K_w < K^* \) Proposition 7 holds as long as \( \lambda \)’s are sufficiently close to
Proposition 7 Assume that $\eta = 1, K_w \neq K^*(s)$ and $F(K, L) = \ln(KL)$. If either

i) $\lambda_1 < \cdots < \lambda_n$ and $K_w < K^*$; or

ii) $\lambda_1 > \cdots > \lambda_n$ and $K_w > K^*$,

then the capital adjustment effect is negative.

We conclude this section with two remarks. First, Proposition 7 shows the negative capital adjustment effect can be associated with either decrease or increase in capital. Second, the negative capital adjustment effect is necessary but not sufficient condition for the post-training profit decline. Clearly, if all $\lambda$’s are very high the the profit improvement effect will dominate the capital adjustment effect and the post-training profit will go up.

4 Ambiguity Aversion and Complexity of Training

4.1 Ambiguity Aversion

In Section 3 we assumed that the microentrepreneur’s utility is the weighted average of the expected profit and the rainy-day profit. In this section we provide a behavioral justification of this assumption based on the concept of ambiguity-aversion.\(^{10}\) We will show that for ambiguity-averse microentrepreneurs, under certain conditions, the objective function will coincide with (2.3). We will then use ambiguity-aversion framework to provide an additional insight on how the efficient training can result in the post-training profit decline.

The literature has documented the role of ambiguity aversion on willingness to switch towards new technologies and practices, and the effect is distinct from risk-aversion. Engle-Warnick et al. (2007) document that farmers in Peru use a traditional variety of potato with low expected yield that, nonetheless, generates enough potatoes to feed a farmer’s family. This is despite the availability of new varieties of potatoes such as the *Papa Caprio* that provide substantial yield improvement. They show that it is ambiguity-aversion and not risk-aversion that was responsible for the crop adoption decision. Similarly, Braham et al. (2014) examined adoption of genetically modified corn and soya beans among 191 Midwestern US grain farmers. Risk preference, measured using a coefficient of relative risk aversion, had no significant impact on adoption. Ambiguity aversion did have a significant effect and expedited adoption of the less ambiguous genetically-modified corn seeds.

Consider an ambiguity-averse microentrepreneur who does not know the underlying state-distribution and instead assumes that it belongs to a set of priors $Q = \{q : q(s) \geq \eta_s, q(s) \geq 0, \sum_s q(s) = 1\}$, where $\eta_s \geq 0$ (LeRoy and Werner, 2001, p. 82). We assume that $(\eta_1, \ldots, \eta_n)$ is

\(^{10}\)We consider the case of risk-aversion in Appendix B.
proportional to correct probabilities, \( \{p(s)\} \), that is \( \eta_s = (1 - \eta) \cdot p(s) \), where \( 1 - \eta = \eta_1 + \ldots + \eta_n \).

The microentrepreneur has maxmin preferences and chooses \( K \) to maximize the expected profit under the worst prior in \( Q \):

\[
\max_{K} \min_{q \in Q} \sum_{s} q(s) \pi(s, K).
\]

It is immediate to verify that the microentrepreneur’s objective function can be re-written as:

\[
U(K) = \min_{q \in Q} \sum_{s} q(s) \pi(s, K) = (1 - \eta) \sum_{s} p(s) \pi(s, K) + \eta \min_{s} \pi(s, K),
\]

which coincides with (2.3). Here \( \{q(s)\} \) is a distribution from \( Q \) and \( \{p(s)\} \) is the objective distribution.

Parameter \( \eta \) has two mathematically equivalent interpretations. The first one, used to derive (4.1), is that \( \eta \) measures the ambiguity of outcomes’ distribution with higher \( \eta \) corresponding to higher ambiguity. When \( \eta = 0 \), \( Q \)’s only element is the objective distribution; when \( \eta = 1 \), \( Q \) contains all possible priors. The second interpretation is that \( \eta \) measures the degree of microentrepreneur’s ambiguity aversion with higher \( \eta \) corresponding to higher ambiguity aversion. When \( \eta = 0 \), the microentrepreneur is risk-neutral; when \( \eta = 1 \), the microentrepreneur is the worst-case profit maximizer. Both interpretations are mathematically equivalent. Economic difference between the two is that, in the former case, the microentrepreneur’s preferences are maxmin and do not depend on \( \eta \); in the latter case, they do.

4.2 Ambiguity Aversion and Complexity of Training

The ambiguity-aversion framework developed above provides an additional insight on how effective training can result in profit’s decline. The behavioral literature on ambiguity aversion has shown that ambiguity aversion can be linked to an individual’s subjective perception of her relative competence (Heath and Tversky, 1991). Fox and Tversky (1995) advanced this idea by developing the comparative ignorance hypothesis. The hypothesis states that ambiguity aversion is produced by a comparison with less ambiguous events, or with more knowledgeable individuals, which makes the notion of competence more salient (see also Fox and Weber, 2002).

In the context of our paper, a microentrepreneur after taking the training faces two alternatives: one is to continue the old practice, which is more familiar to the microentrepreneur; another is to adopt the new practice over which the microentrepreneur can feel less competent and which exposes the microentrepreneur to more knowledgeable individuals, such as trainers or previously trained microentrepreneurs. According to the comparative ignorance hypothesis, the microentrepreneur will then view a new technology as more ambiguous or, using notations of our framework, \( \eta^\text{new} > \eta \).

