Innovation Waves, Investor Sentiment, and Mergers*

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Abstract

We develop a theory of innovation waves, investor sentiment, and merger activity based on uncertainty aversion. Investors must typically decide whether or not to fund an innovative project with very limited knowledge of the odds of success, a situation that is best described as "Knightian uncertainty." We show that uncertainty-averse investors are more optimistic on an innovation if they can also make contemporaneous investments in other innovative ventures. This means that uncertainty aversion makes investment in innovative projects strategic complements, which results in innovation waves. We also show that innovation waves may be sparked by favorable technological shocks in one sector, and then spill over to other contiguous sectors. Thus, innovation waves ripple through the economy amid strong investor sentiment. Finally, we argue that an active M&A market promotes innovative activity and leads to greater innovation rates and firm valuations.

Keywords: Innovation, Ambiguity Aversion, Hot IPO Markets

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Innovation is the most important value driver of modern corporations and a key source of economic growth (Solow, 1957). There are times when innovation is stagnant, but other times when technology leaps forward. Furthermore, investors must typically decide whether or not to fund an innovative project with very limited knowledge of the odds of success, a situation that is best described as "Knightian uncertainty" (Knight, 1921). In this paper, we study the impact of uncertainty aversion on the incentives to innovate, and we show that uncertainty aversion can generate innovation waves that are associated with strong investor sentiment and stock market booms.[1]

There are many reasons why innovation develops in waves. These include fundamental reasons such as random technological breakthroughs in the presence of network externalities. In this paper, we focus on the incentives to create and finance innovations. We argue that innovation waves can be the product of investors' uncertainty aversion. We show that investors' uncertainty aversion creates externalities in innovative activities which may result in innovation waves. We also show that innovation waves are associated with an active M&A market and an equity market boom in technology sectors.

We study an economy with multiple entrepreneurs endowed with project-ideas. Project-ideas are risky and, if successful, may lead to innovations. The innovation process consists of two stages. In the first stage, entrepreneurs must decide whether or not to invest personal resources, such as effort, to innovate. If the first stage of the process is successful, further development of the innovation requires additional investment in the second stage. Entrepreneurs raise funds for the additional investment by selling shares of their firms to uncertainty-averse investors. The second stage of the innovation process is uncertain in that outside investors are uncertain of the exact distribution of the residual success probability of the innovation process. Following Epstein and Schnieder (2011), we model uncertainty aversion by assuming that outside investors are Minimum Expected Utility (MEU) maximizers and that they hold a set of priors, or "beliefs," rather than a single prior as is the case for Subjective Expected Utility (SEU) agents.

In our model, beliefs on the future returns of investments held by uncertainty-averse investors

[1] A positive effect of investor sentiment on innovation has been documented in Aramonte (2015).

are endogenous, and depend on the composition of their portfolios. This implies that uncertainty-averse investors prefer to hold an uncertain asset if they can also hold other uncertain assets, a feature that is denoted as "uncertainty hedging." Because of uncertainty hedging, an investor will be more "optimistic" on an innovation if he/she is able to invest in other innovations as well. Thus, investors have stronger sentiment and, thus, are willing to pay more for equity in a given entrepreneur's firm when other entrepreneurs innovate as well. This means that investors are more willing to fund an entrepreneur's innovation if they can also fund other entrepreneurs at the same time. It also implies that the market value of equity of a new firm will be greater when multiple new firms are on the market as well. Thus, investments in different innovative companies are effectively complements and have positively correlated market valuations.

We show that investors' uncertainty aversion can generate inefficient equilibria where potentially valuable innovation is not pursued. When the initial personal cost to the entrepreneur is sufficiently low, entrepreneurs' dominant strategy is to innovate, irrespective of other entrepreneurs' decisions. Similarly, when the initial personal cost is very large, the dominant strategy is not to innovate. For intermediate levels of the initial personal cost, an entrepreneur is willing to initiate the innovation process only if she expects other entrepreneurs to innovate as well. Thus, multiple equilibria, with and without innovation, may exist. Existence of the inefficient equilibrium without innovation depends on the correlation between the success rates of the innovation processes, that is, on the degree of "relatedness" of the innovation.

Strategic complementarity between innovative activities due to uncertainty aversion may result in innovation waves. An innovation wave may be sparked by a favorable technological shock in one sector that triggers an entrepreneur to initiate an innovation. Because of the innovation in one company, other entrepreneurs now expect more favorable pricing of their equity by investors, inducing them to innovate as well.[2] Thus innovation in one firm can spill over to other firms even in the absence of explicit technological spillover between the two firms. In this way, innovation waves associated with strong investor sentiment ripple through the economy.

Complementarity between innovative activities due to uncertainty aversion may also result in

[2] The externality between firms in the public equity market, and its beneficial effect on firm investment decisions, is also examined in Subrahmanyam and Titman (1999).

technology sector equity market booms. To see this, note that the link between the innovative activities of firms is due to the positive effect of the innovation of one firm on the equity pricing of other firms. This means that innovation in one sector may trigger a positive effect on equity valuations that spills to contiguous sectors. Thus, "equity market booms" in technology markets can materialize. These booms are beneficial since they can spur valuable innovation.

Our paper also has implications for the impact of M&A activity and, more generally, of the ownership structure on innovation rates. Specifically, we propose a new channel, based on uncertainty aversion, in which mergers of innovative firms engaged in related technologies create synergies and spur innovation. In our paper, positive synergies in an acquisition are created endogenously, and are the direct outcome of the beneficial spillover (i.e., externality) of beliefs on innovation processes due uncertainty aversion.[3] In addition, our model predicts that merger activities involving innovative firms will be associated with strong investor sentiment and, thus, greater firm valuations.

Finally, we argue that uncertainty aversion has implications for the composition of venture capital portfolios, and the structure of the venture capital industry. This happens because of the possible beneficial role that venture capitalists can play to remedy a coordination failure that causes the inefficient no-innovation equilibrium.

Our paper rests at the intersection of three strands of literature. First, and foremost, our paper belongs to the rapidly expanding literature on the determinants of innovation and innovation waves (see Fagerberg, Mowery and Nelson, 2005, for an extensive literature review).[4] The critical role of innovation and innovation waves in modern economies has been extensively studied at least since Schumpeter (1939) and (1942), Kuznets (1940), Kleinknecht (1987) and, more recently, Aghion and Howitt (1992). Early research focused mostly on the "fundamentals" behind innovation waves, such as the positive spillover effects across different technologies. More recent research has focused on the link between innovation waves, the availability of financing, and stock market booms. Scharfstein and Stein (1990) suggest that reputation considerations by investment managers may induce them to herd their behavior in the stock market, and thus facilitate the financing of technology firms.

[3] Hart and Holmstrom (2010) develop a model where mergers create value by internalizing externalities, such as coordinating on a technological standard.

[4] See also Chemmanur and Fulghieri (2014) for a discussion of current issues related to entrepreneurial finance and innovation.

Gompers and Lerner (2000) find that higher venture capital valuations are not necessarily linked to better success rates of portfolio companies. Perez (2002) shows that technological revolutions are associated with "overheated" financial markets. Gompers et al. (2008) suggest that increased venture capital funding is the rational response to positive signals on technology firms' investment opportunities. Nanda and Rhodes-Kropf (2013) find that in "hot markets" VCs invest in riskier and more innovative firms. Nanda and Rhodes-Kropf (2014) argue that favorable financial market conditions reduce refinancing risk for VCs, promoting investment in more innovative projects.

To our knowledge, ours is the first paper that models explicitly the role of uncertainty aversion on the innovation process and its impact on innovation waves and stock market valuations. We show that investors' uncertainty aversion can generate innovation waves that are driven by investors' beliefs. In our model, due to uncertainty aversion, investors' beliefs are endogenous, and they respond to the availability of investments in innovative projects. Innovation waves and stock market "exuberance" are jointly determined in equilibrium in a model where investors are sophisticated. In our model, greater investment in innovation is combined with investor optimism and stock market booms.

The second stream of literature is the recent debate on the links between technological innovation and stock market prices. Nichols (2008) shows that an important driver of the stock market run-up experienced in the American economy in the late 1920's was the strong innovative activity by industrial companies which affected the market valuation of corporate "knowledge assets." Pastor and Veronesi (2009) argue that technological revolutions can generate dynamics in asset prices in innovative firms that are observationally similar to assets bubbles followed by a valuation crash. Their paper argues that this "bubble-like" behavior of stock prices is the rational outcome of learning about the productivity of new technologies, where the risk is essentially idiosyncratic, followed by the adoption of the new technologies on large scale, where the risk becomes systematic. Our paper proposes a new explanation for the link between innovative activity and stock market booms. In Pastor and Veronesi (2009) stock market booms (and subsequent crashes) are the outcome of the changing nature of risk that characterizes technological revolutions, from idiosyncratic to systematic, and its impact on discount rates. In our model, periods of strong innovative activity are accompanied by high valuations because innovation waves are, in equilibrium, associated with strong investor sentiment and more optimistic expectations on future expected cash flows from innovations. Thus, our model, which focuses on expected cash flows, complements theirs, that focus on discount rates. Furthermore, similar to Pastor and Veronesi (2009), in our model high valuations imply lower long-term returns.

The third stream of literature focuses on the drivers of merger waves and the impact of M&A activity – and, more generally, of the ownership structure – on the incentives to innovate. High stock market valuations are also associated with strong M&A activity in merger waves (see, for example, Maksimovic and Phillips, 2001, and Jovanovic and Rousseau, 2001). Rhodes-Kropf and Viswanatan (2004) argue that such correlation is the outcome of misvaluation of the true synergies created in a merger in periods when the overall market is overvalued. The impact of M&A activity on corporate innovative activity has been documented by several empirical studies. For example, Bena and Li (2014) argue that the presence of technological overlap between two firms innovative activities is a predictor of the probability of a merger between firms. Sevilir and Tian (2012) show that acquiring innovative target firms is positively related to acquirer abnormal announcement returns and long-term stock return performance. The importance of the presence of technological overlaps between acquiring firms and targets is confirmed by Seru (2014), which finds that innovation rates are lower in diversifying mergers, where the technological benefits of a merger are likely to be absent.

In our model we are able to jointly generate the observed positive correlations between stock market valuations, the level of M&A activity, and innovation rates. Specifically, our paper creates a novel direct link between stock price valuations, M&A activity, and greater innovation rates that is based on investors' uncertainty aversion. Endogeneity of beliefs creates an externality between innovations that is at the heart of the synergy creation in mergers of innovative companies. This externality results in greater innovation rates and innovation waves that are characterized by strong investor sentiment and greater stock market valuations.

The paper is organized as follows. In Section 1, we briefly discuss the model of uncertainty aversion that is at the foundation of our analysis. In Section 2, we introduce the basic model of our paper. In Section 3, we derive the paper's main results. Section 4 examines the impact of mergers on the incentives to innovate. Section 5 shows that our results hold also in the case of process innovation. Section 6 presents the main empirical implications of our model. Section 7 concludes. All proofs are in the Appendix.

1 Uncertainty Aversion

A common feature of current economic models is to assume that all agents know the distribution of all possible outcomes.[5] An implication of this assumption is that there is no distinction between the known-unknown and the unknown-unknown. However, the Ellsberg paradox shows that this implication is not warranted.[6]

In traditional models, economic agents maximize their Subjective Expected Utility (SEU). Given a von-Neumann Morgenstern utility function u and a probability distribution over wealth, μ, each player maximizes

$$U^{e}=E_{\mu}\left[u\left(w\right)\right]. \tag{1}$$

One limitation of the SEU approach is that it cannot account for aversion to uncertainty, or "ambiguity." In the SEU framework, economic agents merely average over the possible probabilities. Under uncertainty aversion, a player does not know the true prior, but only knows that the prior is from a given set, $\mathcal{M}$.

