Unbalanced Growth Slowdown*

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Abstract

Unbalanced growth slowdown is the reduction in aggregate productivity growth that results from the reallocation of economic activity to industries with low productivity growth. We show that unbalanced growth slowdown has considerably reduced past U.S. productivity growth and we assess by how much it will reduce future U.S. productivity growth. To achieve this, we build a novel model that generates the unbalanced growth slowdown of the postwar period. The model makes the surprising prediction that future reductions in aggregate productivity growth do not exceed past ones. The key reason is that the stagnant industries do not take over the economy.

Keywords: Productivity Slowdown; Structural Transformation; Unbalanced Growth.
JEL classification: O41; O47; O51.

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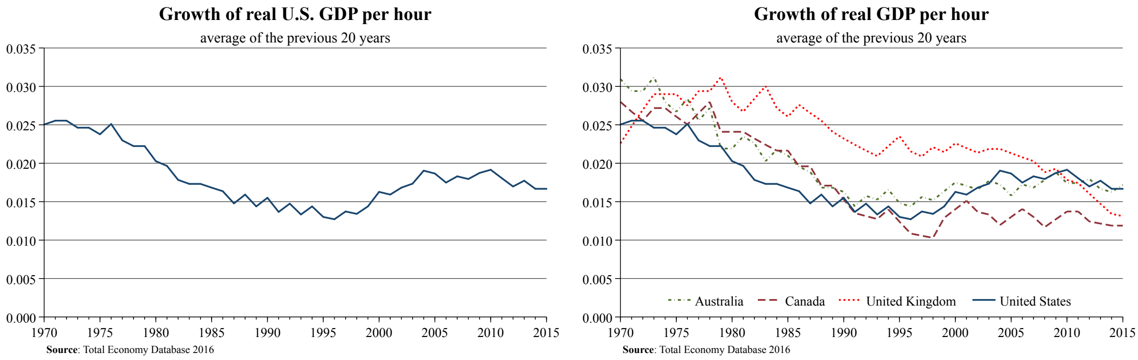
Figure 1: Growth Slowdown in Major English–speaking Countries



1 Introduction

There is ample evidence that long–term economic growth has slowed in the U.S. The left panel of Figure 1 shows this by plotting the average growth rates of U.S. labor productivity during the previous 20 years where labor productivity is measured as real aggregate value added per hour. We can see that labor productivity growth declined by almost a percentage point from an average of about 2.5% during 1950–1970 to a little higher than 1.5% during 1990–2010.[1] Although we focus on the U.S. in this paper, growth slowdown has happened in other rich countries too. The right panel of Figure 1 shows similar downward trends for the three English–speaking countries Australia, Canada, and the U.K. We choose them as comparisons because they are rich countries that after World War II did not experience exceptionally large growth rates as the result of growth miracles (like Japan and South Korea) or of massive reconstruction efforts (like France and Germany).

One of the hotly debated questions of the moment is whether growth slowdown is a temporary or a permanent phenomenon. Fernald and Jones (2014) pointed out that engines of economic growth like education or research and development require the input of time which cannot be increased ad infinitum. This suggests that there is a natural limit to growth and that the slowdown might well be permanent. Gordon (2016) reached the same conclusion, arguing that we picked the "low–hanging fruit" (e.g., railroads, cars, and airplanes) during the "special century 1870–1970" and that more recent innovations pale in comparison.

In this paper, we investigate to which extent growth slowdown results from the interaction between unbalanced growth and structural transformation, which we refer to as *unbalanced growth slowdown*. To explain how unbalanced growth slowdown arises, it is helpful to go back to the observation of Baumol (1967) that modern economic growth has been unbalanced in that labor productivity growth differed widely across industries. Baumol drew particular atten-

[1] Antolin-Diaz et al. (2016) (and several of the reference therein) offer statistical analyses of growth slowdown, confirming the same conclusion that we draw from eyeballing the graph.

tion to the fact that many industries in the service sector experienced low labor productivity growth or even outright stagnation.[2] More recently, Ngai and Pissarides (2007) observed that as economies develop resources are systematically reallocated towards the service industries. Taken to the extreme, their analysis implies that in the limit the service sector with the slowest productivity growth takes over the whole economy.[3] Together, unbalanced growth and structural transformation therefore lead to *unbalanced growth slowdown*.

We begin our analysis by showing that unbalanced growth slowdown was quantitatively important in the private U.S. economy after the second world war. We leave out the government sector because labor productivity of the government sector is not well measured. We use the broad disaggregation into goods (tangible value added) and services (intangible value added) that is common in the literature on structural transformation. Since the service sector comprises most of the U.S. economy and its industries have rather different labor productivity growth, we disaggregate it further. We follow the classification of WORLD KLEMS which distinguishes between market services like financial services and non–market services like residential real estate. Market services tend to have faster productivity growth than non–market services, which essentially stagnated during the last thirty years. To measure the importance of unbalanced growth slowdown in the postwar U.S. private economy, we calculate how large aggregate productivity growth would have been during 1947–2010 for the counterfactual case without structural transformation. Using WORLD KLEMS data and the productivity accounting method of Nordhaus (2002), we find that the annual average growth rates of productivity per efficiency hour would have been between 0.2 and 0.3 percentage points higher than they actually were, depending on the counterfactual that we focus on. These numbers are in the same ball park as those of Nordhaus (2008) and they amount to 20–30% of the one percentage point reduction in average labor productivity growth that is suggested by the left panel of Figure 1. This finding suggests that a sizeable part of the observed growth slowdown can be attributed to unbalanced growth slowdown.

Although unbalanced growth slowdown is empirically important, the literature on structural transformation has all but ignored it. The likely reason for this is that analytically characterizing the equilibrium path of multi–sector models is usually possible only if a generalized balanced growth path exists along which aggregate variables are either constant or grow at constant rates. If aggregate labor productivity grows at a constant rate along the generalized balanced growth path, then unbalanced growth slowdown does not appear to be an issue. Our first contribution is to clarify that this is a misconception. We develop a canonical model of structural transformation and show that it exhibits unbalanced growth slowdown in terms of welfare, in that the growth rate of welfare is declining of time. We then show that whether or not this is picked up by aggregate labor productivity growth depends critically on how one measures real quan-

[2] See Oulton (2001), Nordhaus (2008) and Baumol (2013) for more restatements of this observation.