That the perceived ignorance is relevant to the outcome of business-training is indirectly sup-

\[\text{11Let } s_w(K) \text{ denote the worst state (any worst state if there is more than one) given } K: \pi(s_w(K), K) \leq \pi(t, K) \text{ for any } t. \text{ For a given } K, \text{ the worst prior assigns the smallest probability to all states but the worst, } q_w(s) = \eta_s = (1 - \eta)p(s). \text{ The worst state gets the remaining probability, } q_w(s_w(K)) = \eta_{s_w(K)} + (1 - \eta_1 - \ldots - \eta_n) = (1 - \eta)p(s_w(K)) + \eta.\]
ported by Drexler, Fischer and Schoar (2014)’s result. Drexler et al. find that a simplified rule-of-thumb training produced significant improvement in profits while a standard, but more complex, training did not. The most immediate explanation, which is that microentrepreneurs do not understand the standard training, is not supported by empirical evidence. Most papers report that training does improve knowledge of business practices. In Karlan et al. (2012), business literacy knowledge improved by 0.52 standard deviations after the training. Similarly, Giné and Mansuri (2014) note that “business training did lead to an increase in business knowledge, so lack of understanding is not the issue” (p. 19). The comparative ignorance hypothesis, on the other hand, is consistent with the Drexler, Fischer and Schoar (2014)’s finding. Microentrepreneurs taking a more complex training are more likely to perceive themselves as more ignorant and view new practice as more ambiguous. As we will show in this section, when this is the case, a simplified training can outperform a more complex one.

Consider a microentrepreneur whose current business practice has ambiguity $\eta$. The microentrepreneur takes a business-training course and learns the new practice. As before, the new practice, if adopted, changes the state-profit functions so that $\pi^{new}(s, K) > \pi(s, K)$. However, the microentrepreneur perceives new practice as more ambiguous with $\eta^{new} > \eta$. If the new business practice is adopted, the microentrepreneur’s utility is

$$U^{new}(K) = (1 - \eta^{new})E_{s}\pi^{new}(s, K) + \eta^{new}\pi_{w}^{new}(K).$$

To focus on the effect of a change in $\eta$, we assume that $E_{s}\pi^{new}(s, K^{*\eta^{new}}) \geq E_{s}\pi(s, K^{*})$. That is, if ambiguity of both practices is the same ($\eta^{new} = \eta$) the expected profit goes up. Otherwise, if $E_{s}\pi^{new}(s, K^{*\eta^{new}}) \geq E_{s}\pi(s, K^{*})$ is not satisfied, the expected profit declines due to negative capital adjustment effect, as studied in Section 3, and not due to the new practice being more ambiguous.

The next Proposition shows that practices with higher ambiguity, $\eta$, result in lower utility and lower expected profit. The intuition is straightforward. A more ambiguous practice allows for worse worst-case outcomes, which negatively impacts microentrepreneurs’ utility. As for the expected profit, the fact that it declines with $\eta$ is due to a microentrepreneur’s choice of capital. For higher $\eta$, optimal capital gets closer to $K_{w}$ and further away from the expected-profit maximizing level, $K^{*}$.

**Proposition 8** If $K_{w} < K^{*}$ ($K_{w} > K^{*}$), then $K^{*}_{\eta^{new}}$ is a decreasing (increasing) function of $\eta$. Microentrepreneur’s utility, $(1 - \eta)E_{s}\pi(s, K^{*}_{\eta}) + \eta\pi_{w}(K^{*}_{\eta})$, and expected profit, $E_{s}\pi(s, K^{*}_{\eta})$, are decreasing functions of $\eta$. The worst-case profit, $\pi_{w}(K^{*}_{\eta})$, is an increasing function of $\eta$.

According to Proposition 8, a higher ambiguity of new business practice has a negative effect on the expected profit. We will show that this effect can outweigh a positive effect of the training.

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12 We employ the first interpretation of $\eta$, i.e. it measures the ambiguity of outcomes’ distribution.

13 In the example considered in Section 3.3, the assumption is satisfied for any $\eta$ if $\lambda_{1} = \cdots = \lambda_{n}$. Furthermore, as shown in Section 3, the post-training expected profit goes up for low values of $\eta$, including $\eta = 0$. 
Consider the two inequalities below:

\[
(1 - \eta^{new})E_{s}\pi^{new}(s, K_{\eta^{new}}^{*}) + \eta^{new}\pi^{new}_{w}(K_{\eta^{new}}^{*}) > (1 - \eta)E_{s}\pi(s, K_{\eta}^{*}) + \eta\pi_{w}(K_{\eta}^{*}), \quad (4.2)
\]

\[
E_{s}\pi^{new}(s, K_{\eta^{new}}^{*}) < E_{s}\pi(s, K_{\eta}^{*}). \quad (4.3)
\]

When (4.2) is satisfied, the microentrepreneur is willing to adopt the new practice. When (4.3) is satisfied the post-training expected profit declines. Whether the two inequalities are satisfied or not depends on \(\eta^{new}\).

Let \(\eta^{U} \leq 1\) be a degree of practice ambiguity, such that if \(\eta^{new} = \eta^{U}\), then (4.2) holds with equality, i.e. pre- and post-training utilities are equal. Let \(\eta^{E\pi} \leq 1\) be a degree of practice ambiguity, such that if \(\eta^{new} = \eta^{E\pi}\), then (4.3) holds with equality, i.e. pre- and post-training expected profits are equal.

**Proposition 9** Let \(\eta^{E\pi}\) and \(\eta^{U}\) be as defined above. Then \(\eta < \eta^{E\pi} < \eta^{U}\) and:

i) if \(\eta^{new} \in [\eta, \eta^{E\pi}]\) the microentrepreneur will adopt the new practice and the expected profit will go up;

ii) if \(\eta^{new} \in (\eta^{E\pi}, \eta^{U})\) the microentrepreneur will adopt the new practice but the expected profit will decline;

iii) if \(\eta^{new} > \eta^{U}\) the microentrepreneur will not adopt the new practice.\(^{15}\)

Propositions 8 and 9 show that the training ability to boost the microentrepreneurs’ profit varies depending on their \(\eta^{new}\). It is the strongest for microentrepreneurs with small \(\eta^{new}\), and it decreases as \(\eta^{new}\) goes up. When the difference in practices’ ambiguities is relatively small, \(\eta \leq \eta^{new} < \eta^{E\pi}\), the microentrepreneur will prefer the new practice and the expected profit will go up. Indeed, when \(\eta^{new} = \eta\) the training has positive effect on profit. By continuity, it’s also the case for a small increase in ambiguity, as it is not strong enough to undermine the positive effect of training. When the difference in practices’ ambiguities is intermediate, \(\eta < \eta^{E\pi} < \eta^{new} < \eta^{U}\), the microentrepreneur will adopt the new practice but the expected profit will decline. Now \(\eta^{new}\) is sufficiently high to cause a decline in the expected profit by distorting the capital choice, but is not sufficiently high to prevent the adoption of the new practice. Finally, when the difference in practices’ ambiguities is large, \(\eta < \eta^{E\pi} < \eta^{U} < \eta^{new}\), the microentrepreneur will not switch to a new practice at all. This result can explain the finding of Drexler, Fischer and Schoar (2014). Based on the competence hypothesis, a complex training can result in microentrepreneurs perceiving new practice as more ambiguous than a simple rule-of-thumb training. If \(\eta^{new, simple} < \eta^{E\pi} < \eta^{new} < \eta^{U}\), the training’s positive effect on profit improvement is large enough to outweigh any negative effects due to an increase in \(\eta\).