A common way for modeling uncertainty (or ambiguity) aversion is the Minimum Expected Utility (MEU) approach, promoted in Epstein and Schneider (2011). In this framework, economic

[5] This section draws on Dicks and Fulghieri (2015a). This paper is part of the growing literature studying ambiguity aversion in finance, including including Mukerji and Tallon (2001), Maenhout (2004), Epstein and Schneider (2008) and (2011), Easley and O'Hara (2009) and (2013), Caskey (2009), Illeditsch (2011), Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2013), Garlappi, Giammarino, and Lazrak (2014), Javan – Parker and Liu (2014), Byun (2014), Mele and Sangiorgi (2015), and Dicks and Fulghieri (2015b).

[6] A good illustration of the Ellsberg paradox is actually from Keynes (1921). There are two urns. Urn K has 50 red balls and 50 blue balls. Urn U has 100 balls, but the subject is not told how many of them are red (all balls are either red or blue). The subject will be given \$100 if the color of their choice is drawn, and the subject can choose which urn is drawn from. Subjects typically prefer Urn K, revealing aversion to uncertainty (this preference is shown to be strict if the subject receives \$101 from selecting Urn U but \$100 from Urn K being drawn). To see this, suppose the subject believes that the probability of drawing Blue from Urn U is p_B. If $p_B < \frac{1}{2}$, the subject prefers to draw Red from Urn U. If $p_B > \frac{1}{2}$, the subject prefers to draw Blue from Urn U. If $p_B = \frac{1}{2}$, the subject is indifferent. Because subjects strictly prefer to draw from Urn K, such behavior cannot be consistent with a single prior on Urn U. This paradox provides the motivation for the use of multiple priors. Further, the subject's beliefs motivate the failure of additivity of asset prices: in this example, the subject believes that $p_B + p_R < p_{(B\cup R)} = 1$.

agents maximize

$$U^a = \min_{\mu \in \mathcal{M}} E_\mu \left[u \left(w \right) \right]. \tag{2}$$

As shown in Gilboa and Schmeidler (1989), the MEU approach is a consequence of replacing the Sure-Thing Principle of Anscombe and Aumann (1963) with the Uncertainty Aversion Axiom.[7] This assumption captures the intuition that economic agents prefer risk to uncertainty – they prefer known probabilities to unknown. MEU has the intuitive feature that a player first calculates expected utility with respect to each prior, and then takes the worst-case scenario over all possible priors. In other words, the agent follows the maxim "Average over what you know, then worry about what you don't know."[8]

In this paper, we use the MEU approach with recursively defined utilities, as described in Epstein and Schnieder (2011). Formally, we will model *sophisticated uncertainty-averse* economic agents with consistent planning. In this setting, agents are sophisticated in that they correctly anticipate their future uncertainty aversion. Consistent planning accounts for the fact that agents take into account how they will actually behave in the future.[9] Our results are smooth (a.e.) because we explore a setting where we can apply a minimax theorem.

An important feature of uncertainty aversion that will play a critical role in our paper is that agents may benefit from diversification, a feature that we will refer to as *uncertainty hedging*. This feature can be seen as follows. Consider two random variables, y_k, $k \in \{1, 2\}$, with distribution $\mu_k \in \mathcal{M}$, which is ambiguous to agents. Uncertainty hedging is the property that uncertainty-averse agents prefer to pick the worst-case scenario for a portfolio, rather than choosing the worst-case scenario for each individual asset in its portfolio.[10]

[7] Anscombe and Aumann (1963) is an extention of the Savage (1972) framework: the Anscombe and Aumann framework has both objective and subjective probabilities, while the Savage framework has only subjective probabilities.

[8] Another approach is the smooth ambiguity model developed by Klibanoff, Marinacci, and Mukerji (2005). In their model, agents maximize expected felicity of expected utility. Agents are ambiguity averse if the felicity function is concave.

[9] Siniscalchi (2011) describes this framework as preferences over trees.

[10] Note that, as such, property (3) is reminiscent of the well-known feature that a portfolio of options is worth more than an option on a portfolio and, thus, that writing a portfolio of options is more costly than writing an option on a portfolio.

Theorem 1 *Uncertainty-averse agents prefer uncertainty-hedging:*

$$\begin{aligned} q \min_{\mu \in \mathcal{M}} E_{\mu}\left[u\left(y_1\right)\right] + (1-q) \min_{\mu \in \mathcal{M}} E_{\mu}\left[u\left(y_2\right)\right] &\leq \\ \min_{\mu \in \mathcal{M}} \{qE_{\mu}\left[u\left(y_1\right)\right] + (1-q)E_{\mu}\left[u\left(y_2\right)\right]\}, \quad \textit{for all } q &\in [0,1]. \end{aligned} \tag{3}$$

If agents are SEU, (3) holds as an equality.

This property will play a key role in our model. It implies that uncertainty-averse agents prefer to hold a portfolio of uncertain assets rather than a single uncertain asset, because investors can lower their exposure to uncertainty by holding a diversified portfolio. Alternatively, it suggests that an investor will be more "optimistic" about a portfolio than about a single asset. Thus, uncertainty hedging creates a complementarity between assets for investors so the value investors place on a given asset is increasing in their portfolio exposure to other assets.[11]

A second critical feature of our model is that we do not impose rectangularity of beliefs (as in Epstein and Schneider 2003). Rectangularity of beliefs effectively implies that prior beliefs in the set of admissible priors can be chosen independently from each other.[12] In our model, we assume that the agent faces a restriction on the set of the core beliefs $\mathcal{M}$ over which the minimization problem (2) is taking place. These restrictions are justified by the observation that the nature of the economic problem imposes certain consistency requirements in the set of the core beliefs $\mathcal{M}$. In other words, we recognize that the "fundamentals" of the economic problem faced by the uncertainty-averse agent generates a loss of degree of freedom in the selection of prior beliefs.[13] In alternative, following Epstein and Schneider (2011), lack of rectangularity can be justified by requiring that beliefs in the core-belief set M satisfy a minimum likelihood ratio or, equivalently, a maximum relative entropy with respect to a given set of reference beliefs.

[11] We will show that such portfolio complementarity will induce entrepreneurs to exhibit strategic complementarity in their innovation decisions, resulting in multiple equilibria, because an entrepreneur is more willing to innovate if she believes other entrepreneurs are innovating as well. Dicks and Fulghieri (2015b) shows that uncertainty hedging also causes systemic risk, in that idiosyncratic shocks spread into financial crises.

[12] Rectangularity of beliefs is commonly assumed to guarantee dynamic consistency. However, Aryal and Stauber (2014) show that, with multiple players, rectangularity of beliefs is not sufficient for dynamic consistency.

[13] For example, an uncertainty-averse producer may face uncertainty on the future consumption demand exerted by her customers. The beliefs held by the uncertainty-averse agent on consumer demand must be consistent with basic restrictions, such as the fact that the consumer choices must satisfy an appropriate budget constraint.

2 The Basic Model

We study a two-period model, with three dates, $t \in \{0, 1, 2\}$. The economy has two classes of agents: investors and (two) entrepreneurs. Entrepreneurs are endowed with unique project-ideas that may lead to an innovation. Project-ideas are risky and require an investment both at the beginning of the period, $t = 0$, and at the interim date, $t = 1$, as discussed below; if successful, project-ideas generate a valuable innovation at the end of the second period, $t = 2$. If the project-idea is unsuccessful, it will have zero payoff. For simplicity, we assume initially that there are only two types of project-ideas, denominated by τ, with $\tau \in \{A, B\}$.

Entrepreneurs are penniless and require financing from investors. There is a unit mass of investors. Investors are endowed at the beginning of the first period, $t = 0$, with w_0 units of the riskless asset. The riskless asset can either be invested in one (or both) of the two types of project-ideas, or it can invested in the riskless technology. A unit investment in the riskless technology can be made either at $t = 0$ or $t = 1$, and yields a unit return in the second period, $t = 2$, so that the (net) riskless rate of return is zero.

We assume that project-ideas are specific to each entrepreneur, that is, an entrepreneur can invest in only one type of project-ideas, which will determine entrepreneur's type τ, $\tau \in \{A, B\}$. This assumption captures the notion that project-ideas are creative innovations that can be successfully pursued only by the entrepreneur who generated them.

The innovation process is structured in two stages. To implement a project-idea, and thus "innovate," an entrepreneur must first pay at $t = 0$ a fixed investment k_τ. We interpret the initial investment k_τ as representing all the preliminary personal effort that the entrepreneur must exert in order to generate the idea and make it potentially viable. We will denote the initial personal investment made by the entrepreneur, k_τ, as a "discovery cost" that is necessary for the innovation. The innovation process is inherently risky, and we denote with q_τ the success probability of the first stage of the process. We assume that the first-stage success probabilities of the two project-ideas are correlated. Specifically, we assume that the probability that both entrepreneurs are successful in the first stage is $q_A q_B + r$, while the probability that entrepreneur τ is successful if entrepreneur τ' is not successful is $q_\tau(1 - q_{\tau'}) - r$, with $\tau', \tau \in \{A, B\}$, $\tau' \neq \tau$ and $r \in$

$\left[-\min\left\{q_A q_B, (1-q_A)(1-q_B)\right\}, \min_{\tau} q_\tau (1-q_\tau)\right]$.[14] The parameter r captures the possibility of the presence of similarities between entrepreneurial project-ideas. Thus, the parameter r characterizes the degree of "relatedness" of the innovations.

If the first stage of the innovation process is successful, at $t = 1$ entrepreneurs enter the second stage of the process. In this second stage, the entrepreneur must decide the level of intensity of the innovation process, for example, the level of R&D expenditures. Innovation intensity will affect the ultimate value of the innovation that can be realized at $t = 2$, and that is denoted by y_τ. Innovation intensity is costly, and we assume that an entrepreneur of type τ choosing an innovation intensity y_τ will sustain a cost $c_\tau(y_\tau) = \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}$, where Z_τ represents the productivity of entrepreneur τ's project-idea. To obtain interior solutions, we will assume that the productivity parameters, Z_τ, for the two entrepreneurs are not too dissimilar.[15] The second stage of the innovation process is also risky and, if successful, the innovation will generate at the end of the second period, $t = 2$, the payoff y_τ with probability p_τ, and zero otherwise (if the project fails in the first stage, it is similarly worthless).

We assume that entrepreneurs are impatient and that they will sell at the interim period, $t = 1$, their firms to outside investors at total price V_τ. Investors, however, are uncertain on the success probability of the project-ideas. Following Dicks and Fulghieri (2015a), we assume that the success probability of the second stage of an innovation of type-τ depends on the value of an underlying parameter θ, and is denoted by $p_\tau(\theta)$. Outside investors are uncertainty-averse and treat the parameter θ as uncertain, and believe $\theta \in C \equiv \left[\hat{\theta}_0, \hat{\theta}_1\right] \subset [\theta_0, \theta_1]$, where C represents the set of "*core beliefs*." For analytical tractability, we assume that $p_A(\theta) = e^{\theta-\theta_1}$ and $p_B(\theta) = e^{\theta_0-\theta}$.[16] In this specification, increasing the value of the parameter θ increases the success probability of type-A project-ideas and decreases the success probability of type-B project-ideas. This means that

[14] It can be quickly verified that the correlation of the first-stage projects is $r\left[q_A(1-q_A)q_B(1-q_B)\right]^{-\frac{1}{2}}$.

[15] Formally, we assume that $\frac{Z_A}{Z_B} \in \left(\frac{1}{\psi}, \psi\right)$ where ψ will be defined later. This assumption guarantees that if both first-stage projects are successful, entrepreneurs execute innovation intensity levels so that investors have interior beliefs in equilibrium.