[3] Herrendorf et al. (2014) review the literature on structural transformation.

tities. Specifically, the model has a generalized balanced growth path along which aggregate labor productivity grows at a constant rate if one measures real quantities in "the usual model way". This involves expressing the variables of a given period in units of a numeraire from the same period. In other words, the model way uses a different numeraire and different relative prices in each period. In contrast, "the NIPA way" of calculating real quantities uses fixed prices from a base period that do not change between two periods. We will show that this difference is critical: although our model displays balanced growth if real quantities are calculated in the model way, it displays unbalanced growth slowdown if real quantities are calculated in the NIPA way.[4] Having clarified this, we restrict our model to generate the unbalanced growth slowdown of labor productivity in the postwar U.S. and use the restricted model to assess by how much unbalanced growth and structural transformation will slow down labor productivity growth in the next half century. To put the bottom line upfront, this will yield the surprising conclusion that although unbalanced growth slowdown has been quite a drag on postwar U.S. growth, it will not become more of a challenge to the future growth performance. The reason for this conclusion is that, in contrast to the implication that is usually derived in the literature, it will turn out that the slowest growing sector won't take over our entire model economy in the limit. This will restrain the future effect of unbalanced growth slowdown.

To guide which features to put into our canonical model of structural transformation, we document key stylized facts about unbalanced growth and structural transformation between goods and services and between market and non–market services. The usual patterns hold between goods and total services: the shares of goods in total expenditure and total hours worked decline; the labor productivity of goods grows more strongly than that of total services; the price of goods relative to total services reflects this and declines. We then establish the following novel patterns about the two subsectors of the service sector: the shares of non–market services in the hours and expenditures of total services increase until about 1980 after which they remain roughly constant; labor productivity of market services grows over the whole period by less than that of goods and by more than that of non–market services; labor productivity in non–market services grows somewhat until around 1980 and stagnates afterwards; the price of non–market relative to market services reflects this: it initially increases until around 1980 and then it increases strongly. Together, these stylized facts imply that when the labor productivity of non–market services starts to stagnate around 1980 the shares of non–market services stop increasing. This is the crucial observation for what is to come, because it suggests that the slow–growing non–market services are *not* taking over the entire service sector.

Our canonical model has three sectors, which produce goods, market services, and non–

[4]When we refer to the NIPA way of calculating real quantities, we mean calculating real quantities via chain indexes, which conforms to the best practice used by the BEA. A chain index is the geometric average of the Laspeyres index and the Paasche index, which are both fixed–price indexes that use either the fixed prices of the initial period or the subsequent period. We emphasize that although it might sound similar, using chain indexes is completely different from using relative prices that change from period to period.

market services. There is exogenous, sector–specific technological progress. Preferences are described by the non–homothetic CES utility function that has recently been proposed by Comin et al. (2015) in the context of structural transformation and that implies that income effects do not disappear in the limit when consumption grows without bound. This feature is consistent with the existing evidence [Boppart (2016) and Comin et al. (2015)], and it is potentially crucial in the present context because we are after the limit behavior of the economy. The novelty of our model compared to the literature on structural transformation is that we allow the elasticity of substitution between goods and total services to differ from the elasticity between market and non–market services. To achieve this, we nest two non–homothetic CES utility functions: an outer layer aggregates goods and total services; an inner layer aggregates market and non–market services into total services. This allows the model to do two things: keep the well established feature of preferences that goods and total services are complements, see for example Herrendorf et al. (2013); match the fact that the share of non–market services did not increase when their relative price increased strongly after 1980, for which market and non–market services must be substitutes, instead of complements.

Assuming that the recent sectoral labor productivity growth continues into the future, our model implies that unbalanced growth slowdown will be at most as large in the future as in the past. The reason for this surprising finding is that in our model the stagnating non–market services do not take over the entire the entire economy. This comes about because the substitutability between market and non–market services puts a limit on how much future growth slowdown may occur in our model. If the relative price of non–market services increases without bound because non–market services stagnate, then households substitute market services for non–market services. This does not happen in existing models of structural transformation which impose a common elasticity of substitution among goods and all services and find that then goods and services are complements.

Our work is related to several papers arguing that the service sector has become so large and heterogenous that it is useful to disaggregate it into subsectors; see for example Baumol et al. (1985), Jorgenson and Timmer (2011), and Duarte and Restuccia (2016). Our work is also related to several papers that measured cross–country gaps in sectoral TFP or labor productivity; see for example Duarte and Restuccia (2010), Herrendorf and Valentinyi (2012) and Duarte and Restuccia (2016). Instead of focusing on cross–sections of countries, we focus on the evolution of U.S. labor productivity over time. The most closely related paper to ours is Duarte and Restuccia (2016), which features in both sets of papers. Duarte and Restuccia use the 2005 cross section of the International Comparisons Program of the Penn World Table to estimate sectoral productivity gaps between rich and poor countries. They distinguish between traditional and modern services, which roughly corresponds to our distinction between non–market and market services. They find that the largest cross–country productivity gaps are in goods and modern services and the smallest cross–country gaps are in traditional services. This cross sectional evidence nicely complements our time series evidence from the U.S. that labor productivity in non–market services grows less than in the other two sectors. If we took a rich and a poor country with that feature and looked at the productivity differences in a given sector, then our findings would imply that over time larger productivity differences between these countries emerge in goods and market services than in non–market services.

The remainder of the paper is organized as follows. In the next Section, we present evidence that structural transformation has led to growth slowdown in the postwar U.S. In Section 3, we develop our model. In the next section, we characterize under what conditions our model leads to unbalanced growth slowdown. In section 5, we calibrate our model and use it to predict how much future growth slowdown results from structural transformation. Section 6 concludes and an Appendix contains the detailed description of our data work, the proofs of our results, and additional evidence.

2 Unbalanced Growth Slowdown in the Postwar U.S. Data

We use WORLD KLEMS as our data source, because it offers information about both raw hours and adjusted hours that take into account differences in human capital ("efficiency hours").[5] Since we will study counterfactuals that reallocate workers with potentially different levels of human capital across sectors, having information about efficiency hours will allow us to perform important robustness checks. We focus on the private U.S. economy. This implies that we leave out the two industries of the government sector, namely compulsory social security as well as public administration and defense, whose value added is hard to measure. We split the private economy into the standard broad sectors goods and services. The goods sector comprises agriculture, construction, manufacturing, mining, and utilities and the service sector comprises the remaining industries.