\(^{14}\)If \(\lambda^{'}s\) are large enough then it is possible that either \(\eta^{U} \leq 1\) or \(\eta^{E\pi} \leq 1\) do not exist. In this case, the positive effect of the training on profit improvement is large enough to outweigh any negative effects due to an increase in \(\eta\).

\(^{15}\)If \(\eta^{U} \leq 1\) does not exist the condition \(\eta^{new} \in (\eta^{E\pi}, \eta^{U})\) becomes \(\eta^{new} \in (\eta^{E\pi}, 1]\), and the microentrepreneur always adopts the new practice. If \(\eta^{E\pi} \leq 1\) does not exist then the microentrepreneur adopts the new practice and post-training expected profit always goes up.
\( \eta^{\text{new, complex}} < \eta^U \), we will observe the profit increase after a simple training and the profit decrease after a complex training.

Proposition 9 generates two testable predictions regarding the likelihood of adopting new business practice and the post-training profit. First, it is microentrepreneurs with extreme degree of ambiguity aversion towards a new practice who are less likely to adopt it. Second, among the microentrepreneurs who adopt the new practice, it is those with higher ambiguity aversion towards the new practice that are more likely to experience a decline in post-training profit.\(^{16}\)

5 Concluding Remarks

Most experts agree that there is a need to improve business knowledge among small- and microentrepreneurs as entrepreneurs in developing countries are often unaware of even the basic business practices such as keeping personal and business finances separate, or keeping records of their transactions and inventory. The business training programs aimed at microentrepreneurs around the world date back as far as the seventies. Yet, the effect of these training programs is not as strong as one would hope. Dar and Tzannatos (1999), as well as an updated review by Betcherman et al. (2004), find that the impact of training programs has wide variation with some programs demonstrating a positive effect, while others have no effect or even a negative effect. In a more recent work, McKenzie and Woodruff (2014) reach a similar conclusion, though they argue that a lack of impact on profit could be caused by methodological issues such as sample size or heterogeneity.

This paper provides a theoretical framework to understand a mixed impact of business-training. In it we rely on a holistic view of a microentrepreneur as someone whose livelihood and ambitions are more complex than just being an entrepreneur. First, the microentrepreneur has several sources of income in addition to income from business activities. Second, the microentrepreneur has other goals in addition to maximizing total income. The impact of business training, however, is narrow: it makes business activities more profitable.

We show the training effect is heterogeneous and depends on microentrepreneur’s ambition. This is consistent with the observation that “BDPs have been more successful for some entrepreneurs than the others” (Verrest, 2013, p. 58). For microentrepreneurs with strong profit-maximizing ambition the training effect is positive. For other microentrepreneurs, however, it is possible that adoption of new business practices will result in a lower expected profit. Most importantly, we show that the reason behind profit decline is the mismatch in the BDPs focus on growing microentrepreneur’s business, and a wider context in which the microentrepreneur operates. It is only when both conditions are satisfied (two sources of income and two ambitions) when the training can have a negative impact.

\(^{16}\)In Section 3 we argued that all microentrepreneurs are willing to adopt the new practice. The difference is that now we allow microentrepreneurs to perceive ambiguity of old and new business practices differently. According to Proposition 9, as long as the difference in ambiguity is not extremely high, it is still the case that microentrepreneurs will adopt the new practice. However, for extremely high difference, microentrepreneurs might choose not to adopt the new practice at all.
Finally, we develop a behavioral justification of our model linking microentrepreneur’s objective to ambiguity-aversion. We then use the perceived ignorance hypothesis to show that another source that undermines training efficiency is the perception of new business practices. If the new practice is viewed as more ambiguous then it can either prevent the switch to the new practice or is more likely lead to lower expected profit. We use this to explain why a simplistic rule-of-thumb training in Drexler et al. (2014) worked better than a more complex one.

Overall, our paper highlights the importance of having a holistic view on microfinance clients. While this is not a novel insight for empirical literature, to the best of our knowledge this is one of the first theoretical papers that shows the importance of the holistic view; and how using it affects the model’s predictions.

6 Appendix A: Proofs

Proof of Proposition 1: Differentiability follows from differentiability of $F$. By the implicit function theorem applied to (2.2)

\[
\frac{\partial L^*}{\partial K} = -\frac{sF''_{LK}}{F''_{LL}}.
\]

By the envelope theorem applied to $\pi(s, K)$ we have

\[
\pi'_K(s, K) = sF'_K(K, L^*) - R,
\]

and

\[
\pi''_{KK} = sF''_{KK}(K, L^*) + sF''_{KL}(K, L^*) \frac{\partial L^*}{\partial K} = sF''_{KK} - sF''_{KL} \frac{F''_{LK}}{F''_{LL}} < 0,
\]

where the last inequality follows from concavity of $F$. This proves concavity of $\pi(s, K)$. Finally, given that

\[
\pi'_I(s, K) = sF'_I(K, L^*) - R,
\]

and given Inada conditions there exists $0 < K < +\infty$ that satisfies $\pi'_K(s, K) = 0$. By concavity of $\pi(s, K)$ such $K$ is unique. That proves single-peakedness. ■

Proof of Proposition 2: Take two states such that $s > t$. By the envelope theorem:

\[
\pi'_K(s, K) = sF'_K(K, L^*_s) - R;
\]

\[
\pi'_K(t, K) = sF'_K(K, L^*_t) - R.
\]
Then
\[
\pi'_K(s, K) - \pi'_K(t, K) = sF'_K(K, L_s^*) - tF'_K(K, L_t^*) = t(F'_K(K, L_s^*) - F_K(K, L_t^*)) + (s - t)F'_K(K, L_s^*) = tF''_{KL}(K, \tilde{L})(L_s^* - L_t^*) + (s - t)F'_K(K, L_s^*),
\]
where \( \tilde{L} \in [L_t^*, L_s^*] \). A sufficient condition for the expression above to be positive is \( L_s^* \geq L_t^* \).