[16] This assumption allows us to dispense with rectangularity of beliefs in a tractable way, but is not necessary. Our paper's main results go through for $\{p_A, p_B\} \in C$, as long as the core belief set C is a strictly convex, compact set with a smooth boundary. If the core of beliefs is the set of distributions that are sufficiently close to a (given) reference belief, measured by relative entropy, the core of beliefs will be strictly convex as the lower level set of a strictly convex function.

a greater value of θ is "favorable" for innovation A and "unfavorable" for innovation B.[17] Note however that, for a given value of the parameter θ, the probability distributions $p_\tau(\theta)$, $\tau \in \{A, B\}$, are independent.[18] We will also assume that the core of beliefs is symmetric, so that $\theta_1 - \hat{\theta}_1 = \hat{\theta}_0 - \theta_0$, and we set $\theta^e \equiv \frac{1}{2}(\theta_0 + \theta_1)$. We will at times benchmark the behavior of uncertainty-averse investors with the behavior of uncertainty-neutral, or SEU, investors, and we will assume that uncertainty-neutral investors believe that $\theta = \theta^e$, differently from uncertainty-averse investors who believe that $\theta \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$.

Payoffs are determined as follows. If entrepreneur τ innovates, and the first stage of the innovation process is successful, he develops an innovation with a (potential) value y_τ. At the interim date, $t = 1$, entrepreneurs sell their entire firm to outside investors for a value V_τ, which thus represents her payoff from the innovation. In turn, an uncertainty-averse investor can purchase a fraction ω_τ of firm τ, with $\tau \in \{A, B\}$, and thus holding the residual value $w_0 - \omega_A V_A - \omega_B V_B$ in the risk-free asset. To avoid (uninteresting) corner solutions, we assume that the endowment is risk-free asset is sufficiently large that the budget constraint will not be nonbinding in equilibrium: $w_0 > \omega_A V_A + \omega_B V_B$. Investors' final payoff will then depend on their holdings of the risk-free asset and on the success/failure of each innovation at the second stage and on their holdings in the innovation, ω_τ. Finally, we assume that, while outside investors are uncertainty averse with respect to the parameter θ, there are no other sources of uncertainty (as opposed to "risk") in the economy,[19] and that all agents (investors and entrepreneurs) are otherwise risk-neutral.

2.1 Endogenous Beliefs

An important implication of uncertainty aversion is that the investor's belief at the interim date on the parameter θ depends on their overall exposure to the source of risk in the economy and, thus, on the structure of their portfolios.[20] If an investor decides to purchase a proportion ω_τ of

[17] For example, the parameter θ captures the uncertainty on consumers' preference between two competing products, say, Apple's iPhone and Samsung's Galaxy. More generally, the parameter θ can be interpreted as representing uncertainty on relevant technological and/or commercial value of competing innovations.

[18] Our model can easily be extended to the case where, given θ, the realization of the asset payoffs at the end of the period are correlated.

[19] If there is uncertainty on q or r, entrepreneurs will assume the worst, selecting $q_{\min}$ and $r_{\min}$, because entrepreneurs' payoffs are increasing in q and r.

[20] For additional discussion, see Dicks and Fulghieri (2015a).

entrepreneur τ's firm, with innovation intensity y_τ, the investor will hold a risky portfolio that we denote as $\Pi = \{\omega_A y_A, \omega_B y_B, w_0 - \omega_A V_A - \omega_B V_B\}$. Because investors are uncertainty-averse (since, they believe $\theta \in C$) but otherwise risk-neutral, a portfolio Π provides the investor with utility

$$U(\Pi) \quad = \min_{\theta \in C} \left\{ e^{\theta - \theta_1} \omega_A y_A + e^{\theta_0 - \theta} \omega_B y_B + w_0 - \omega_A V_A - \omega_B V_B \right\}.$$

Because of uncertainty aversion, the investor's belief at $t = 1$ on the state of the economy, θ^a, is the solution to the minimization problem

$$\theta^a(\Pi) = \arg\min_{\theta \in C} U(\Pi),$$

and is characterized in the following lemma.

Lemma 1 *Let*

$$\tilde{\theta}^a(\Pi) = \theta^e + \frac{1}{2} \ln \frac{\omega_B y_B}{\omega_A y_A}. \tag{4}$$

For a given portfolio $\Pi = \{\omega_A y_A, \omega_B y_B, w_0 - \omega_A V_A - \omega_B V_B\}$*, an uncertainty-averse agent holds beliefs on the uncertain parameter* θ

$$\theta^a(\Pi) = \begin{cases} \hat{\theta}_0 & \tilde{\theta}^a(\Pi) \leq \hat{\theta}_0 \\ \tilde{\theta}^a(\Pi) & \tilde{\theta}^a(\Pi) \in \left(\hat{\theta}_0, \hat{\theta}_1\right) \\ \hat{\theta}_1 & \tilde{\theta}^a(\Pi) \geq \hat{\theta}_1 \end{cases}. \tag{5}$$

Lemma 1 shows that an investor's beliefs are endogenous and depend crucially on the composition of her portfolio, Π. Thus, we will refer to $\theta^a(\Pi)$ as the "portfolio-distorted" beliefs. We will say that the agent has "*interior beliefs*" when $\theta^a \in \left(\hat{\theta}_0, \hat{\theta}_1\right)$, in which case, the agent's beliefs are equal to $\tilde{\theta}^a(\Pi)$ as in (4). Otherwise, we will say that the investor holds "*corner beliefs*."

Note that the beliefs of an uncertainty-averse investor depend essentially on the composition of her portfolio Π, as follows.

Lemma 2 *Holding the exposure to type-*τ' *innovation risk,* $\omega_{\tau'} y_{\tau'}$*, constant, an increase in an investor's exposure to type-*τ *innovation risk,* $\omega_\tau y_\tau$*, with* $\tau \neq \tau'$*, induces the investor to hold* *portfolio distorted beliefs, θ^a, that are (weakly) less favorable to type-τ innovations, for $\tau \in \{A, B\}$. In addition, portfolio-distorted beliefs θ^a are homogeneous of degree zero in the holding of the risky innovations, $\{\omega_A y_A, \omega_B y_B\}$.*

Lemma 2 shows that when a investor has a relatively greater proportion of her portfolio invested in innovation τ, $\omega_\tau y_\tau > \omega_{\tau'} y_{\tau'}$, she will be relatively more pessimistic about the return on that innovation. This happens because a greater exposure to the risk generated by an innovation of type τ makes the investor more concerned about the priors that are less favorable to that innovation. Thus, the investor will give more weight to the states of nature that are less favorable for that innovation. In other words, the investor will be more "pessimistic" on the success probability of that innovation. Correspondingly, the investor will become more "optimistic" with respect to the other innovation, type $\tau^{'}$. Proportional changes in an investor's position in both risky innovations will not affect her beliefs.

Lemma 2 also shows an interesting implication of Lemma 1. Suppose entrepreneur of type A decides to innovate, but entrepreneur B decides not to innovate. Because $y_B = 0$, by Lemma 1, we have that $\theta^a(\Pi) = \hat{\theta}_0$ for any $\omega_A y_A > 0$. Correspondingly, if entrepreneur B decides to innovate, but entrepreneur A does not, we have that $\theta^a(\Pi) = \hat{\theta}_1$. Similar situations emerge if only one entrepreneur has a successful first-stage project-idea, while the other entrepreneur fails. In this case, at the interim date, $t = 1$, investors hold more pessimistic beliefs about the successful innovation than if both entrepreneurs have a successful first-stage project-idea. This means that investors, when facing only one innovation, will be more pessimistic on that innovation than when facing both innovations.

In our model, portfolio-distorted beliefs determine investors' expectations on the ultimate success probability of the innovation processes in the economy, and thus characterize investors' "sentiment" toward innovations. An important implication of Lemma 1 that will play a key role in our analysis is that investor sentiment about one innovation will crucially depend on the availability of other innovations in the economy, and their innovation intensity. In particular, an investor will be more optimistic about an innovation success probability, and she values it more, if she will be able to also invest in the other innovation. Thus, investors' beliefs create an externality for entrepreneurs, in that an entrepreneur's successful innovation will be more valuable if other entrepreneurs have successful innovations as well. In other words, if both entrepreneurs innovate, and their innovations are successful, investor sentiment toward both innovations improves making both innovations more valuable. The spillover effect from one innovation to another is driven by investors' beliefs, that is by their sentiment.

3 The Innovation Decision

We will solve the model recursively. First, we find the choice by entrepreneurs that are successful at the first stage of the innovation process of the optimal innovation intensity, y_τ, and the value V_τ that investors are willing to pay at the interim date for innovations. Next, we solve for the initial choice by entrepreneurs on whether or not to initiate the innovation process by incurring the initial discovery cost k_τ. As a benchmark, we start the analysis by characterizing the two entrepreneurs' innovation decisions when investors are uncertainty-neutral SEU agents, then we consider the case where investors are MEU uncertainty-averse agents.

The implementation of the second stage of the innovation process requires entrepreneurs to raise capital from investors by selling equity in the capital markets at $t = 1$. For simplicity, we assume that an entrepreneur of type τ sells her entire firm to investors, uses the proceeds to pay for the intensity costs $c_\tau(y_\tau)$, and pocket the difference. We assume that y_τ is observable and contractible with outside investors, thus ruling out moral hazard. In this case, the choice of innovation intensity y_τ by a type-τ entrepreneur depends on the price that outside investors are willing to pay for her firm, that is, on the market value of the equity of the firm. This, in turn, depends on the beliefs held by investors on the success probability of the innovation, $p_\tau(\theta)$.

Lemma 3 *Given investors' beliefs and risk-neutrality, entrepreneurs' firms are priced at their expected value, that is,* $V_\tau = p_\tau(\theta^a)y_\tau$ *for uncertainty-averse investors, and* $V_\tau = p_\tau(\theta^e)\, y_\tau$ *for uncertainty-neutral investors, with* $\tau \in \{A, B\}$*. In equilibrium, it is (weakly) optimal for investors to hold a balanced portfolio:* $\omega_A^* = \omega_B^*$ *for both type of investors (SEU and MEU).*

Lemma 3 shows that, given our assumption of universal risk-neutrality, investors price equity at its expected value, given their beliefs. Investors' beliefs, however, depend on their attitude toward uncertainty, that is whether they are uncertainty-neutral investors or uncertainty-averse investors. Endogeneity of beliefs is critical because it will lead to different market valuation of equity, and thus, different behavior by entrepreneurs. In addition, it is weakly optimal for investors to hold balanced portfolios. SEU investors are indifferent on their portfolio composition, because of risk neutrality. In contrast, uncertainty-averse investors strictly prefer a balanced portfolio, due to uncertainty-hedging (see Theorem 1). For notational simplicity, we normalize investors' portfolio holding and set $\omega_A^* = \omega_B^* = 1$.[21]

3.1 The Uncertainty-Neutral Case

As a benchmark, we start with the simpler case in which investors are uncertainty-neutral. When investors are uncertainty-neutral, equity prices depend only on their prior $\theta^e = \frac{1}{2}(\theta_0 + \theta_1)$ and on the level of innovation intensity, y_τ, chosen by the firm, giving

$$V_\tau^S = e^{\frac{1}{2}(\theta_0 - \theta_1)} y_\tau, \quad \text{for} \quad \tau \in \{A, B\}. \tag{6}$$

Equation (6) shows that equity value for an innovation of a type τ depends only the investors' beliefs of the success probability of the second stage of the innovation process, $p_\tau(\theta^e) = e^{\frac{1}{2}(\theta_0 - \theta_1)}$, and its level of innovation intensity, y_τ, and it does not depend on the innovation intensity decision of the other firm, $y_{\tau'}$, for $\tau' \neq \tau$. This means that, under uncertainty neutrality, there are no interactions between the choice of the innovation intensities by the two entrepreneurs. In this case, if the first stage of the project-idea was successful, entrepreneur τ's chooses the level of innovation intensity for the second stage, y_τ, by solving

$$\max_{y_\tau} \quad \mathcal{U}_\tau^S \equiv V_\tau^S - c_\tau(y_\tau) = e^{\frac{1}{2}(\theta_0 - \theta_1)} y_\tau - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}. \tag{7}$$

[21] This is WLOG optimal if there is one unit mass of investors.