Baumol et al. (1985) observed that the service sector "contains some of the economys most progressive activities as well as its most stagnant". To capture the heterogeneity in the productivity of the service industries, we follow the sector classification of WORLD KLEMS and disaggregate aggregate services into market and non–market services. As the names suggest, market services are traded in reasonably undistorted markets whereas non–market services are either not traded in markets at all or are trade in markets that are subject to heavy government intervention. Market services comprise the following industries: business services except for real estate activities; distribution; finance; personal services; and post and telecommunication. Non–market services comprise the following industries: education; health and social work; and real estate activities. The cleanest example of non–market services is real estate activities, be-

[5]See Jorgenson et al. (2013) for a description of the data set.

cause much of its value added comes from the imputed rents for owner–occupied housing that are not traded in markets by definition.

Like all disaggregation schemes, the disaggregation of services into market and non–market services involves arbitrary judgement. We therefore chose to adopt a widely used disaggregation scheme that underlies WORLD KLEMS, instead of coming up with our own scheme. The disaggregation into market and non–market services is natural to use in our context, because it groups the services for which we do and do not have reasonably reliable market prices into two different groups. This implies that most services for which unmeasured quality improvements are likely to be a severe problem are in non–market services.

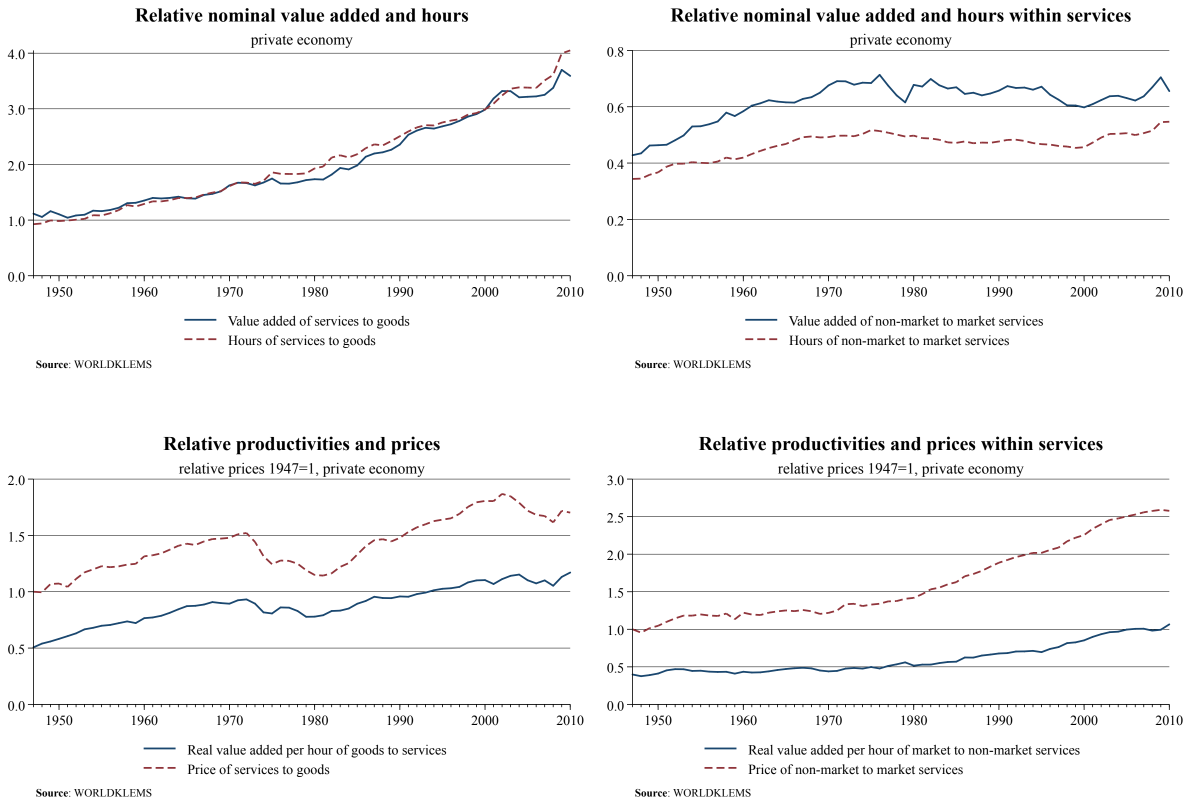
Instead of disaggregating aggregate services into market and non–market services, Duarte and Restuccia (2016) disaggregated them into traditional and modern services to study the differences in sectoral productivities in a cross section of countries. The defining criterion they used was how the relative price of a service category changed with the level of GDP per capita across countries. It it increases (decreases) then they called the service category traditional (modern). Their distinction between traditional and modern services corresponds broadly to WORLD KLEMS' distinction between non–market and market services. The exception is personal services which are included in Duarte and Restuccia's traditional services and in WORLD KLEMS' market services. We established that our conclusions would not change in quantitatively important ways if we disaggregated the service sector into traditional and non–traditional services.

2.1 Structural transformation in the data

Figure 2 shows the behavior of our sectors in the postwar U.S. economy.[6] The left panel plots the standard distinction between goods and services while the right panel plots the new distinction between market and non–market services. The figures in the upper panel plot ratios of sectoral efficiency hours and sectoral nominal expenditures. The upper–left figure shows that usual pattern that the ratios of goods relative to aggregate services increased. The upper–right figure shows that the novel pattern that the ratios of non–market to market services increased until about 1980 after which they remained roughly constant. The figures in the lower panel plot relative labor productivities and relative prices. The lower–left figure shows the usual pattern that the labor productivity of goods relative to services and the price of services relative to goods increased for most of the postwar period. The lower–right panel shows the novel pattern that both the labor productivity of market relative to non–market services and the price of non–market relative to market services increased somewhat until about 1980 and increased strongly afterwards.