If either \( L_s^* \) or \( L_t^* \) or both are corner solutions then \( L_s^* \geq L_t^* \). If neither is a corner solution, then they satisfy the FOCs:

\[
-w_s + sF'_L(K, L_s^*) = 0, \\
-w_t + tF'_L(K, L_t^*) = 0.
\]

Given that \( F'_L(K, \cdot) \) is a decreasing function of \( L \), we have that

\[
L_s^* > L_t^* \iff F'_L(K, L_s^*) < F'_L(K, L_t^*) \iff w_t - \frac{t}{s}w_s > 0.
\]

This proves both statements of the proposition. ■

**Proof of Proposition 3:** The worst-case profit, \( \pi_w(K) \), is a concave function because it’s a minimum of concave state-profit functions, \( \pi(s, K) \). The utility, \( U(K) \) is a linear combination of the expected profit and the worst-case profit functions and, therefore, it is concave as well. Concave functions can be either monotone or single-peaked. In the case of \( U(K) \) and \( \pi_w(K) \), the former is impossible since all state-profit functions are increasing for \( K < K^*(1) \) and are decreasing for \( K > K^*(n) \). Finally, it is a property of concave functions of a single real variable that they are continuous and admit left and right-derivatives that are non-increasing. ■

**Proof of Proposition 4:** That \( K^*_\eta \) should satisfy

\[
U'(K^*_\eta-) \geq 0 \geq U'(K^*_\eta+)
\]

follows from the fact that \( U(K) \) is a concave function. That proves the first part of the proposition.

If \( K_w = K^*(\sigma) \), where \( \sigma \in \arg \min_s \pi(s, K^*(s)) \) then we are done. Assume now that \( K_w \neq K^*(\sigma) \). Let \( S_w(K) \) be the set of all worst states given \( K \), \( S_w(K) = \{ s : \pi(s, K) \leq \pi(t, K) \text{ for all } t \neq s \} \). \( S_w(K) \) is non-empty and has one or more elements in it.

First, we show that if \( K_w \neq K^*(\sigma) \), then \( S_w(K_w) \) has at least two elements in it. Proof by contradiction. Assume to the contrary that \( S_w(K_w) \) has exactly one element, \( s' \). Then, on one hand, \( K_w \neq K^*(s') \). Otherwise, if \( K_w = K^*(s') \) then

\[
\pi(s', K^*(\sigma)) < \pi(s', K^*(s')) \leq \pi(\sigma, K^*(s')) < \pi(\sigma, K^*(\sigma)),
\]

(6.1)

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which contradicts the definition of $\sigma$. Here, the first inequality follows from the fact that $K^*(s')$ is optimal given $s'$; the second inequality holds because $S_w(K_w) = \{s'\}$ and $K_w = K^*(s')$; the last inequality follow from the fact that $K^*(\sigma)$ is optimal given $\sigma$. On the other hand, it must be the case that $K_w = K^*(s')$. Indeed, by continuity, $s'$ is the unique worst-case states in the neighborhood of $K_w$. Then, $\pi(s', K_w)$ can be neither strictly increasing nor strictly decreasing at $K_w$. In the former case, $K$ just above $K_w$ will deliver higher worst-case profit. In the latter case, $K$ just below $K_w$ will deliver higher worst-case profit. Thus, $K_w = K^*(s')$. We reached a contradiction; therefore, $s'$ is not unique state given $K_w$.

Let $s$ be the lowest and $s'$ be the highest states in $S_w(K_w)$. Recall that we consider the case where $K_w \neq K^*(\sigma)$. Therefore, one then can apply the same reasoning as in (6.1) to show that $\pi'_K(s, K_w) \neq 0$ and $\pi'_K(s', K_w) \neq 0$. Furthermore, it cannot be the case that both $\pi'_K(s, K_w) > 0$ and $\pi'_K(s', K_w) > 0$. Assume not, $\pi'_K(s, K_w) > 0$ and $\pi'_K(s', K_w) > 0$. Then $K_w < K^*(s) < K^*(s')$. State $s$ is the smallest worst-case state given $K_w$. Therefore, by weak complementarity $K^*(s') > K_w$ for every $s'' \in S_w(K_w)$, which in turn implies $\pi'_K(s'', K_w) > 0$. By continuity, in a sufficiently small neighborhood of $K_w$, only states from $S_w$ can be the worst states.$^{17}$ But then, $K$ slightly above $K_w$ will result in a higher worst-case profit. Similarly, it cannot be the case that $\pi'_K(s, K_w) < 0$ and $\pi'_K(s', K_w) < 0$.

Thus, $\pi'_K(s, K_w)$ and $\pi'_K(s', K_w)$ have different signs and neither is equal to zero. By weak complementarity, it has to be the case that $\pi'_K(s, K_w) < 0 < \pi'_K(s', K_w)$, which completes the proof. ■

**Proof of Proposition 5:** When business activity is the only source of income, the state-profit functions become: $\pi(s, K) = sF(K, T) - RK$, since all of the labor is used for the business activity. For notational simplicity, we will omit the second argument (labor) within this proof, as it is always equal to $T$. The post-training state-profit function is $\pi^{new}(s, K) = sF(\lambda, K) - RK$.