From (7) it immediately follows that the optimal innovation intensity, y_τ, chosen by entrepreneur τ, is equal to

$$y_\tau^* \equiv \left[e^{\frac{1}{2}(\theta_0-\theta_1)} Z_\tau\right]^{\frac{1}{\gamma}}, \tag{8}$$

By direct substitution of y_τ^* into (7),[22] we obtain that the ex-ante expected payoff for entrepreneur τ from initiating the innovation process, and thus incurring discovery cost k_τ, is equal to

$$E\mathcal{U}_\tau^S = q_\tau \frac{\gamma}{1+\gamma} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}} - k_\tau.$$

Thus, entrepreneur τ innovates at $t = 0$ if $E\mathcal{U}_\tau^S \geq 0$, leading to the following theorem.

Theorem 2 *When investors are uncertainty-neutral, entrepreneurs of type τ innovate iff*

$$k_\tau \leq k_\tau^S \equiv q_\tau \frac{\gamma}{1+\gamma} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}}, \quad \tau \in \{A, B\},$$

and the innovation processes of the two entrepreneurs are independent.

Theorem 2 shows that when investors are uncertainty neutral, the investment decisions by the two entrepreneurs are effectively independent from each other, with no spillover effects. When investors are uncertainty averse, however, the innovation processes of the two firms are interconnected, as shown below.

3.2 Uncertainty Aversion and Innovation

We now derive optimal innovation decisions when investors are uncertainty averse. In this case, from Lemma 1, we know that the beliefs held by investors on the success probability of the second stage of each innovation process, $p_\tau(\theta^a)$, depend on the overall risk exposure of their portfolios. Specifically, beliefs held by uncertainty-averse investors are endogenous, and depend on the innovation intensities chosen by both firms, y_τ, and on their relative portfolio investment in the two firms, ω_A/ω_B. However, from Lemma 3, uncertainty-averse investors choose a balanced portfolio with $\omega_A = \omega_B$,

[22] Because $\frac{\partial^2 U_\tau^S}{\partial y_\tau^2} = -\frac{\gamma}{Z_\tau} y_\tau^{\gamma-1} < 0$, first-order conditions are sufficient for a maximum.

which means that, in equilibrium, the market value of equity of each firms depends only on the level of innovation intensity chosen by both forms, y_τ, as follows.

Lemma 4 *If investors are uncertainty averse, the market value of entrepreneur τ's firm is*

$$V_\tau^U(\Pi) = \begin{cases} e^{\hat{\theta}_1-\theta_1}y_\tau & y_\tau \le e^{2\left(\theta^e-\hat{\theta}_1\right)}y_{\tau'} \\ e^{\frac{1}{2}(\theta_0-\theta_1)}y_\tau^{\frac{1}{2}}y_{\tau'}^{\frac{1}{2}} & y_\tau \in \left(e^{2\left(\theta^e-\hat{\theta}_1\right)}y_{\tau'}, e^{2\left(\theta^e-\hat{\theta}_0\right)}y_{\tau'}\right) \\ e^{\hat{\theta}_0-\theta_1}y_\tau & y_\tau \ge e^{2\left(\theta^e-\hat{\theta}_0\right)}y_{\tau'} \end{cases}, \tag{9}$$

where y_τ is the innovation intensity selected by entrepreneur of type-τ, with $\tau, \tau' \in \{A, B\}$, $\tau \neq \tau'$.

Lemma 4 shows that, when investors are uncertainty averse, the market value of equity of one firm depends on the level of innovation intensity chosen by its entrepreneur as well as on the level chosen by the other firm. The interaction between the market values of the equity of the two firms, caused by investors' beliefs, creates a strategic externality between the two entrepreneurs, which will be critical in the analysis below.

Note that the linkage between the market value of the two firms occurs through investors' beliefs. Consider a firm of type τ: from Lemma 1 an increase of firm-τ' innovation intensity, $y_{\tau'}$, will increase the relative exposure of investors to firm-τ' risk relative to firm-τ risk, making (all else equal) investors relatively more optimistic about firm-τ success probability and, correspondingly, relatively more pessimistic about about firm-τ' success probability.

Lemma 4 also implies that an increase of the level of innovation intensity in one firm, y_τ, has two opposing effects on it value V_τ^U. The first is the positive direct effect that greater innovation intensity has on the ultimate value of the innovation. This positive effect can however be mitigated (in the case of "interior beliefs") by a second negative effect that an increase in innovation intensity has, all else equal, on investors' beliefs. This implies that firm value is a (weakly) increasing function of the innovation intensities of both firms.

Finally, note that if one of the two firms does not innovate or the innovation is not successful in the first stage, the level of innovation intensity for that firm is necessarily equal to zero. From Lemma 4 this implies that the market value of equity of the other firm will be determined at the worst case scenario for that firm, that is $V_\tau(\Pi) = \min_\theta p_\tau(\theta)\, y_\tau$.

We can now determine the optimal level of innovation intensity for entrepreneur τ. If the first stage of the project-idea was successful, entrepreneur τ's chooses the level of innovation intensity for the second stage, y_τ, by solving

$$\max_{y_\tau} \mathcal{U}_\tau^U \equiv V_\tau(\Pi) - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}, \tag{10}$$

where $\Pi = \{y_A, y_B, w_0 - V_A - V_B\}$ (since investors optimally set $\omega_A = \omega_B = 1$) and $V_\tau(\Pi)$ is given in (9). To simplify the exposition, in what follows we assume that the two types of firms are not too dissimilar. Specifically, we assume that the values Z_A and Z_B are not too far away from each other: $\frac{Z_A}{Z_B} \in \left(\frac{1}{\psi}, \psi\right)$ where $\psi \equiv \frac{1}{4} e^{2\left(\theta^e - \hat{\theta}_0\right)(\gamma+1)} \left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$. We make this assumption to ensure that if both firms have successful first-stage projects, they find it optimal to chose levels of innovation intensity $\{y_A, y_B\}$ that in equilibrium result in interior beliefs for the investors.

The solution to problem (10) depends on whether one or both firms decide to initiate the innovation process and pay the discovery costs k_τ and, if they do so, whether they are successful at the first stage of the innovation process. Thus, there are four possible states of the world that we need to analyze: (i) when both entrepreneurs had a successful first stage, state SS; (ii) when only one entrepreneur has a successful first-stage, state SF with the symmetric FS state, (iii) when both entrepreneur fail in the first stage and no innovation can take place, state FF. Since the last state FF is trivial, we now focus on the first two.

3.2.1 Only One Firm Has Successful First-Stage Project, State SF

Consider first the case in which only entrepreneur of type-τ had a successful first-stage project-idea, state SF. For future reference, note that this state may emerge either because the other entrepreneur of type-τ', with $\tau' \neq \tau$, has not initiated the innovation process (that is, she did not sustain the discovery cost $k_{\tau'}$), or because the first stage of the started innovation process was unsuccessful.

Lemma 5 *If only entrepreneur of type-τ has a successful first stage project-idea (state SF), she* *selects innovation intensity equal to*

$$y_\tau^{U,SF} = \left[e^{\hat{\theta}_0-\theta_1} Z_\tau\right]^{\frac{1}{\gamma}}; \tag{11}$$

the market value of the entrepreneur's firm is equal to

$$V_\tau^{U,SF} = e^{\left(\hat{\theta}_0-\theta_1\right)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}}, \tag{12}$$

giving a continuation utility for the entrepreneur equal to

$$\mathcal{U}_\tau^{U,SF} \equiv e^{\left(\hat{\theta}_0-\theta_1\right)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}} \frac{\gamma}{1+\gamma}. \tag{13}$$

If only one entrepreneur successfully develops a first-stage project, there will only be one type of uncertain innovation available to investors. In this case, from Lemma 1 investors will believe the worst-case scenario about that innovation type, resulting in pessimistic beliefs and low equity valuations. Therefore, the entrepreneur will chose a low level of innovation intensity, consistent with the endogenously pessimistic beliefs held by investors.

3.2.2 Both Firms Have Successful First-Stage Projects, State SS

If both entrepreneurs have successful first-stage projects, market valuation is given in Lemma 4, which leads to the following lemma.

Lemma 6 *Let $\frac{Z_A}{Z_B} \in \left(\frac{1}{\psi}, \psi\right)$. If both entrepreneurs innovate and have a successful first stage (state SS), they select innovation intensities equal to*

$$y_\tau^{U,SS}(y_{\tau'}) = \left[\frac{Z_\tau}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} (y_{\tau'})^{1/2}\right]^{\frac{1}{\gamma+\frac{1}{2}}}, \quad \text{with } \tau \neq \tau', \text{ and } \tau, \tau' \in \{A, B\}. \tag{14}$$

Lemma 6 establishes that there is strategic complementarity in entrepreneurs' production decisions. In particular, an entrepreneur's choice of innovation intensity, $y_\tau^{U,SS}(y_{\tau'})$, is an increasing function of the other entrepreneur's innovation intensity, $y_{\tau'}$. The strategic complementarity originates in investors' uncertainty aversion and belief endogeneity. From Lemma 1 and Lemma 4, we know that the beliefs of uncertainty-averse investors on the success probability of the second stage of an innovation process and, thus, their market valuations at the interim date, depend on the innovation intensities chosen by both entrepreneurs. Thus, because of the effect on beliefs, investors perceive innovations effectively as complements. This complementarity is then transferred from investors' beliefs to entrepreneurs' innovation decisions.

We can now determine the equilibrium levels of innovation intensities chosen by the two entrepreneurs in the SS state.

Theorem 3 *If both entrepreneurs innovate and have successful first-stage projects, state SS, the equilibrium level of innovation intensities for an entrepreneur of type τ, with $\tau \in \{A, B\}$, is*

$$y_{\tau}^{U,SS} = \left[\frac{1}{2} e^{\frac{1}{2}(\theta_0 - \theta_1)} Z_{\tau}^{\frac{2\gamma+1}{2\gamma+2}} Z_{\tau'}^{\frac{1}{2\gamma+2}}\right]^{\frac{1}{\gamma}}. \tag{15}$$

In equilibrium, firm value for each firm is

$$V_{\tau}^{U,SS} = 2^{-\frac{1}{\gamma}} e^{\frac{1}{2}(\theta_0 - \theta_1)\frac{1+\gamma}{\gamma}} \left(Z_{\tau} Z_{\tau'}\right)^{\frac{1}{2\gamma}}, \tag{16}$$

and continuation utility is equal to

$$\mathcal{U}_{\tau}^{U,SS} = 2^{-\frac{1}{\gamma}} e^{\frac{1}{2}(\theta_0 - \theta_1)\frac{1+\gamma}{\gamma}} \left(Z_{\tau} Z_{\tau'}\right)^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}. \tag{17}$$

The following corollary compares the equilibrium values when one or both entrepreneurs have successful first-stage projects.