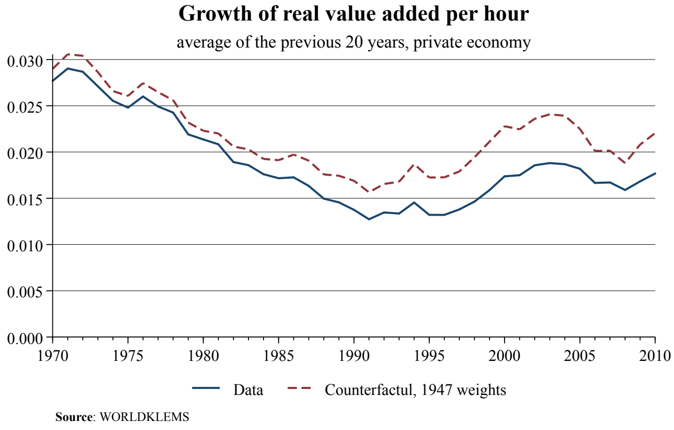
[6] All figures in the text use raw hours worked. The patterns are qualitatively similar for raw hours and efficiency hours. The corresponding figures for efficiency hours can be found in Appendix D.

Figure 2: Postwar U.S. Structural Transformation – Hours Worked



The slow productivity growth of many service industries may in part come from the fact that quality improvements in services are not properly measured. Triplett and Bosworth (2003), for example, wrote that "perhaps the services industries were never sick, it was just, as Griliches (1994) has suggested, that the measuring thermometer was wrong". Mis–measured quality may translate into mis–measured aggregate productivity slowdown, although Byrne et al. (2016) and Syverson (2016) argued that this is not likely to be the case for the recent productivity slowdown since the early 2000s. We will nonetheless take the numbers from WOLD KLEMS at face value in this paper and pretend that there are no mismeasured quality issues. Our estimates of unbalanced growth slowdown therefore provide an upper bound for the actual unbalanced growth slowdown; if there are unmeasured quality improvements in services, then the future growth slowdown will be smaller than our estimate. This way of proceeding is informative in our context because our key finding will be that unbalanced growth slowdown will be limited in the future.

Figure 3: IS THIS FIGURE FOR 30 SECTORS? IT SHOULD BE! Actual versus Counterfactual U.S. Growth Slowdown



2.2 Unbalanced growth slowdown in the data

To assess whether unbalanced growth of sectoral labor productivity and structural transformation let to a quantitatively important growth slowdown, we compare the actual growth rates of real value added per hour with two counterfactual growth rates. The first would have resulted if no structural transformation had taken place and the sectoral composition had not changed since 1947. The second counterfactual growth rate would have resulted if structural transformation had taken place all at once in 1947 and the sectoral composition had jumped to that of 2010. To eliminate short–term fluctuations, we calculate rolling averages of growth rates. We choose 20 years because the WORLD KLEMS data for the U.S. start in 1947 and the beginning of the growth slowdown is usually dated at 1970; see for example Gordon (2016). Calculating the counterfactuals is not as straightforward as one might think because WORLD KLEMS is build around Törnqvist indexes that are not additive.[7] To deal with this complication, we use the productivity accounting method of Nordhaus (2002); see Appendix A for the details.

Figure 3 depicts the results, suggesting that the growth slowdown would have been considerably less pronounced if there had not been any structural transformation and the sectoral weights had been fixed at their 1947 levels. Table **??** reports the growth slowdown of productivity per raw hour and per efficiency hour. The table contains numbers for both the original 30–sector split from the data and the 3–sector split we adopt. The table has three important implications: there is a sizeable unbalanced growth slow–down of around 0.3 percentage points; our 3–sector split captures around two–thirds of that unbalanced growth slowdown; the size of the unbalanced growth slowdown is similar for efficiency hours and for raw hours. We draw two conclusions from this. First, WORLD KLEMS' disaggregation into market and non–market

[7]Törnqvist and Chain Indexes are both flexible weight indexes. For the data work, they two gives very similar results.

Table 1: Actual vs. Counterfactual Productivity Growth

sector composition	growth of value added per hour (in %)				
actual			2.07		
counterfactual	13 sectors	fast growing / slow growing	traditional / non–traditional	market / non–market	skilled / unskilled
fixed at 1947	2.27	2.31	2.31	2.31	2.32
fixed at 2010	1.70	1.73	1.74	1.75	1.75

Table 2: Actual vs. Counterfactual Productivity Growth

sector composition	growth of value added per hour (in %)				
actual			2.1		
counterfactual	13 sectors	fast growing / slow growing	traditional / non–traditional	market / non–market	skilled / unskilled
fixed at 1947	2.3	2.3	2.3	2.3	2.3
fixed at 2010	1.7	1.7	1.7	1.8	1.8

services captures most of unbalanced growth slowdown, although it is very simple and remains analytically tractable. Second, it is acceptable in the current context to focus on productivity per raw hour, instead of on productivity per efficiency house. Focusing on productivity per raw hour has the advantages of being simpler and of making our results more easily comparable with those of other studies.

Given that we used the productivity accounting method of Nordhaus, it is natural to compare our results with his which were based on the comparison between the two counterfactuals with fixed initial and fixed final sector weights. Nordhaus (2008) found that the difference between them amounts to 0.6 percentage points. The second column of Table 3 shows that for 30 sectors and productivity per raw hour, we find 0.7 percentage points. Possible reasons for the discrepancy is that Nordhaus (2008) investigated the shorter period 1948–2001 and used data from the BEA. Moreover, he reported the productivity effect from reallocating labor among industries with different growth rates of sectoral productivity, whereas we report the combined productivity effects from reallocating labor among industries with different growth *rates* and *levels* of sectoral productivity. These two different effects are sometimes referred to as the "Baumol Effect" and the "Denison Effect"; see Nordhaus (2008) for a more detailed discussion of them.

ROBUSTNESS

After having established that structural transformation led to an economically significant unbalanced growth slowdown, we now build a model to capture this. Afterwards, we will calibrate the model and use it to assess by how much unbalanced sectoral productivity growth

Table 3: Value Added per Hour vs. Efficiency Hour for 13 Sectors

sector composition	growth of value added (in %)	
	per hour	per efficiency hour
actual	2.07	1.67
counterfactual fixed at 1947	2.27	1.89
counterfactual fixed at 2010	1.70	1.22

Table 4: Value Added per Hour vs. Efficiency Hour for 13 Sectors

sector composition	growth of value added (in %)	
	per hour	per efficiency hour
actual	2.1	1.7
counterfactual fixed at 1947	2.3	1.9
counterfactual fixed at 2010	1.7	1.2

and structural transformation will reduce future productivity growth.