For every $K$, the worst case state, therefore, is $s = 1$. The microentrepreneur’s utility is

$$(1 - \eta) \sum_s p(s)(sF(K) - RK) + \eta(F(K) - RK).$$

Consider, first, the case of $\eta = 1$ and $\lambda_1 = \cdots = \lambda_n = \lambda$. The new optimal capital level, $K^{new}_w$, is determined by the FOC

$$\lambda F'_K(\lambda K) - R = 0,$$

and $K^{new}_w$ depends on $\lambda$. One can show that $\lambda K^{new}_w$ is an increasing function of $\lambda$:

$$\frac{\partial(\lambda K^{new}_w)}{\partial \lambda} = K^{new}_w - \frac{F'_K + (\lambda K)F''_{KK}}{\lambda^2 F''_{ KK}} = \frac{(\lambda^2 - \lambda)K^{new}_w F''_{KK} - F'_K}{\lambda^2 F''_{KK}} > 0. \quad (6.2)$$

$^{17}$For every $t \in S_w(K_w)$ and every $t' \in S_w(K_w)$, it is the case that $\pi(t, K_w) > \pi(t', K_w)$. Then, for any $K$ sufficiently close to $K_w$, it is also the case that $\pi(t, K) > \pi(t', K)$. Therefore, $t \notin S_w(K_w)$ cannot be the worst-case state for $K$ that are sufficiently close to $K_w$. 

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Here we took into account that $F'_K > 0$, $F''_{KK} < 0$ and $\lambda > 1$.

We can show now that the post-training profit will increase in every state, which will imply that the expected profit will increase as well. For state 1 it is obvious, as $s = 1$ is the rainy day state, which is what the microentrepreneur maximizes. Thus for $s = 1$ we have

$$F(\lambda K^*_{w}) - F(K_w) - R(K^*_{w} - K_w) > 0. \quad (6.3)$$

For state $s > 1$ the profit increases if and only if

$$sF(\lambda K^*_{w}) - sF(K_w) - R(K^*_{w} - K_w) > 0.$$ 

As we established earlier $\lambda K^*_{w}$ is an increasing function of $\lambda$. Thus $\lambda K^*_{w} > K_w$ and, therefore, $F(\lambda K^*_{w}) > F(K_w)$. Then

$$sF(\lambda K^*_{w}) - sF(K_w) - R(K^*_{w} - K_w) > F(\lambda K^*_{w}) - F(K_w) - R(K^*_{w} - K_w) > 0,$$

where the last inequality comes from (6.3). We have established that post-training profit will increase in every state, which means that so will the expected profit.

Above we considered the case $\lambda_1 = \cdots = \lambda_n$. The case $\lambda_1 \leq \cdots \leq \lambda_n$ is now trivial since $F(\lambda_1 K^*_{w}) \geq F(\lambda_1 K^*_{w}) > F(K_w)$. Naturally, for higher $\lambda_s$ the profit improvement in states $s > 1$ becomes even higher.

Finally, consider the case of $\eta < 1$. The microentrepreneur’s utility is

$$(1 - \eta)E_s\pi^\text{new}(s, K) + \eta\pi^\text{new}(1, K), \quad (6.4)$$

where, again we use the fact that state 1 is the worst-case state. It was established earlier that the microentrepreneur’s post-training utility will go up:

$$(1 - \eta)E_s\pi^\text{new}(s, K^{*\text{new}}) + \eta\pi^\text{new}(1, K^{*\text{new}}) > (1 - \eta)E_s\pi(s, K^*_\eta) + \eta\pi(1, K^*_\eta),$$

which implies that

$$(1 - \eta)(E_s\pi^\text{new}(s, K^{*\text{new}}) - E_s\pi(s, K^*_\eta)) > \eta(\pi(1, K^*_\eta) - \pi^\text{new}(1, K^{*\text{new}})).$$

The expression on the LHS is a change in the expected profits.

If $\pi(1, K^*_\eta) - \pi^\text{new}(1, K^{*\text{new}}) > 0$ then we are done as then the post-training expected profit will increase. If not, then $\pi^\text{new}(1, K^{*\text{new}}) - \pi(1, K^*_\eta) > 0$ which can be re-written as

$$(F(\lambda_1 K^{*\text{new}}) - F(K^*_\eta)) - R(K^{*\text{new}} - K^*_\eta) > 0. \quad (6.5)$$
It means that the post-training profit in the first state will go up. Then, as long as \( \lambda_1 K^*_{\text{new}} - K^*_\eta > 0 \), one can show that for any state \( s \):

\[
s(F(\lambda_s K^*_{\text{new}}) - F(K^*_\eta)) - R(K^*_{\text{new}} - K^*_\eta) > (F(\lambda_1 K^*_{\text{new}}) - F(K^*_\eta)) - R(K^*_{\text{new}} - K^*_\eta) > 0.
\]

This means that the post-training profit will increase in every state which implies that the post-training expected profit will increase as well.

Thus the last thing to show is that \( \lambda_1 K^*_{\text{new}} - K^*_\eta > 0 \). Let \( Z = \lambda_1 K^*_{\text{new}} \). From the FOC

\[
0 = (1 - \eta) E_s [s \lambda_s F'(\lambda_s K^*_{\text{new}}) - R] + \eta (\lambda_1 F'(\lambda_1 K^*_{\text{new}}) - R)
\]

follows

\[
0 = (1 - \eta) E_s \left[ \frac{s \lambda_s F'(\lambda_s Z)}{\lambda_1} - \frac{R}{\lambda_1} \right] + \eta \left( F'(Z) - \frac{R}{\lambda_1} \right) > \left(1 - \eta\right) E_s \left[ s F'(Z) - \frac{R}{\lambda_1} \right] + \eta \left( F'(Z) - \frac{R}{\lambda_1} \right),
\]

where the inequality follows from the fact that \( \lambda F'(\lambda K) \) is an increasing function of \( \lambda \).\(^{18}\) Let \( K^*_\eta(R/\lambda_1) \) be optimal pre-training capital level given the interest rate \( R/\lambda_1 \). From the derivation above follows that \( Z > K^*_\eta(R/\lambda_1) \). Finally, from the implicit function theorem follows that optimal capital is a decreasing function of \( R \). Therefore, \( K^*_\eta(R/\lambda_1) > K^*_\eta \), as \( K^*_\eta \) is the optimal capital level given higher interest rate, \( R \). Combining the two inequalities we get that \( \lambda_1 K^*_{\text{new}} = Z > K^*_\eta(R/\lambda_1) > K^*_\eta \), which completes the proof. \( \blacksquare \)