Corollary 1 *An entrepreneur is better off when also the other entrepreneur has a successful first-stage projects: $U_{\tau}^{U,SS} > U_{\tau}^{U,SF}$. If entrepreneurs productivities are not too dissimilar, $\frac{Z_{\tau'}}{Z_{\tau}} \in \left(\frac{1}{\psi_1}, \psi_1\right)$, equity values are higher when both entrepreneurs have successful first-stage projects: $V_{\tau}^{U,SS} > V_{\tau}^{U,SF}$. In addition, if entrepreneurs productivities are sufficiently close together, $\frac{Z_{\tau'}}{Z_{\tau}} \in \left(\frac{1}{\psi_2}, \psi_2\right)$, entrepreneurs innovate with greater intensity when both have successful first-stage projects: $y_{\tau}^{U,SS} > y_{\tau}^{U,SF}$. Finally, $\psi_2 < \psi_1 < \psi$.*

An important implication of Corollary 1 is that, if entrepreneurs' productivities are not too dissimilar, because of the complementarity of innovations generated by uncertainty aversion, investors value one type of innovation more when they can invest also in the other type of innovation, yielding $V_\tau^{U,SS} > V_\tau^{U,SF}$.

3.3 The Innovation Decision

In the previous sections we have shown that investors' uncertainty aversion affects equity valuations and generates strategic complementarity in the interim choice of innovation intensity, y_τ. The interim strategic complementarity of the choice of innovation intensity generates a strategic complementarity also in the entrepreneurs' decisions to innovate at the beginning of the innovation process, $t = 0$, that is, to incur the discovery cost k_τ.

If entrepreneur τ' chooses to innovate, the expected utility for entrepreneur τ from sustaining at $t = 0$ the initial discover cost k_τ and, thus, initiating the innovation process is

$$\begin{aligned} E\mathcal{U}_\tau^{U,I} &= (q_\tau q_{\tau'} + r)\mathcal{U}_\tau^{U,SS} + (q_\tau(1 - q_{\tau'}) - r)\mathcal{U}_\tau^{U,SF} - k_\tau \\ &= (q_\tau q_{\tau'} + r)2^{-\frac{1}{\gamma}} e^{\frac{1}{2}(\theta_0 - \theta_1)\frac{1+\gamma}{\gamma}} (Z_\tau Z_{\tau'})^{\frac{1}{2\gamma}} \frac{2\gamma + 1}{2\gamma + 2} \\ &\quad + (q_\tau(1 - q_{\tau'}) - r)e^{\left(\hat{\theta}_0 - \theta_1\right)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}} \frac{\gamma}{1+\gamma} - k_\tau, \end{aligned}$$

for $\tau, \tau' \in \{A, B\}$ and $\tau \neq \tau'$. Conversely, if entrepreneur τ' does not innovate at $t = 0$, the expected for entrepreneur τ from choosing to innovate at $t = 0$ is

$$E\mathcal{U}_\tau^{U,N} = q_\tau \mathcal{U}_\tau^{U,SF} - k_\tau = q_\tau e^{\left(\hat{\theta}_0 - \theta_1\right)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}} \frac{\gamma}{1+\gamma} - k_\tau.$$

We can now characterize the equilibrium of the innovation decision at the beginning of the period, $t = 0$.

Theorem 4 *There are threshold levels $\{\underline{k}_\tau, \bar{k}_\tau\}_{\tau \in \{A,B\}}$ (defined in the appendix) with $\underline{k}_\tau < \bar{k}_\tau$, such that:*

(i) if $k_\tau \leq \underline{k}_\tau$, entrepreneur of type τ always innovates; (ii) if $k_\tau \geq \bar{k}_\tau$, entrepreneur of type τ

never innovates; (iii) If $k_\tau \in (\underline{k}_\tau, \bar{k}_\tau)$ entrepreneur of type τ innovates if $k_{\tau'} \leq \underline{k}_{\tau'}$, and she does not innovate if $k_{\tau'} \geq \bar{k}_{\tau'}$; (iv) if $k_\tau \in (\underline{k}_\tau, \bar{k}_\tau)$ for both $\tau \in \{A, B\}$, there are multiple equilibria, one where both entrepreneurs innovate and one where neither innovate. The equilibrium where both entrepreneurs innovate dominates the equilibrium where neither of the entrepreneurs innovate.

For very small levels of discovery costs, $k_\tau \leq \underline{k}_\tau$, it is a dominant strategy for entrepreneur τ to innovate. For very large levels of discovery costs, $k_\tau \geq \bar{k}_\tau$, it is a dominant strategy for entrepreneur τ to not innovate. For intermediate levels of discovery costs, $k_\tau \in (\underline{k}_\tau, \bar{k}_\tau)$, entrepreneur τ wishes to innovate only if the other entrepreneur innovates as well. Theorem 4 shows this strategic complementarity in entrepreneurs' innovation decisions.

When both entrepreneurs have intermediate levels of the discovery cost, there are multiple equilibria, with and without innovation. In this case, entrepreneurs face a classic "assurance game," in which there is a Pareto-dominant equilibrium, where both entrepreneurs innovate, yet there is also an inefficient, Pareto-inferior equilibrium, where neither entrepreneur innovates. Multiplicity of equilibria depends on the fact that it is profitable for one entrepreneur to innovate only if he expects the other entrepreneur to innovate as well. Such multiplicity of equilibria in the innovation game is the direct outcome of investors' uncertainty aversion.

We conclude this section by characterizing the impact of the model's parameters on the threshold levels $\{\underline{k}_\tau, \bar{k}_\tau\}_{\tau \in \{A,B\}}$.

Corollary 2 *The threshold levels $\{\bar{k}_\tau\}_{\tau \in \{A,B\}}$ are increasing functions of $q_\tau, q_{\tau'}, Z_\tau, Z_{\tau'}$ and r, and the threshold levels $\{\underline{k}_\tau\}_{\tau \in \{A,B\}}$ are increasing functions of q_τ and Z_τ.*

Corollary 2 has the interesting implication that an increase in one firm's probability of success, q_τ, makes not only that firm, but also other firms, more willing to attempt first-stage discovery of a product-idea. This follows because the strategic complementarity induced by uncertainty aversion. In the absence of uncertainty aversion, an increase in the probability of discovery affects only that entrepreneur, with no effect on other entrepreneurs. Corollary 2 also shows that entrepreneurs are more willing to innovate if her innovation is more related to other entrepreneurs' innovations, that is r is greater. This happens because greater degree of relatedness increases the probability that both project-ideas are simultaneously successful in the first-stage, increasing the market value of the innovations. Finally, Corollary 2 also shows that an increase in productivity of an entrepreneur increases not only that entrepreneur's willingness to innovate, but also makes other entrepreneurs willing to innovate as well.

4 Acquiring Innovation

In the previous sections, we have shown that investors' uncertainty aversion creates externalities across innovations. These externalities are due to endogeneity of investor beliefs, and create the possibility of value dissipation due to coordination failures. This means that there may be gains from internalizing such externalities via acquisitions.

There are two externalities at work in our model. The first externality is due to the valuation spillover discussed in Lemma 2. This happens because, for any given set of choices of innovation intensities, $\{y_\tau, y_{\tau'}\}$, the two firms are more valuable to uncertainty-averse investors when they are held in the same portfolio than when they are owned separately.

The second externality is due the strategic complementarity between the choices of innovation intensity y_τ, that we discussed in Lemma 4: the market value of an individual firm, V_τ^U, firm is an increasing function of the innovation intensity chosen by both firms, $\{y_\tau, y_{\tau'}\}$, through its effect on investors' beliefs. When a firm chooses their own optimal level of innovation intensity, they ignore the positive externality that choice has on the other firm's choice, leading to a loss of social surplus.

We extend our analysis by examining the effect of the strategic complementarity between innovation intensities. We modify the basic model as follows. If both entrepreneurs are successful in the first stage, we now allow for the possibility that at the interim date, $t = 1$, both entrepreneurs merge their firms in a new firm.[23] After the merger, the entrepreneurs jointly determine the innovation intensity, y_τ, for both innovation processes $\tau \in \{A, B\}$. After the selection of the innovation intensities y_τ, the merged firm will again sell all its equity in the public equity market. The two innovations processes may be sold to the public equity market either as a single multi-divisional

[23] Alternatively, the merger between the two firms may be initiated by a third firm which may acquire the innovation from both entrepreneurs.

firm, or as two independent firms.[24]

After the merger of the first-stage innovations, the problem of the merged firm is to maximize the combined value of the two innovation projects. By identical reasoning to the proof of Lemma 3, the merged firm will value the projects at $V_\tau = p_\tau(\theta_I)\, y_\tau$, for $\tau \in \{A, B\}$, where θ_I is the investors' belief when the merged firm is sold on the public equity market. Thus, the merged firm's objective at this stage is now to solve

$$\max_{\{y_A, y_B\}} \quad \mathcal{U}^M = p_A(\theta_I)\, y_A + p_B(\theta_I)\, y_B - c_A(y_A) - c_B(y_B).$$

Note first that, if investors are uncertainty neutral, $\theta_I = \theta^e$, and the choice of y_A and y_B are independent of each other. In this case, the merged firm solves the same problem as the original uncertainty-neutral entrepreneurs (7): $U^M = U^S_A + U^S_B$. This implies that the optimal levels of innovation intensity chosen by the merged firm are again given by (8), that is, the values the entrepreneurs would choose if the two firms were independent. Thus, if investors are uncertainty neutral, the merger does not add value with respect to what entrepreneurs can do independently.

In contrast, if investors are uncertainty averse, $\theta_I = \theta^a$ which, from (5), depends on the choice of both y_A and y_B. As shown in Lemma 4, for interior beliefs (which we will show is the case in equilibrium), we now have that

$$V_A = V_B = e^{\frac{1}{2}(\theta_0 - \theta_1)} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}}.$$

This implies that the maximization problem of the merged firm becomes

$$\max_{y_A y_B} \ \mathcal{U}^M = 2e^{\frac{1}{2}(\theta_0 - \theta_1)} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}} - \frac{1}{Z_A(1+\gamma)} y_A^{1+\gamma} - \frac{1}{Z_B(1+\gamma)} y_B^{1+\gamma},$$

leading to the following theorem.

Theorem 5 *If investors are uncertainty averse, the merged firm will select a greater innovation intensity at both firms*

$$y_\tau^M \equiv \left[e^{\frac{1}{2}(\theta_0 - \theta_1)} Z_{\tau'}^{\frac{1}{2\gamma+2}} Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} \right]^{\frac{1}{\gamma}} > y_\tau^{U,SS} \ ,$$

[24] Remember that, if the two innovations are sold in two separate firms, from Lemma 3, investors will optimally invest in both firms.

and will have a greater value than these firms would have as a stand-alone:

$$V^M = 2e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} [Z_A Z_B]^{\frac{1}{2\gamma}} > V_A^{U,SS} + V_B^{U,SS}.$$

Theorem 5 shows that a merger can add value to the innovative process by merging both firms from the original entrepreneurs and then choosing an innovation intensity at both firms that is greater than the one that the entrepreneurs would chose individually. Because of the positive externality between investment levels y_τ, inefficiently low levels of investment occur when each entrepreneur maximizes his own payoff. By merging, the post acquisition firm internalizes the spillover effects of investment, leading to greater firm valuation.

We now examine the impact of the possibility of a merger at the interim date $t = 1$ on the entrepreneurs' ex-ante incentives to innovate, that is, to sustain at $t = 0$ the discovery cost k_τ. The initial decision to innovate by an entrepreneur will depend on the terms at which the entrepreneur anticipates the merger with take place. The acquisition price, in turn, will depend on the allocation of the surplus generated by the acquisition, that is, on how the synergies are divided between the two entrepreneurs.

The allocation of the synergies created in the merger occurs through bargaining, and we will assume that the two entrepreneurs will split the surplus equally. Thus, if both innovations are successful in the first stage, entrepreneur τ earns

$$v_\tau = \mathcal{U}_\tau^{U,SS} + \frac{1}{2}\left(\mathcal{U}^M - \mathcal{U}_A^{U,SS} - \mathcal{U}_B^{U,SS}\right).$$

The incentives to pay the initial discover cost are discussed in the following.