3 Model

3.1 Environment

There are three sectors. In each sector, value added is produced from labor:

$$Y_{it} = A_{it} H_{it} \tag{1}$$

$i = g, m, n$ is the sector index which stands for goods, market services, and non–market services. Y_i, A_i, and H_i denote value added, total factor productivity, and labor of sector i. The linear functional form implies that labor productivity equals TFP, that is, $Y_{it}/H_{it} = A_{it}$. We use it because it is simple and captures the essence of the role that technological progress plays for structural transformation; see Herrendorf et al. (2015) for further discussion.

There is a measure one of identical households. Each household is endowed with one unit of labor that is inelastically supplied and can be used in all sectors. As a result, GDP and GDP per capita, GDP per worker, and GDP per hour ("labor productivity") are all equal in our model.

Utility is described by two nested, non–homothetic CES utility functions. The utility from the consumption of goods and (aggregate) services, C_{gt} and C_{st}, is given by:

$$C_t = \left(\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g - 1}{\sigma_c}} C_{gt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s - 1}{\sigma_c}} C_{st}^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}} \tag{2a}$$

Services are given by a non–homothetic CES aggregator of market and non–market services, C_{mt} and C_{nt}:

$$C_{st} = \left(\alpha_m^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_m - 1}{\sigma_s}} C_{mt}^{\frac{\sigma_s - 1}{\sigma_s}} + \alpha_n^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_n - 1}{\sigma_s}} C_{nt}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \tag{2b}$$

$\alpha_g, \alpha_s, \alpha_m, \alpha_n \geq 0$ are weights; $\sigma_c, \sigma_s \geq 0$ are elasticities of substitution; $\varepsilon_g, \varepsilon_s, \varepsilon_m, \varepsilon_n > 0$ capture income effects. We follow Comin et al. (2015) and assume that in each aggregator one good is a luxury good whereas the other one is a necessity: $\min\{\varepsilon_g, \varepsilon_s\} < 1 < \max\{\varepsilon_g, \varepsilon_s\}$ and $\min\{\varepsilon_m, \varepsilon_n\} < 1 < \max\{\varepsilon_m, \varepsilon_n\}$.

The non–homothetic CES utility function (2) goes back to the work of Hanoch (1975) and has recently been used in the context of structural transformation by Comin et al. (2015) and by Sposi (2016). For $\varepsilon_i = 1$ it reduces to the standard CES utility that implies homothetic demand functions for each consumption good. For $\varepsilon_i \neq 1$, the level of consumption affects the relative weight that is attached to the consumption goods. Although in this case there is no closed–form solution for utility as a function of the consumption goods, it turns out that the implied non–homothetic demand functions remain tractable. Moreover, the income effects that are implied by the non–homothetic demand functions do not disappear in the limit as consumption grows without bound, which is consistent with the available evidence [see Boppart (2016) and Comin et al. (2015)]. Having long–run income effects is potentially important in our context because growth slowdown in the far future depends on the limit behavior of the economy. A standard Stone–Geary utility specification would not capture long–run income effects because, as consumption grows without bound, it converges to a homothetic utility function.[8]

The non–homothetic CES aggregators make economic sense only if they are monotonically increasing in each of the arguments. To ensure that this is the case, we restrict the parameters as follows:

Assumption 1

- $\min\{\varepsilon_g, \varepsilon_s\} \leq 1 \leq \max\{\varepsilon_g, \varepsilon_s\}$ *and* $\sigma_c < \min\{\varepsilon_g, \varepsilon_s\}$ *or* $\max\{\varepsilon_g, \varepsilon_s\} < \sigma_c$.

[8] We should mention that a disadvantage of the functional form (2) is that it is not aggregable in the Gorman sense; if there is a distribution of households with different consumption expenditure instead of a continuum of measure one of identical households, then it is not obvious how to derive the aggregate demand for the different consumption goods from the decisions of a representative household. While that is not a crucial limitation for our application with a representative household, it is likely to be an issue in environments with a non–degenerate cross section of households.

- $\min\{\varepsilon_m, \varepsilon_n\} \leq 1 \leq \max\{\varepsilon_m, \varepsilon_n\}$ *and* $\sigma_s < \min\{\varepsilon_m, \varepsilon_n\}$ *or* $\max\{\varepsilon_m, \varepsilon_n\} < \sigma_s$.

Appendix B proves that these restrictions have the desired effect:

Lemma 1 *Assumption 1 is necessary and sufficient for*

$$\frac{\partial C_t(C_{gt}, C_{st})}{\partial C_{it}} > 0 \quad \forall i \in \{g, s\} \; \forall C_{gt}, C_{st} \geq 0$$
$$\frac{\partial C_{st}(C_{mt}, C_{nt})}{\partial C_{jt}} > 0 \quad \forall j \in \{m, n\} \; \forall C_{mt}, C_{nt} \geq 0$$

We complete the description of the environment with the resource constraints:

$$C_{it} \leq Y_{it}, \quad i = g, m, n \tag{3a}$$
$$H_t = H_{gt} + H_{st} = H_{gt} + H_{mt} + H_{nt} \leq 1 \tag{3b}$$

3.2 Equilibrium

We focus on competitive equilibrium. In the competitive equilibrium of many multi–sector models, the nominal labor productivities per hour are equalized across sectors. To capture the fact that this is not borne out by our data, we introduce a sector–specific tax, τ_{it}, that firms have to pay per unit of wage payments. The proceeds of the tax are lump–sum rebated through a transfer $T_t = \sum_{i=g,m,n} \tau_{it} w_t H_{it}$ to households. The problem of firm $i = g, m, n$ then becomes:

$$\max_{H_{it}} P_{it} A_{it} H_{it} - (1 + \tau_{it}) w_t H_{it}$$

The first–order condition implies that

$$\frac{P_{jt}}{P_{gt}} = \frac{(1 + \tau_{jt}) A_{gt}}{(1 + \tau_{gt}) A_{jt}}, \quad j = m, n \tag{4}$$

Using this and the production function, (1), we obtain that the relative sectoral labor productivities equal the relative taxes:

$$\frac{P_{jt} C_{jt} / H_{jt}}{P_{gt} C_{gt} / H_{gt}} = \frac{1 + \tau_{jt}}{1 + \tau_{gt}}, \quad j = m, n \tag{5}$$

In other words, the taxes drive a wedge between the expenditure ratio and the hours ratio. As a result, our model captures that the nominal sectoral labor productivities are different. This is important in our context, because the effects of structural transformation on aggregate productivity depend on the differences in both the growth *rates* and the *levels* of sectoral productivity ("Baumol Effect" and "Denisson Effect").