**Proof of Proposition 7**: When \( F(K, L) = \ln(KL) \) then the training affects profit functions in state by adding a constant \( \ln \lambda_s \). That means that \( K^*_{\text{new}}(s) = K^*(s) \) and \( K^*_{\text{new}} = K^* \). Next we study the impact of the training on \( K_w \). If \( K_w \neq K^*(s) \), then by Proposition 4 there exist two states, \( s < s' \) such that \( \pi(s, K_w) = \pi(s', K_w) \leq \pi(t, K_w) \) for all other states \( t \), and \( \pi^t(s, K_w) < 0 < \pi^t(s', K_w) \).

Consider first the case when \( \lambda_1 < \cdots < \lambda_n \). Then \( \pi^t(s, K_w) < \pi^t(t, K_w) \) for every \( t > s \). Thus, the new smallest worst-state given \( K_w \), denote it \( s^w_{\text{new}} \), is less or equal than \( s \), \( s^w_{\text{new}} \leq s \). Given weak-complementarity, \( s^w_{\text{new}} \leq s \), and Proposition 4, the new state-profit function for \( s^w_{\text{new}} \) is decreasing at \( K_w \). Therefore, \( K > K_w \) cannot be the new optimal worst-case capital: \( \pi^w_{\text{new}}(K) = \min_t \lambda_t \pi(t, K) \leq \lambda_1 \pi(s^w_{\text{new}}, K) \leq \lambda_{s^w_{\text{new}}} \pi(s^w_{\text{new}}, K_w) = \min_t \lambda_t (t, K_w) = \pi^w_{\text{new}}(K_w) \). The first inequality comes from the fact that the lowest profit given \( K \) is less or equal than the profit at state \( s^w_{\text{new}} \). The second inequality comes from the fact that \( \pi(s^w_{\text{new}}, \cdot) \) declines when \( K < K_w \).

Moreover, \( K_w \) is no longer the optimal worst-case capital either. Since \( \pi^w_{\text{new}}(t, K_w) > \pi^w_{\text{new}}(s, K_w) \) for any \( t > s \), it must be the case that all worst states for \( K_w \) are less than or equal to \( s \). By weak

\(^{18}\) Recall that we assumed that \( F'_K + K F''_{KK} \geq 0 \). Taking derivative of \( \lambda F'(\lambda K) \) with respect to \( \lambda \) we get

\[
\frac{\partial \lambda F'(\lambda K)}{\partial \lambda} = F'_K(\lambda K) + (K \lambda) F''_{KK}(\lambda K) \geq 0.
\]
complementarity, the corresponding state-profit functions are decreasing at $K_w^\ast$. Then, $K$ slightly below $K_w$ will give strictly higher worst-case profit, and $K_w$ is no longer optimal. That proves that $K_w^{new} < K_w^\ast$.

The case $\lambda_1 > \cdots > \lambda_n$ is similar. If $K_w \neq K^\ast(s)$ there are two worst-case states $s < s'$ that correspond to $K_w$ and such that $\pi'_K(s, K_w) < 0 < \pi'_K(s', K_w)$. For any $t < s'$, it is now the case that $\pi^{new}(t, K_w) > \pi^{new}(s', K_w)$. Thus, all new worst-case states given $K_w$ are greater or equal than $s'$. For every $t \geq s'$, the state-profit functions are increasing at $K_w$ and, therefore, the new worst-case capital must be strictly greater than $K_w$.

We established that $K_w^{new} < K_w < K^\ast = K^{new} \leq K_w < K_w^{new}$ when $\lambda_1 < \cdots < \lambda_n$ and $K^\ast = K^{new} < K_w < K_w^{new}$ when $\lambda_1 > \cdots > \lambda_n$. Thus in both scenarios the capital adjustment effect is negative. ■

**Proof of Proposition 8**: We will prove the proposition for the case $K_w < K^\ast$ only. The case $K_w > K^\ast$ is similar.

Since $U(K)$ is a concave function of $K$, it has left and right derivatives and the left derivative is greater or equal than the right derivative. For a given $\eta$, the necessary and sufficient condition for $K_w^\eta$ to maximize $U(K)$ is

\[(1 - \eta)E_s\pi'_K(s, K_w^\eta) + \eta \cdot (\pi_w(K_w^\eta -))'_K \geq 0 \geq (1 - \eta)E_s\pi'_K(s, K_w^\eta) + \eta \cdot (\pi_w(K_w^\eta +))'_K. \tag{6.6}\]

If the utility function is differentiable at $K_w^\eta$, then (6.6) becomes

\[(1 - \eta)E_s\pi'_K(s, K_w^\eta) + \eta \cdot (\pi_w(K_w^\eta))'_K = 0. \tag{6.7}\]

By concavity, when $K^\ast > K_w$ then $K^\ast \geq K_w^\eta \geq K_w$ for every $\eta$. This is because when $K > K^\ast (K < K_w)$, both the expected profit and the worst-case profit have negative (positive) left and right derivatives.