Theorem 6 *There are threshold levels $\{\underline{K}_\tau, \bar{K}_\tau\}_{\tau\in\{A,B\}}$ (defined in the appendix) with $\underline{K}_\tau < \bar{K}_\tau$: (i) if $k_\tau \leq \underline{K}_\tau$, entrepreneur of type τ always innovates; (ii) if $k_\tau \geq \bar{K}_\tau$, entrepreneur of type τ never innovates; (iii) If $k_\tau \in \left(\underline{K}_\tau, \bar{K}_\tau\right)$ entrepreneur of type τ innovates if $k_{\tau'} \leq \underline{K}_{\tau'}$, and she does not innovate if $k_{\tau'} \geq \bar{K}_{\tau'}$; (iv) if $k_\tau \in \left(\underline{K}_\tau, \bar{K}_\tau\right)$ for both $\tau \in \{A, B\}$, there are multiple equilibria, one where both entrepreneurs innovate and one where neither innovate. The equilibrium where both entrepreneurs innovate dominates the equilibrium where neither of the entrepreneurs innovate.*

Finally $\bar{k}_\tau < \bar{K}_\tau$: the possibility of a merger induces entrepreneurs to innovate more ex-ante.

Theorems 5 and 6 have the interesting implication that an active M&A market promotes innovative activity and leads to greater innovation rates, stronger investor sentiment, and higher firm valuations. The synergies created in the merger are a direct consequence of endogeneity of beliefs that is due to uncertainty aversion. A merger allows entrepreneurs to internalize the positive impact that the choice of the innovation intensity in one innovation has on other innovations, and leads to greater innovation rates. Thus, the merger of innovations endogenously promotes stronger investor sentiment and leads to greater valuations.

5 Process Innovation

An important distinction that has been identified in the literature on innovation is the difference between "product innovation" and "process innovation."[25] Product innovation refers to the generation of a new product that did not exist before, while process innovation involves the improvement of an already existing product. Process innovation is interpreted broadly as involving the improvement of any part of the production process of an existing product, which typically results in efficiency gains due to productivity increases and/or cost reductions.

The innovation process that we have considered so far is in our analysis is well suited to describe the case of "product innovation," whereby a firm invest resources, such as R&D, to develop an innovative product. If the R&D is successful, the firm obtains a new product, while if the R&D is not successful, the innovation process has no value.

In this section we show that our analysis extends very easily to the case of process innovation. We model process innovation by assuming that, by paying at $t = 0$ a fixed cost of κ_τ, a firm can increase the productivity of its second-stage innovation process from Z_τ to IZ_τ $(1 < I < \psi)$. In addition, we assume that the first stage of the innovation process is not risky, $q_\tau = 1$, for $\tau \in \{A, B\}$. The rest of the model unfolds as before.

The following theorem characterizes the equilibrium innovation decision by the two firms.

[25] The distinction between process innovation and product innovation goes back at least to Utterback and Abernathy (1975). More recent work includes Klepper (1996), among many others.

Theorem 7 *There are threshold levels $\{\underline{\kappa}_\tau, \bar{\kappa}_\tau\}_{\tau\in\{A,B\}}$ (defined in the appendix) with $\underline{\kappa}_\tau < \bar{\kappa}_\tau$, such that: (i) if $\kappa_\tau \leq \underline{\kappa}_\tau$, firm τ always innovates; (ii) if $\kappa_\tau \geq \bar{\kappa}_\tau$, firm τ never innovates; (iii) If $\kappa_\tau \in (\underline{\kappa}_\tau, \bar{\kappa}_\tau)$ firm τ innovates if $\kappa_{\tau'} \leq \underline{\kappa}_{\tau'}$, and does not innovate if $\kappa_{\tau'} \geq \bar{\kappa}_{\tau'}$; (iv) if $\kappa_\tau \in (\underline{\kappa}_\tau, \bar{\kappa}_\tau)$ for both $\tau \in \{A, B\}$, there are multiple equilibria, one where both firms innovate and one where neither innovate. The equilibrium where both firms innovate dominates the equilibrium where neither of the firms innovate. There are strategic complementarities in process innovation iff investors are uncertainty averse.*

Similar to the case of product innovation, if firm τ' does innovate, it is optimal for firm τ to spend the cost κ_τ, and thus implement the process innovation, if $\kappa_\tau < \bar{\kappa}_\tau$; in contrast, if firm τ' does not innovate, it is optimal for firm τ to spend the cost κ_τ, and implement the process innovation, if $\kappa_\tau < \underline{\kappa}_\tau < \bar{\kappa}_\tau$. For intermediate values of the initial fixed cost $\kappa_\tau \in [\underline{\kappa}_\tau, \bar{\kappa}_\tau]$ there are multiple equilibria, generating again an assurance game. The presence of multiple equilibria is again a direct consequence of the strategic complementarities created by investors' aversion to uncertainty. If, on the contrary, investors are uncertainty neutral, $\underline{\kappa}_\tau = \bar{\kappa}_\tau$, and the innovation processes in the two firms are independent from each other.

6 Empirical Implications

Our paper has several novel empirical implications.

1. Innovation waves. The strategic complementarity between entrepreneurs' innovation decisions creates in our model the possibility of innovation waves. An innovation wave occurs if an entrepreneur's decision to initiate the innovation process, and thus to undertake the first stage of her project-idea, has the effect of inducing also the other entrepreneur to do the same. This can happen, for example, when a positive shock in the project idea of one entrepreneur lowers the discovery cost from a high level, $k_\tau > \bar{k}_\tau$, to a low level, $k_\tau < \underline{k}_\tau$, while the other entrepreneur faces a moderate discovery cost, $k_{\tau'} \in \left(\underline{k}_{\tau'}, \bar{k}_{\tau'}\right)$, $\tau \neq \tau'$. Because the first entrepreneur faces a high discovery cost, it is not profitable for that entrepreneur to initiate the innovation process, which will discourage the other entrepreneur from innovating as well. If the discovery costs of the first entrepreneur are subject to a shock and decrease to a low level, $k_\tau < \underline{k}_\tau$, it now becomes optimal for her to initiate the innovation process. This decision makes it profitable for the other entrepreneur to innovate as well, in anticipation of the possibility of higher equity prices if both entrepreneurs are successful. Thus, a positive idiosyncratic shock to the technology of an entrepreneur spills over to the other entrepreneur, triggering an innovation wave. Similar results hold for the productivity of innovation, Z_τ, and the probability of success, q_τ. Note that the beneficial spillover effect is more likely to occur the greater the degree of relatedness of the two technologies (the greater the value of r).

2. Innovation waves, investor sentiment, and hot IPO markets. In our model, the market value of an entrepreneur's firm is greater when there are two firms in the market, rather than only one. This is because uncertainty-averse investors are more optimistic when they can invest in the equity of both firms, rather than in one firm only, leading to higher equity valuations. Given the discussion on point 1 above, this means that innovation waves will be associated with strong investor sentiment toward innovations and, thus, booms in the equity markets of technology firms. This means that innovation waves are associated with hot IPO markets, which are followed by lower stock returns. In additions, innovation waves and hot IPO markets are more likely to occur in related industries.

3. Innovation waves and venture capitalists. An additional implication of our model is a new role for venture capitalists. If discovery costs fall in the intermediate range, $k_\tau \in \left(\underline{k}_\tau, \bar{k}_\tau\right)$, entrepreneurs face an "assurance game" in that each entrepreneur will be willing to incur the discovery cost and innovate only if she is assured that also the other entrepreneur will do the same. Lacking such assurance, entrepreneurs may be confined to the inefficient equilibrium with no innovation. In this setting, a venture capitalist may indeed play a positive role by addressing the coordination failure among entrepreneurs. By investing in both firms, the venture capitalist can help coordination among entrepreneurs and lead to greater innovation. In addition, as discussed above, coordination among entrepreneurs' innovative activities will be associated with greater equity market valuations. These observations imply that venture capital activity will be associated with innovation waves and greater equity valuations.

4. Innovation, investor sentiment and merger activity. Our paper presents a new channel in which merger activity can generate synergies and spur innovative activity. In our paper, synergistic gains are the direct outcome of the spillover effects of beliefs on innovation. In the post-merger firm, innovators will internalize the beneficial effect of beliefs on innovation intensity, leading to greater innovation rates for the merged firms. In addition, our model predicts that merger activities involving innovative firms will be associated with strong investor sentiment and greater firm valuations.

7 Conclusion

In this paper, we show that uncertainty aversion generates innovation waves. Uncertainty aversion causes investors to treat different uncertain lotteries as complements, a property that we refer to as uncertainty hedging. Uncertainty hedging by investors produces strategic complementarity in entrepreneurial behavior, producing innovation waves. Specifically, when one entrepreneur has a successful first-stage project, equity valuation, entrepreneur utility, and the intensity of innovation increase for other entrepreneurs as well. Thus, entrepreneurs are more willing to innovate if they expect other entrepreneurs are going to innovate as well, resulting in multiple equilibria. Our model can thus explain why there are some periods when investment in innovation is "hot," and venture capitalists are more willing to invest in risky investment projects tainted by significant uncertainty. Finally, if both innovations are successful, a mergers can add value because the positive spillover effects of innovation due to uncertainty hedging. Thus, our model predicts simultaneous innovation waves, merger waves, and positive investor sentiment.

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A Appendix

Proof of Theorem 1. Let $V_\mu = qE_\mu [u(y_1)] + (1-q)E_\mu [u(y_2)]$, and define $\mu_1 = \arg\min E_\mu [u(y_1)]$, $\mu_2 = \arg\min E_\mu [u(y_2)]$, and $\mu_q = \arg\min V_\mu$. Thus, $E_{\mu_1}[u(y_1)] \le E_{\mu_q}[u(y_1)]$ and $E_{\mu_2}[u(y_2)] \le E_{\mu_q}[u(y_2)]$, so $qE_{\mu_1}[u(y_1)] + (1-q) E_{\mu_2}[u(y_2)] \le qE_{\mu_q}[u(y_1)] + (1-q) E_{\mu_q}[u(y_2)] = \min V_\mu$. Thus, (3) holds. Because uncertainty-neutral agents can be modeled as uncertainty-averse agents with a singleton for their core of beliefs, the inequality holds with equality in the absence of uncertainty aversion. ■

Proof of Lemma 1. Define $u(\theta;\Pi) = e^{\theta-\theta_1}\omega_A y_A + e^{\theta_0-\theta}\omega_A y_B + w_0 - \omega_A V_A - \omega_B V_B$, so that $U(\Pi) = \min_{\theta\in C}\{u(\theta;\Pi)\}$. Thus, $u_\theta = e^{\theta-\theta_1}\omega_A y_A - e^{\theta_0-\theta}\omega_B y_B$, and $u_{\theta\theta} = e^{\theta-\theta_1}\omega_A y_A + e^{\theta_0-\theta}\omega_B y_B$. Because $u_{\theta\theta} > 0$, u is convex in θ, so first order conditions are sufficient for a minimum. $u_\theta = 0$ iff $\theta = \tilde{\theta}^a$ where

$$\tilde{\theta}^a(\Pi) = \frac{1}{2}(\theta_0 + \theta_1) + \frac{1}{2}\ln\frac{\omega_B y_B}{\omega_A y_A}.$$

If $\tilde{\theta}^a(\Pi) \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$, $\theta^a = \tilde{\theta}^a$ (because $\tilde{\theta}^a$ minimizes u). If $\tilde{\theta}^a < \hat{\theta}_0$, $u_\theta > 0$ for all $\theta \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$, so $\theta^a = \hat{\theta}_0$. Similarly, if $\tilde{\theta}^a > \hat{\theta}_1$, $u_\theta < 0$ for all $\theta \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$, so $\theta^a = \hat{\theta}_1$. Therefore, (5) is the worst-case scenario for the investor. ■