It is also crucial to capture that labor productivity growth is strongest in the goods sector and weakest in the non–market service sector. To ensure tractability, we assume for the analytical part that sectoral taxes and sectoral labor productivity growth are constant.

Assumption 2 *Taxes are constant and technological progress is constant and uneven across sectors:* $\widehat{A}_{jt} = \gamma_j$ *where* $j \in \{g, m, n\}$ *and*

$$\widehat{A}_{jt} \equiv \frac{A_{jt}}{A_{jt-1}}$$

denote growth factors and γ_j *are constants with* $1 < \gamma_n < \gamma_m < \gamma_g$.

Note that equation (4) and the assumption that taxes are constant imply that

$$\widehat{\left(\frac{P_{jt}}{P_{it}}\right)} = \frac{\gamma_i}{\gamma_j}, \quad i, j \in \{g, m, n\} \tag{6}$$

To solve the household problem, we split it in two layers. The "inner layer" is to allocate a given quantity of service consumption between the consumption of market and non–market services:

$$\min_{C_{mt}, C_{nt}} P_{mt}C_{mt} + P_{nt}C_{nt} \quad \text{s.t.} \quad \left(\alpha_m^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_m - 1}{\sigma_s}} C_{mt}^{\frac{\sigma_s - 1}{\sigma_s}} + \alpha_n^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_n - 1}{\sigma_s}} C_{nt}^{\frac{\sigma_s - 1}{\sigma_s}}\right)^{\frac{\sigma_s}{\sigma_s - 1}} \geq C_{st}$$

Appendix B shows that the first–order conditions imply that

$$\frac{P_{nt}C_{nt}}{P_{mt}C_{mt}} = \frac{\alpha_n}{\alpha_m}\left(\frac{P_{nt}}{P_{mt}}\right)^{1-\sigma_s} C_t^{\varepsilon_n - \varepsilon_m} \tag{7a}$$

$$P_{st} = \left(\alpha_m C_t^{\varepsilon_m - 1} P_{mt}^{1-\sigma_s} + \alpha_n C_t^{\varepsilon_n - 1} P_{nt}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}. \tag{7b}$$

where P_{st} is price index of aggregate services. The "outer layer" is to allocate a given quantity of total consumption between the consumption of goods and services:[9]

$$\min_{C_{gt}, C_{st}} P_{gt}C_{gt} + P_{st}C_{st} \quad \text{s.t.} \quad \left(\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g - 1}{\sigma_c}} C_{gt}^{\frac{\sigma_c - 1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s - 1}{\sigma_c}} C_{st}^{\frac{\sigma_c - 1}{\sigma_c}}\right)^{\frac{\sigma_c}{\sigma_c - 1}} \geq C_t$$

[9]The given quantity of total consumption is determined by the endowment of labor and the technology: $C_t = A_{gt}/P_t$. This is shown in the proof of Lemma 2 in Appendix B. A particular household takes A_{gt}/P_t as given because A_{gt} is exogenous and P_t is the aggregate price index that is independent of its actions.

Appendix B shows that the first–order conditions imply

$$\frac{P_{st}C_{st}}{P_{gt}C_{gt}} = \frac{\alpha_s}{\alpha_g}\left(\frac{P_{st}}{P_{gt}}\right)^{1-\sigma_c} C_t^{\varepsilon_s-\varepsilon_g} \tag{8a}$$

$$P_t = \left(\alpha_g C_t^{\varepsilon_g-1} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s-1} P_{st}^{1-\sigma_c}\right)^{\frac{1}{1-\sigma_c}} \tag{8b}$$

$$P_t C_t = \left(\alpha_g C_t^{\varepsilon_g-\sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s-\sigma_c} P_{st}^{1-\sigma_c}\right)^{\frac{1}{1-\sigma_c}} \tag{8c}$$

where P_t is the aggregate price index and $P_t C_t \equiv \sum_{i=g,m,n} P_{it} C_{it}$.

4 Unbalanced Growth Slowdown in the Model

We are now ready to study how unbalanced growth slowdown arises in our model. We will proceed in two steps. We will first identify parameter restrictions under which our model gives rise to the stylized facts of structural transformation and then study unbalanced growth slowdown in two special cases in which we can obtain analytical solutions.

4.1 Structural transformation

We begin with the structural transformation between goods and services. Dividing (8a) for periods $t+1$ and t by each other, we obtain:

$$\widehat{\left(\frac{P_{st}C_{st}}{P_{gt}C_{gt}}\right)} = \widehat{\left(\frac{P_{st}}{P_{gt}}\right)}^{1-\sigma_c} \widehat{C}_t^{\varepsilon_s-\varepsilon_g} \tag{9}$$

The first term on the right–hand side is the relative price effect and the second term is the income effect. We make the standard assumption that goods and aggregate services are complements, goods are necessities, and services are luxuries:[10]

Assumption 3 $0 < \sigma_c < 1$ *and* $\varepsilon_g < 1 < \varepsilon_s$.

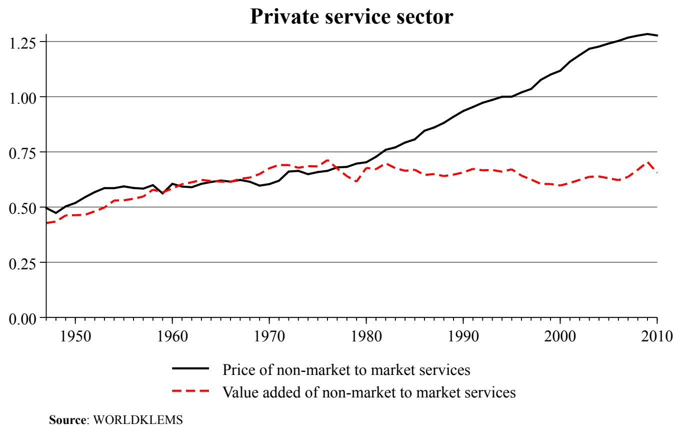
Expression (9) shows that our model then generates the observed structural transformation from goods to services if P_{st}/P_{gt} and C_t both grow. We will impose additional restrictions below that make sure that this is the case.