We can now prove that $K_w^\eta$ is a decreasing function of $\eta$.\(^{19}\) In the proof, we will use the fact that $(\pi_w)'_K(K_w^\eta +) \leq (\pi_w)'_K(K_w^\eta -) < 0$ when $K_w^\eta > K_w$, and $E_s\pi'_K(s, K_w^\eta) > 0$ when $K_w^\eta < K^\ast$. The proof is by contradiction. Assume not. Then there exist $\eta_1 < \eta_2$ such that $K_w^\eta_1 < K_w^\eta_2$. That means that

\[
0 \geq (1 - \eta_1)E_s\pi'_K(s, K_w^\eta_1) + \eta_1 \cdot (\pi_w(K_w^\eta_1 +))'_K
\]
\[
> (1 - \eta_1)E_s\pi'_K(s, K_w^\eta_2) + \eta_1 \cdot (\pi_w(K_w^\eta_2 +))'_K
\]
\[
> (1 - \eta_2)E_s\pi'_K(s, K_w^\eta_2) + \eta_2 \cdot (\pi_w(K_w^\eta_2 -))'_K,
\]

which, given (6.6), means that $K_w^\eta$ cannot be optimal given $\eta_2$. Here, the first inequality follows from the fact that $K_w^\eta_1$ is optimal given $\eta_1$, see (6.6); the second inequality follows from the fact

\(^{19}\)It is not a strictly decreasing function of $\eta$. When the worst-case state is unique, as in (6.7), it is a strictly increasing function of $\eta$. When it is not unique, as in (6.6), it is weakly decreasing. That is, there is a range of $\eta$'s that would correspond to the same optimal $K_w^\eta$.  

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that the utility function is concave and $K^*_\eta_1 < K^*_\eta_2$; the last inequality follows from the fact that $\eta_2 > \eta_1$ and that the derivative of $\pi_w$ is negative (because $K^*_\eta_2 > K^*_\eta_1 \geq K_w$), while the derivative of the expected profit is positive.

The rest of the proposition is straightforward. Given that the worst-case profit $\pi_w(\cdot)$ is a concave function that reached its maximum at $K_w$, given that $K^*_\eta \geq K_w$ and that $K^*_\eta$ is a decreasing function of $\eta$, we can conclude that the worst-case profit, $\pi_w(\cdot)$, is an increasing function of $\eta$. Similarly, given that the expected profit, $E_s\pi(s, \cdot)$, is a concave function with the maximum at $K^*$, given that $K^* \geq K^*_\eta$, and that $K^*_\eta$ is a decreasing function of $\eta$, we can conclude that the expected profit is a decreasing function of $\eta$.

Finally, the utility function is a weakly decreasing function of $\eta$. Consider $\eta_1 < \eta_2$ and let $K^*_\eta_1$ and $K^*_\eta_2$ be corresponding optimal capital levels. Then

$$(1 - \eta_1)E_s\pi(s, K^*_\eta_1) + \eta_1 \pi_w(K^*_\eta_1) \geq (1 - \eta_1)E_s\pi(s, K^*_\eta_2) + \eta_1 \pi_w(K^*_\eta_2)$$

$$> (1 - \eta_2)E_s\pi(s, K^*_\eta_2) + \eta_2 \pi_w(K^*_\eta_2).$$

The first inequality follows from the fact that $K^*_\eta_1$ is optimal given $\eta_1$. The second inequality follows from the fact that the expected profit is greater than the worst-case profit, $E_s\pi(s, K) > \pi_w(K)$, and that $\eta_1 < \eta_2$. This completes the proof of the proposition. ■

**Proof of Proposition 9:** From Proposition 8 follows that $\eta^{E\pi} > \eta$. Indeed, when $\eta^{new} = \eta$ the LHS of (4.3) is greater than the RHS. This is because we assumed that without increase in $\eta$, the post-training expected profit will go up. The LHS of (4.3) is a decreasing function of $\eta^{new}$ and, therefore, (4.3) holds as equality when $\eta^{new} = \eta^{E\pi} > \eta$.

It also follows from Proposition 8 that $\eta^U > \eta^{E\pi}$. Indeed, when $\eta^{new} = \eta^{E\pi}$, (4.2) becomes:

$$(1 - \eta^{E\pi})E_s\pi^{new}(s, K^*_{\eta^{E\pi}}) + \eta^{E\pi} \pi_w^{new}(K^*_{\eta^{E\pi}}) > (1 - \eta)E_s\pi(s, K^*_\eta) + \eta \pi_w(K^*_\eta),$$

and it is satisfied. By definition of $\eta^{E\pi}$, expected profits (first terms of the LHS and the RHS in (6.8)) are equal. By Proposition 8, the worst-case profit on the LHS is higher since $\eta^{E\pi} > \eta$ and $\pi_w(K^*_\eta)$ is an increasing function of $\eta$. It was established in Proposition 8 that the LHS of (4.2) is a decreasing function of $\eta^{new}$ and, therefore, $\eta^U > \eta^{E\pi}$.

That $\eta^U > \eta^{E\pi} > \eta$, combined with the fact that the LHSs of (4.2) are (4.3) are decreasing functions of $\eta^{new}$, completes the proof of Proposition 9. ■

7 Appendix B: Risk-Aversion

In this section we check the robustness of our results to the case when the microentrepreneur is risk averse. For risk averse microentrepreneurs the objective function is

$$\max_K E_s \left[ u(w_s(T - L) + sF(K, L) - RK) \right],$$

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where \( u(\cdot) \) is a concave Bernoulli function.

We will measure the strength of the livelihood-oriented ambition by the degree of absolute risk-aversion. When ARA is low, preferences are close to risk-neutral preferences. We interpret it as having strong business-oriented ambition and it is similar to the case of \( \eta \) being close to zero. When ARA is close to infinity, preferences are close to maxmin preferences. We interpret it as having strong livelihood-oriented ambition and it is similar to the case of \( \eta \) being close to 1.

As before, the training improves the productivity of capital in each state so that the new production function is \( sf(\lambda sK, L) \), where \( 1 \leq \lambda_1 \leq \cdots \leq \lambda_n \).

**Proposition 10** For microentrepreneurs with sufficiently strong business-oriented ambition (low risk-aversion), the training will always result in higher post-training expected profit. For microentrepreneurs with sufficiently strong livelihoods-oriented ambition (high risk-aversion), the training can result in post-training profit decline.

This proposition is presented without the proof because the proof simply repeats the reasoning from Section 3. When the microentreprenuer is risk-neutral, \( u(x) = x \). Then by Proposition 6 the post-training expected profit will go up. When the ARA is sufficiently low, one can use continuity to show that the expected profit will go up as well. When the microentrepreneur is infinitely risk-averse her preferences become maxmin preferences and she maximizes the worst-case profit \( \pi_w(K) \). This corresponds to the case of \( \eta = 1 \) from Section 3, and one can use the exact same example as in Section 3.3 to show that the post-training expected profit can decline.