Proof of Lemma 3. Each investor's objective function is $U(\Pi) = \min_{\theta\in C} u(\theta;\Pi)$ where $u(\theta;\Pi) = e^{\theta-\theta_1}\omega_A y_A + e^{\theta_0-\theta}\omega_B y_B + w_0 - \omega_A V_A - \omega_B V_B$. Thus, for $\tau \in \{A, B\}$,

$$\frac{dU}{d\omega_\tau} = \frac{\partial u}{\partial \omega_\tau} + \frac{\partial u}{\partial \theta}\frac{d\theta}{d\omega_\tau}$$

If investors are uncertainty-neutral, they believe $C = \{\theta^e\}$, so the second term disappears ($\theta = \theta^e$, so it is constant). If investors are uncertainty averse, θ^a solves the minimization problem, so either $\frac{\partial u}{\partial\theta} = 0$ (an interior solution) or $\frac{d\theta}{d\omega_A} = 0$ (a corner solution). Thus, $\frac{\partial u}{\partial\theta}\frac{d\theta}{d\omega_\tau} = 0$, so that $\frac{dU}{d\omega_\tau} = \frac{\partial u}{\partial\omega_\tau}$ for $\tau \in \{A, B\}$.

$$\frac{\partial u}{dy_A} = e^{\theta^a-\theta_1}y_A - V_A$$

and

$$\frac{du}{dy_B} = e^{\theta_0-\theta^a}y_B - V_B$$

Thus, market clearing requires that $V_A = e^{\theta^a-\theta_1}y_A$ and $V_B = e^{\theta_0-\theta^a}y_B$. Because $p_A(\theta^a) = e^{\theta^a(\Pi)-\theta_1}$ and $p_B(\theta^a) = e^{\theta_0-\theta^a(\Pi)}$, it follows that $V_A = p_A(\theta^a)y_A$ and $V_B = p_B(\theta^a)y_B$ (The proof is identical for SEU, with θ^e instead of θ^a). Note that it is WLOG optimal for all investors to set $\omega_A = \omega_B = 1$, because innovations are priced at expected value given market beliefs. Further, if investors are uncertainty-averse, they will hold identical positions in the risky portfolio (formally, $\frac{\omega_A}{\omega_B}$ is constant across all investors), because there would be gains from trade if they did not. ■

Proof of Lemma 4. Solve the problem in three cases: $\theta^a(\Pi) = \hat{\theta}_0$, $\theta^a(\Pi) = \hat{\theta}_1$, and $\theta^a(\Pi) \in \left(\hat{\theta}_0, \hat{\theta}_1\right)$.

From Lemma 1, $\theta^a(\Pi) = \hat{\theta}_0$ iff $\tilde{\theta}^a(\Pi) \le \hat{\theta}_0$ iff $y_A \ge e^{2\left(\theta^e-\hat{\theta}_0\right)}y_B$. Thus, if $y_A \ge e^{2\left(\theta^e-\hat{\theta}_0\right)}y_B$, $V_A = p_A\left(\hat{\theta}_0\right)y_A$ and $V_B = p_B\left(\hat{\theta}_0\right)y_B$. Similarly, $\theta^a(\Pi) = \hat{\theta}_1$ iff $\tilde{\theta}^a(\Pi) \ge \hat{\theta}_1$ iff $y_A \le e^{2\left(\theta^e-\hat{\theta}_1\right)}y_B$. Thus, if $y_A \le e^{2\left(\theta^e-\hat{\theta}_1\right)}y_B$, $V_A = p_A\left(\hat{\theta}_1\right)y_A$ and $V_B = p_B\left(\hat{\theta}_1\right)y_B$. Finally, from Lemma 1, $\theta^a(\Pi) \in \left(\hat{\theta}_0, \hat{\theta}_1\right)$ iff $\tilde{\theta}^a(\Pi) \in \left(\hat{\theta}_0, \hat{\theta}_1\right)$ iff $y_A \in \left(e^{2\left(\theta^e-\hat{\theta}_1\right)}y_B, e^{2\left(\theta^e-\hat{\theta}_0\right)}y_B\right)$. Because $\theta^a(\Pi) = \tilde{\theta}^a(\Pi)$ on this region, $p_\tau(\theta^a(\Pi)) = e^{\frac{1}{2}(\theta_0-\theta_1)}y_{\tau'}^{\frac{1}{2}}y_\tau^{-\frac{1}{2}}$,which implies that the market values entrepreneur τ's firm at $V_\tau = e^{\frac{1}{2}(\theta_0-\theta_1)}y_{\tau'}^{\frac{1}{2}}y_\tau^{\frac{1}{2}}$. The piecewise function immediately follows because p_A is increasing in θ but p_B is decreasing in θ, and because the core of beliefs is symmetric: $\theta_1 - \hat{\theta}_1 = \hat{\theta}_0 - \theta_0$. There is strategic complementarity in production because $\frac{\partial V_\tau}{\partial y_{\tau'}} \ge 0$ for $\tau' \ne \tau$, with strict inequality for $y_A \in \left(e^{2\left(\theta^e-\hat{\theta}_1\right)}y_B, e^{2\left(\theta^e-\hat{\theta}_0\right)}y_B\right)$. ■

Proof of Lemma 5. Suppose that only entrepreneur A has a successful first-stage project-idea (the case with entrepreneur B follows symmetrically), so $y_B = 0$. By Lemma 1, $\tilde{\theta}^a = -\infty$, so $\theta^a = \hat{\theta}_0$. By Lemma 3, $p_A(\theta_0) =$

$e^{\hat{\theta}_0-\theta_1}$, so $V_A = e^{\hat{\theta}_0-\theta_1} y_A$. Thus, entrepreneur A's payoff is

$$\mathcal{U}_A = e^{\hat{\theta}_0-\theta_1} y_A - \frac{1}{Z_A(1+\gamma)} y_A^{1+\gamma}.$$

Note that $\frac{\partial \mathcal{U}_A}{\partial y_A} = e^{\hat{\theta}_0-\theta_1} - \frac{1}{Z_A} y_A^{\gamma}$, and $\frac{\partial^2 \mathcal{U}_A}{\partial y_A^2} = -\frac{\gamma}{Z_A} y_A^{\gamma-1} < 0$, so FOCs are sufficient for a maximum. Thus, entrepreneur A selects $y_A^{U,SF} = \left[e^{\hat{\theta}_0-\theta_1} Z_A\right]^{\frac{1}{\gamma}}$, sells for $V_A^{U,SF} = e^{\left(\hat{\theta}_0-\theta_1\right)\frac{1+\gamma}{\gamma}} Z_A^{\frac{1}{\gamma}}$, and earns continuation payoff $\mathcal{U}_A^{U,SF} = e^{\left(\hat{\theta}_0-\theta_1\right)\frac{1+\gamma}{\gamma}} Z_A^{\frac{1}{\gamma}} \frac{\gamma}{1+\gamma}$. ■

Proof of Lemma 6. Suppose it is optimal for entrepreneurs to produce output resulting in interior beliefs: $y_A \in \left(e^{\theta_0-\theta_1} y_B, e^{\theta_1-\theta_0} y_B\right)$, which will be optimal because the assumptions on Z_A and Z_B. For $\tau \in \{A, B\}$ and $\tau' \neq \tau$, when entrepreneur τ' produces $y_{\tau'}$, entrepreneur τ produces y_τ and earns continuation utility

$$\mathcal{U}_\tau = e^{\frac{1}{2}(\theta_0-\theta_1)} y_\tau^{\frac{1}{2}} y_{\tau'}^{\frac{1}{2}} - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}.$$

Thus, $\frac{\partial \mathcal{U}_\tau}{\partial y_\tau} = \frac{1}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} y_\tau^{-\frac{1}{2}} y_{\tau'}^{\frac{1}{2}} - \frac{1}{Z_\tau} y_\tau^{\gamma}$ and $\frac{\partial_\tau^2 \mathcal{U}}{\partial y_\tau^2} = -\frac{1}{4} e^{\frac{1}{2}(\theta_0-\theta_1)} y_\tau^{-\frac{3}{2}} y_\tau^{\frac{1}{2}} - \frac{\gamma}{Z_\tau} y_\tau^{\gamma-1}$. Because $\frac{\partial_\tau^2 \mathcal{U}}{\partial y_\tau^2} < 0$, FOCs are sufficient for a local maximum. Thus, $y_\tau = \left[\frac{Z_\tau}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} y_{\tau'}^{\frac{1}{2}}\right]^{\frac{1}{\gamma+\frac{1}{2}}}$.

On this region, optimal output by one firm is strictly increasing in the output of the other firm. Inspection of the revenue function from Lemma 4 immediately shows that entrepreneur A's problem is locally concave almost everywhere, the exception being at $y_A = e^{\theta_1-\hat{\theta}_0} y_B$. Because there is a kink at $y_A = e^{\theta_1-\hat{\theta}_0} y_B$, there may be multiple critical points, resulting in a discontinuous best response function. Thus, there are some parameter values for which there is no pure strategy equilibrium. However, it can be verified (after messy calculations) that so long as $\frac{Z_A}{Z_B} \in \left(\frac{1}{\psi}, \psi\right)$ where $\psi = \frac{1}{4} e^{2\left(\theta^e-\hat{\theta}_0\right)(\gamma+1)} \left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$, if both firms enter, there is a unique equilibrium – it is optimal for the firms to produce output levels such that investors have interior beliefs. Thus, on this region, entrepreneurs' best response functions satisfy $y_\tau^{U,SS}(y_{\tau'}) = \left[\frac{Z_\tau}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} y_{\tau'}^{\frac{1}{2}}\right]^{\frac{1}{\gamma+\frac{1}{2}}}$. ■

Proof of Theorem 3. In equilibrium, the two entrepreneurs select innovation intensity optimally, given the intensity the other entrepreneur is innovating. From Lemma 6, the best response functions are $y_\tau^{U,SS}(y_{\tau'}) = \left[\frac{Z_\tau}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} y_{\tau'}^{\frac{1}{2}}\right]^{\frac{1}{\gamma+\frac{1}{2}}}$. Because entrepreneur τ' also selects intensity optimally, selecting $y_{\tau'}^{U,SS}(y_\tau)$, it follows that

$$y_\tau = \left[\frac{Z_\tau}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} \left[\frac{Z_{\tau'}}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} y_\tau^{\frac{1}{2}}\right]^{\frac{1}{2\gamma+1}}\right]^{\frac{1}{\gamma+\frac{1}{2}}}.$$

After some messy calculations, this holds iff

$$y_\tau^{U,SS} = \left[\frac{1}{2} e^{\frac{1}{2}(\theta_0-\theta_1)} Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} Z_{\tau'}^{\frac{1}{2\gamma+2}}\right]^{\frac{1}{\gamma}}$$

for $\tau \in \{A, B\}$ and $\tau' \neq \tau$.

Because the market price is $V_\tau^{U,SS} = e^{\frac{1}{2}(\theta_0-\theta_1)} y_\tau^{\frac{1}{2}} y_{\tau'}^{\frac{1}{2}}$, it follows that

$$V_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{\gamma+1}{\gamma}} \left[Z_\tau Z_{\tau'}\right]^{\frac{1}{2\gamma}}.$$

Similarly, entrepreneur τ earns continuation utility $\mathcal{U}_\tau^{U,SS} = V_\tau^{U,SS} - \frac{1}{Z_\tau(1+\gamma)} y_\tau^{1+\gamma}$, which can be expressed as

$$\mathcal{U}_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2},$$

for $\tau \in \{A, B\}$ and $\tau' \neq \tau$. Thus, there are strategic complementarities in production and profit. ■

Proof of Corollary 1. Recall $\mathcal{U}_\tau^{U,SS} = \frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0 - \theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}$ and $\mathcal{U}_\tau^{U,SF} = e^{\left(\hat{\theta}_0 - \theta_1\right)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}} \frac{\gamma}{1+\gamma}$. Thus, $\mathcal{U}_\tau^{U,SS} > \mathcal{U}_\tau^{U,SF}$ iff $\frac{Z_{\tau'}}{Z_\tau} > \frac{1}{\psi}$ where $\psi = \frac{1}{4} e^{2\left(\theta^e - \hat{\theta}_0\right)(\gamma+1)} \left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$. Recall that we assumed $\frac{Z_{\tau'}}{Z_\tau} \in \left(\frac{1}{\psi}, \psi\right)$ where $\psi = \frac{1}{4} e^{2\left(\theta^e - \hat{\theta}_0\right)(\gamma+1)} \left(1 + \frac{1}{2\gamma}\right)^{2\gamma}$, so this is always satisfied – entrepreneurs are better off when other entrepreneurs have a successful first-stage project.