We continue with the structural transformation between market and non–market services. Combining Equations (5), (6) and (7a), we obtain:

$$\widehat{\left(\frac{P_{nt}C_{nt}}{P_{mt}C_{mt}}\right)} = \left(\frac{\gamma_m}{\gamma_n}\right)^{1-\sigma_s} \widehat{C}_t^{\varepsilon_n-\varepsilon_m} \tag{10}$$

[10]See Kongsamut et al. (2001), Ngai and Pissarides (2007), and Herrendorf et al. (2013) for justifications of these assumptions.

Figure 4: Relative Prices and Expenditures in Services



We assume that market and non–market services are substitutes, market services are a necessity, and non–market services are a luxury:

Assumption 4 $1 < \sigma_s$ *and* $\varepsilon_m < 1 < \varepsilon_n$.

Assumption 4 implies that the relative price effect, which is the first term on the right–hand side of Equation (10), decreases the expenditure ratio of non–market to market services because productivity growth is slower in non–market services. The income effect, which is the second term on the right–hand side, increases the expenditure ratio of non–market to market services if C_t increases. Combining these effects, our model can replicate the patterns of structural transformation within the service sector as summarized by Figure 4. Until around 1980 the price of non–market relative to market services increased along with the corresponding expenditure ratio. Our model replicates this pattern if the the income effect dominates the relative price effect before 1980. For this to happen, non–market services must be luxuries. After 1980, the increase in the price of non–market relative to market services accelerated while the expenditure ratio remained roughly constant. Our model replicate this pattern if the income effect offsets the relative price effect after 1980. For this to happen, the two service subcategories must be substitutes and the acceleration in the relative price increase after 1980 must sufficiently strengthen the price effect.

Alternative parameter constellations would not be able to generate the observed patterns. To see this, note first that the income effects do not change in 1980 but work in the same directions during the whole period. So the change in the relative expenditure share pattern must be coming from the acceleration in the relative prices. If the services were complements, then the expenditure of non–market relative to market services would have increased by more after 1980 than before 1980, which is counterfactual. Given that the two services must be substitutes, non–market services must be a luxury and market services must be a necessity. If the opposite was true and market services were luxuries and non–market services were necessities, then the expenditure of market relative to non–market services would have increased during the whole period, which again is counterfactual.

We are left with the task of ensuring that C_t and P_{st}/P_{gt} both grow, which we have assumed but not proved so far. We need to impose further restrictions on the parameters to show this. First, the elasticity of consumption expenditures with respect to real consumption is non–negative. Appendix B shows that a sufficient condition is:

Assumption 5

$$\frac{\varepsilon_g - \sigma_c}{1 - \sigma_c} < \frac{\varepsilon_n - 1}{1 - \sigma_s} + \frac{\varepsilon_s - \sigma_c}{1 - \sigma_c} \tag{11}$$

We can now ensure that the growth of aggregate consumption is positive and finite:

Lemma 2 *If Assumptions 1–5 hold, then the growth of aggregate consumption is bounded from below and above:* $1 < \underline{C} \leq \widehat{C}_t \leq \bar{C}$ *where*

$$\underline{C} \equiv \gamma_n^{\frac{(1-\sigma_c)(1-\sigma_s)}{(\varepsilon_m-1)(1-\sigma_c)+(\varepsilon_s-\sigma_c)(1-\sigma_s)}} \tag{12a}$$

$$\bar{C} \equiv \gamma_g^{\frac{1-\sigma_c}{\varepsilon_g-\sigma_c}} \tag{12b}$$

Lastly, we need to ensure that the price of aggregate services relative to goods increases. We need two additional assumptions:

Assumption 6 *The growth factors of sector labor productivity satisfy:*

$$\gamma_m < \gamma_n^{1+\frac{(1-\sigma_c)(\varepsilon_m-\varepsilon_n)}{(\varepsilon_m-1)(1-\sigma_c)+(\varepsilon_s-\sigma_c)(1-\sigma_s)}} \tag{13a}$$

$$\gamma_n < \gamma_g^{1-\frac{(1-\sigma_c)(\varepsilon_n-1)}{(\varepsilon_g-\sigma_c)(\sigma_s-1)}} \tag{13b}$$

Note that a sufficient condition for (13b) to be satisfied is that

$$\frac{\varepsilon_n - 1}{\sigma_s - 1} < \frac{\varepsilon_g - \sigma_c}{1 - \sigma_c} \tag{14}$$

Lemma 3 *Suppose that Assumptions 1–6 hold. Then the price of services relative to goods increases over time,* $\widehat{P}_{st} > 1$.

We conclude this subsection by summarising the patterns of structural transformation in our model. We start with goods and services:

Proposition 1 *If Assumptions 1–6 hold, then along the equilibrium path the expenditure and employment shares of the goods sector are monotonically decreasing and converge to zero as time goes to infinity.*

In other words, our model generates the standard pattern of the structural transformation between goods and services that is familiar from many two–sector models of structural transformation: the goods sector shrinks and eventually the service sector takes over the model economy.

Regarding the structural transformation within the service sector, Lemma 2 and Equation (10) immediately imply:

Proposition 2 *If Assumptions 1–6 hold, then along the equilibrium path*

$$\frac{\gamma_m}{\gamma_n} < \underline{C}^{\frac{\varepsilon_n-\varepsilon_m}{\sigma_s-1}} \implies \widehat{\left(\frac{P_{nt}C_{nt}}{P_{mt}C_{mt}}\right)} > 1 \tag{15a}$$

$$\frac{\gamma_m}{\gamma_n} > \bar{C}^{\frac{\varepsilon_n-\varepsilon_m}{\sigma_s-1}} \implies \widehat{\left(\frac{P_{nt}C_{nt}}{P_{mt}C_{mt}}\right)} < 1 \tag{15b}$$

In other words, if the productivity growth in the two service sectors is "sufficiently close", then the income effect will dominate and non–market services take over in the limit. In contrast, if productivity in the two service sectors is "sufficiently different", then the price effect will dominate and market services will take over in the limit. The fact that both cases are possible in the limit suggest that there is a third knife–edge case in which the two forces exactly offset each other and the share of market services remains constant. Which of the three cases prevails in the long run for plausible parameter values is a quantitative question which we will answer below by simulating a calibrated version of our model.