In a special case when the training has the same impact in every state and a microentrepreneur’s utility exhibit decreasing absolute risk-aversion, one can extend the statement of Proposition 5 to the case of risk-averse agents.

**Proposition 11** Assume that \( u(\cdot) \) exhibit decreasing absolute risk-aversion and that \( \lambda_1 = \cdots = \lambda_n > 1 \). Assume also that the microentreprenuer’s only source of income is business-activity. Then the post-training profit will increase.

The proof is somewhat technical, but the intuition is straightforward. For a given \( K \) the pre-training lottery is \( L_1 = \left( F(K) - RK, p(1), \ldots, nF(K) - RK, p(n) \right) \) and the post-training lottery is \( L_2 = \left( F(\lambda K) - RK, p(1), \ldots, nF(\lambda K) - RK, p(n) \right) \). The outcomes of post-training lottery, \( L_2 \), become more spread out because of the factor \( \lambda \), thereby making \( L_2 \) riskier. Given that, nonetheless, risk averse agent will prefer \( L_2 \) to \( L_1 \) it must be the case that \( L_2 \) has higher expected payoff. The actual proof follows this idea while taking into account that the post-training value of \( K \) changes.

**Proof.** When the only source of income is business, the microentrepreneur’s utility is

\[
E_s\left( u(sF(\lambda K) - RK) \right),
\]
where we omit the second argument (labor), which is always equal to $T$. Optimal capital level, $K_{u}^{new}$, satisfies the first-order condition

$$E_s\left\{u'(s\lambda K_{u}^{new} - RK_{u}^{new})(s\lambda F'_{K}(\lambda K_{u}^{new}) - R)\right\} = 0. \tag{7.1}$$

Let $Z = \lambda K_{u}^{new}$. Then (7.1) becomes

$$\lambda E_s\left\{u'\left(s\frac{R}{\lambda} Z\right) \left(s\frac{F'_{K}(Z) - R}{\lambda}\right)\right\} = 0.$$

One can interpret $Z$ as optimal pre-training capital level given the interest rate $R_{\lambda} = R/\lambda$, where $R_{\lambda} < R$. One can verify that optimal capital level is a decreasing function of the interest rate. Indeed, consider the pre-training FOC

$$E_s\left\{u'(sF(K) - RK)(sF'(K) - R)\right\} = 0,$$

from here

$$\frac{\partial K}{\partial R} = -\frac{E_s\{u''(sF(K) - RK)(-K)(sF'(K) - R) - u'(sF(K) - RK)\}}{E_s\{u''(sF(K) - RK)(sF'(K) - R)^2 + u'(sF(K) - RK)sF''(K)\}} \tag{7.2}.$$ 

Let $RA(x)$ be the absolute risk-aversion at point $x$ and, by assumption, it is a decreasing function of $x$. Let $\tilde{s}$ be the highest state such that $sF'(K) - R < 0$. Then for all $s > \tilde{s}$

$$u''(sF(K) - RK) \geq -RA(\tilde{s}F(K) - RK) \cdot u'(sF(K) - RK),$$

and therefore

$$u''(sF(K) - RK)(sF'(K) - RK) \geq -RA(\tilde{s}F(K) - RK) \cdot u'(sF(K) - RK)(sF'(K) - RK).$$

For all $s \leq \tilde{s}$

$$u''(sF(K) - RK) \leq -RA(\tilde{s}F(K) - RK) \cdot u'(sF(K) - RK),$$

and, therefore

$$u''(sF(K) - RK)(sF'(K) - RK) \geq -RA(\tilde{s}F(K) - RK) \cdot u'(sF(K) - RK)(sF'(K) - RK),$$

where I used the fact that when $s \leq \tilde{s}$ then $sF'(K) - R < 0$. Thus,

$$E_s[u''(sF(K) - RK)(sF'(K) - RK)] \geq -RA(\tilde{s}F(K) - RK)E_s[u'(sF(K) - RK)(sF'(K) - RK)] = 0.$$
The first term in the numerator of (7.2) is $E_s[u''(sF(K) - RK)(sF'(K) - RK)]$ times $(-K)$ and, therefore, is negative. It implies that the numerator of (7.2) is negative and, therefore, $\partial K/\partial R < 0$.

Thus for the post-training capital, $Z = \lambda K_u^{new} > K_u$, where $K_u$ is the optimal pre-training capital. Given $K_u$, the pre-training lottery is $L_1 = \{F(K_u) - RK_u, p(1), \ldots, nF(K_u) - RK_u, p(n)\}$ and the post-training lottery is $L_2 = \{F(\lambda K_u^{new}) - RK_u^{new}, p(1), \ldots, nF(\lambda K_u^{new}) - RK_u^{new}, p(n)\}$.

One can show that since $K_u < \lambda K_u^{new}$ the latter lottery is riskier. Indeed, riskiness does not depend on adding or subtracting a constant so it is enough to show that lottery $L'_2 = \{F(\lambda K_u^{new}), p(1), \ldots, nF(\lambda K_u^{new}), p(n)\}$ is riskier than $L'_1 = \{F(K_u), p(1), \ldots, nF(K_u), p(n)\}$. Here we added $RK_u^{new}$ to $L_2$ and $RK_u$ to $L_1$. Notice that $L_2 = \frac{F(\lambda K_u^{new})}{F(K_u)} L_1$ and $F(\lambda K_u^{new})/F(K_u) > 1$. Therefore, by Corollary 10.5.6 in LeRoy and Werner (2001), $L_2$ is riskier.

Finally, given that $L_2$ is riskier than $L_1$ but a microentrepreneur prefers $L_2$ to $L_1$ it must be the case that the expected payoff of $L_2$ is higher (Theorem 10.5.2, LeRoy and Werner (2001)) meaning that the post-training expected profit goes up.

References