Recall that $V_\tau^{U,SS} = 2^{-\frac{1}{\gamma}} e^{\frac{1}{2}(\theta_0 - \theta_1)\frac{\gamma+1}{\gamma}} \left[Z_\tau Z_{\tau'}\right]^{\frac{1}{2\gamma}}$ and $V_\tau^{U,SF} = e^{\left(\hat{\theta}_0 - \theta_1\right)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{\gamma}}$. After some messy algebra, it can be shown that $V_\tau^{U,SS} > V_\tau^{U,SF}$ iff $\frac{Z_{\tau'}}{Z_\tau} > 4 e^{2\left(\hat{\theta}_0 - \theta^e\right)(1+\gamma)}$. Define $\psi_1 = \frac{1}{4} e^{2\left(\theta^e - \hat{\theta}_0\right)(1+\gamma)}$.

Finally, $y_\tau^{U,SS} = \left[\frac{1}{2} e^{\frac{1}{2}(\theta_0 - \theta_1)} Z_\tau^{\frac{2\gamma+1}{2\gamma+2}} Z_{\tau'}^{\frac{1}{2\gamma+2}}\right]^{\frac{1}{\gamma}}$ and $y_\tau^{U,SF} = \left[e^{\hat{\theta}_0 - \theta_1} Z_\tau\right]^{\frac{1}{\gamma}}$. After some messy algebra, it can be shown that $y_\tau^{U,SS} > y_\tau^{U,SF}$ iff $\frac{Z_{\tau'}}{Z_\tau} > 4^{\gamma+1} e^{2\left(\hat{\theta}_0 - \theta^e\right)(1+\gamma)}$. Thus, define $\psi_2 = \left[\frac{1}{4}\right]^{\gamma+1} e^{2\left(\theta^e - \hat{\theta}_0\right)(1+\gamma)}$. Further, because $\gamma > 0$, it follows immediately that $\psi_2 < \psi_1 < \psi$. ■

Proof of Theorem 4. If only entrepreneur τ innovates, he earns payoff $E\mathcal{U}_\tau^{U,N} = q_\tau \mathcal{U}_\tau^{U,SF} - k_\tau$ (Lemma 5). Thus, if an entrepreneur does not expect the other entrepreneur to innovate, he will innovate iff $k_\tau \leq \underline{k}_\tau \equiv q_\tau \mathcal{U}_\tau^{U,SF}$. Conversely, if the other entrepreneur innovates, entrepreneur τ earns payoff $E\mathcal{U}_\tau^{U,I} = (q_\tau q_{\tau'} + r)\mathcal{U}_\tau^{U,SS} + \left[q_\tau\left(1 - q_{\tau'}\right) - r\right]\mathcal{U}_\tau^{U,SF} - k_\tau$ if he innovates as well. Thus, if the other entrepreneur innovates, entrepreneur τ will innovate iff $k_\tau \leq \bar{k}_\tau \equiv (q_\tau q_{\tau'} + r)\mathcal{U}_\tau^{U,SS} + \left[q_\tau\left(1 - q_{\tau'}\right) - r\right]\mathcal{U}_\tau^{U,SF}$. By Corollary 1, $\mathcal{U}_\tau^{U,SF} < \mathcal{U}_\tau^{U,SS}$, so it follows that $\underline{k}_\tau < \bar{k}_\tau$ (because the coefficients on the terms in $\bar{k}_\tau$ sum to q_τ). ■

Proof of Corollary 2. Comparative Statics follow immediately from inspection of the expressions for $\underline{k}_\tau$ and $\bar{k}_\tau$, and because $\mathcal{U}_\tau^{U,SS}$ is increasing in Z_τ and $Z_{\tau'}$, and $\mathcal{U}_\tau^{U,SF}$ is increasing in Z_τ. ■

Proof of Theorem 5. The merged firm seeks to maximize the combined value of the two projects. By identical reasoning to Lemma 3, $V_A = p_A(\theta_I)\, y_A$ and $V_B = p_B(\theta_I)\, y_B$, where θ_I is the market belief at $t = 1$ on θ. Thus, the merged firm's objective is

$$\mathcal{U}^M = p_A(\theta_I)\, y_A + p_B(\theta_I)\, y_B - c_A(y_A) - c_B(y_B).$$

In contrast, if investors are uncertainty averse, $\theta_I = \theta^a$, which depend on the choice of y_A and y_B. As shown in Lemma 4, $V_A = V_B = e^{\frac{1}{2}(\theta_0 - \theta_1)} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}}$, so the objective function of the merged firm becomes

$$\mathcal{U}^M = 2e^{\frac{1}{2}(\theta_0 - \theta_1)} y_A^{\frac{1}{2}} y_B^{\frac{1}{2}} - \frac{1}{Z_A(1+\gamma)} y_A^{1+\gamma} - \frac{1}{Z_B(1+\gamma)} y_B^{1+\gamma} - X_A - X_B.$$

$$\frac{\partial \mathcal{U}^M}{\partial y_\tau} = e^{\frac{1}{2}(\theta_0 - \theta_1)} y_\tau^{-\frac{1}{2}} y_{\tau'}^{\frac{1}{2}} - \frac{1}{Z_\tau} y_\tau^\gamma$$

for $\tau \in \{A, B\}$ and $\tau' \neq \tau$. This implies[26] that

$$y_\tau = \left[e^{\frac{1}{2}(\theta_0 - \theta_1)} y_{\tau'}^{\frac{1}{2}} Z_\tau\right]^{\frac{1}{\gamma + \frac{1}{2}}},$$

so

$$y_\tau^M = \left[e^{\frac{1}{2}(\theta_0 - \theta_1)} Z_{\tau'}^{\frac{1}{2\gamma+2}} Z_\tau^{\frac{2\gamma+1}{2\gamma+2}}\right]^{\frac{1}{\gamma}}.$$

Thus, each project within the merged firm has value $V_\tau^M = e^{\frac{1}{2}(\theta_0 - \theta_1)\frac{1+\gamma}{\gamma}} \left[Z_{\tau'} Z_\tau\right]^{\frac{1}{2\gamma}}$. ■

Proof of Theorem 6. If entrepreneur τ does not expect entrepreneur τ' to innovate, he innovates iff $k_\tau \leq \underline{K}_\tau \equiv q_\tau \mathcal{U}_\tau^{U,SF}$, the same cutoff as without the possibility of a merger. However, if entrepreneur τ expects entrepreneur τ' to innovate, he innovates iff $k_\tau \leq \bar{K}_\tau \equiv (q_\tau q_{\tau'} + r)\, v_\tau + \left[q_\tau\left(1 - q_{\tau'}\right) - r\right]\mathcal{U}_\tau^{U,SF}$. Because $v_\tau = \mathcal{U}_\tau^{U,SS} + \frac{1}{2}\left(\mathcal{U}^M - \mathcal{U}_A^{U,SS} - \mathcal{U}_B^{U,SS}\right)$, if $v_\tau > \mathcal{U}_\tau^{U,SS}$, $\bar{K}_\tau > \bar{k}_\tau$, so the cutoff will be larger when mergers are possible, resulting in more innovation. Thus, it is sufficient to show that $\mathcal{U}^M > \mathcal{U}_A^{U,SS} + \mathcal{U}_B^{U,SS}$.

[26] Because the cost functions are convex, the problem is globally concave, so first-order conditions are sufficient.

Because $V^M = V_A^M + V_B^M$, the merged firm earns utility

$$\mathcal{U}^M = 2\frac{\gamma}{1+\gamma} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} \left[Z_A Z_B\right]^{\frac{1}{2\gamma}}.$$

Each entrepreneur could earn utility $\mathcal{U}_\tau^{U,SS} = \frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}$ if they did not merge, so

$$\begin{aligned}
\mathcal{U}_A^{U,SS} + \mathcal{U}_B^{U,SS} &= 2\frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_A^{\frac{1}{2\gamma}} Z_B^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2} \\
&= \mathcal{U}^M \frac{1}{2^{\frac{1}{\gamma}}} \frac{2\gamma+1}{2\gamma}
\end{aligned}$$

To see that $\frac{1}{2^{\frac{1}{\gamma}}}\frac{2\gamma+1}{2\gamma} \in (0,1)$ for all $\gamma \in (0,\infty)$, define $x = \frac{1}{\gamma}$, and $f(x) = 2^{-x}\frac{\frac{2}{x}+1}{\frac{2}{x}} = 2^{-x-1}(2+x)$. Note that $\lim_{x\to 0^+} f(x) = 1$, and $\lim_{x\to+\infty} f(x) = 0$, and $f'(x) = 2^{-x-1}\left[1-(2+x)\ln 2\right]$, which is strictly negative because $2\ln 2 \approx 1.3863 > 1$. Therefore, $\frac{1}{2^{\frac{1}{\gamma}}}\frac{2\gamma+1}{2\gamma} < 1$ for all $\gamma \in (0,\infty)$. Thus, the merger adds value, because $\mathcal{U}^M > \mathcal{U}_A^{U,SS} + \mathcal{U}_B^{U,SS}$. Because surplus is divided evenly, entrepreneur τ receives utility $v_\tau = \mathcal{U}_\tau^{U,SS} + \frac{1}{2}\left(\mathcal{U}^M - \mathcal{U}_A^{U,SS} - \mathcal{U}_B^{U,SS}\right)$. ∎

Proof of Theorem 7. Suppose, for the sake of discussion, that process innovation is successful with probability 1. As shown in Theorem 3, if neither firm innovates, firm τ receives utility

$$\mathcal{U}_\tau^{U,N} = \frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2},$$

while, if either firm τ or firm τ' innovates,

$$\mathcal{U}_\tau^{U,S} = I^{\frac{1}{2\gamma}} \frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2},$$

and if both firms innovate, they

$$\mathcal{U}_\tau^{U,B} = I^{\frac{1}{\gamma}} \frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}.$$

Thus, if firm τ' does not innovate, firm τ executes process innovation iff $\mathcal{U}_\tau^{U,S} - \kappa_\tau > \mathcal{U}_\tau^{U,N}$, or equivalently, iff

$$\kappa_\tau < \underline{\kappa}_\tau \equiv \left(I^{\frac{1}{2\gamma}} - 1\right) \frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}.$$

Similarly, if firm τ' innovates, firm τ executes process innovation iff $\mathcal{U}_\tau^{U,B} - \kappa_\tau > \mathcal{U}_\tau^{U,S}$, or equivalently, iff

$$\kappa_\tau < \bar{\kappa}_\tau \equiv I^{\frac{1}{2\gamma}} \left(I^{\frac{1}{2\gamma}} - 1\right) \frac{1}{2^{\frac{1}{\gamma}}} e^{\frac{1}{2}(\theta_0-\theta_1)\frac{1+\gamma}{\gamma}} Z_\tau^{\frac{1}{2\gamma}} Z_{\tau'}^{\frac{1}{2\gamma}} \frac{2\gamma+1}{2\gamma+2}.$$

Because $I > 1$, $\underline{\kappa}_\tau < \bar{\kappa}_\tau$. ∎