4.2 Growth slowdown

After having established that our model can qualitatively generate the patterns of structural transformation that we observe in the data, we now turn to the question whether it generates growth slowdown along the equilibrium path. We start by exploring what we can say about balanced growth of welfare in our model. Given that we have a continuum of measure one of identical households, the natural welfare measure is C_t. Appendix B shows that:

Proposition 3 *If Assumptions 1–6 hold, then the growth factor of welfare, $\widehat{C_t}$, decreases along the equilibrium path.*

In other words, our model generates an unbalanced growth slowdown in terms of welfare. This should not come as a surprise given that the interaction of preferences and uneven technological progress reallocates labor from the faster growing goods sector to the slower growing service sectors. In practice, researchers use GDP per capita as a proxy for welfare because GDP per capita is so widely available over time and across countries. Note that the way in which we have set up our model implies that GDP per capita equals labor productivity, so we will use the two concepts interchangeably in what follows.

We now answer the obvious question whether standard measures of GDP per capita capture the unbalanced growth slowdown in welfare. The answer is more nuanced than one might initially think. We will show that it depends critically on which measure of GDP per capita is used and on what the relative sizes of the taxes are. We will derive analytical results in two special cases, which serve as useful benchmarks to sharpen our intuition: the growth factors are constant but uneven across sectors and taxes are zero ("uneven technological progress and zero taxes");[11] the growth factors are the same in all sectors and taxes are constant but uneven across sectors ("even technological progress and unequal taxes"). We will deal with the general case ("uneven technological progress and non–zero taxes") in the subsequent quantitative section.

4.2.1 Uneven technological progress and zero taxes

If taxes are zero, it turns out that whether or not our model leads to unbalanced growth slowdown in terms of GDP per capita depends on the way in which GDP capita is calculated. There are two possibilities. The literature on multi–sector models chooses the units of a current–period numeraire $j \in \{g, m, n\}$:

$$\widehat{Y}_t^{nj} \equiv \frac{\sum_{i=g,m,n} \frac{P_{it}}{P_{jt}} C_{it}}{\sum_{i=g,m,n} \frac{P_{it-1}}{P_{jt-1}} C_{it-1}}$$

This means that the numeraire differs between two adjacent periods: in period t it is C_{jt} whereas in $t-1$ it is C_{jt-1}. In contrast, NIPA statisticians calculate GDP per capita by using chain indexes, that is, the geometrically weighted average of the Laspeyres and the Paasche quantity indexes.

$$\widehat{Y}_t^{ch} = \sqrt{\frac{\sum_{i=g,m,n} P_{it-1} C_{it}}{\sum_{i=g,m,n} P_{it-1} C_{it-1}} \cdot \frac{\sum_{i=g,m,n} P_{it} C_{it}}{\sum_{i=g,m,n} P_{it} C_{it-1}}}$$

Note that it does not matter whether one calculates GDP from a chained quantity index like the one above or from nominal GDP and a chained price index. This is shown in the next lemma, which is proven again in Appendix B.

Lemma 4 *Chained quantity and price indexes satisfy:*

$$\widehat{Y}_t^{ch} \times \widehat{P}_t^{ch} = \widehat{P_t Y_t}$$

[11] Note that technically it is enough if taxes are the same in all sectors. But there is no important difference in our context between all taxes being equal to zero and all taxes being the same.

We start with characterizing the growth of GDP per capita when we use a current–period numeraire. Appendix B shows that:

Proposition 4 *Suppose that $\tau_{it} = \tau_t$ for $i = g, m, n$ and that we use a current–period numeraire $j = g, m, n$ to calculate real GDP. If Assumptions 1–2 hold, then the growth factor of real GDP per capita is constant: $\widehat{Y}_t^{nj} = \gamma_j$.*

The fact that the growth of real GDP per capita in units of a current–period numeraire is constant greatly simplifies the characterization of the equilibrium path, because it can be calculated as a balanced growth path. This is particularly helpful in model versions with capital. The disadvantage is that the growth factor of real GDP per capita depends on the choice of the numeraire, which is sometimes called the Gerschenkron Effect. In addition, constant growth of real GDP per capita is in sharp contrast to the unbalanced growth slowdown of welfare.

Chain indexes avoid the Gerschenkron Effect and have the additional advantage that the growth rate of real GDP per capita is independent of the base year. In addition, it turns out that they can capture the unbalanced growth slowdown in terms of welfare:[12]

Proposition 5 *Suppose that $\tau_{it} = \tau_t$ for $i = g, m, n$ and that we use the chain index to calculate real GDP. then the equilibrium growth of real GDP per capita*

- *changes with the sectoral composition of the economy ("unbalanced growth");*
- *slows over time if $\widehat{H}_{nt} \geq 0$.*

Proposition 4 provides a sufficient condition under which our model generates growth slowdown in terms of GDP per capita: the forces of structural transformation have to play out in such a way that labor is reallocated to the non–market service sector, $\widehat{H}_n \geq 0$. To develop intuition for this condition we proceed in two steps. First, fix the allocation of aggregate services between market and non–market services. The reallocation from goods to aggregate service then slows down aggregate productivity growth, because productivity growth is larger in the goods sector than in the service subsector. This is an example of what Nordhaus (2002,2008) called the "Baumol Effect". Consider now the additional reallocation within aggregate services. Since market services have higher productivity growth than non–market services, reallocation towards market services increases the productivity growth of aggregate services. For growth slowdown to happen, this effect must not be too strong. A sufficient condition for this is that this reallocation is absent, which is the case if the hours share of non–market services does not decline.

In sum, whether or not our model exhibits unbalanced growth slowdown in terms of GDP per capita depends critically on which of the two methods we use to calculate GDP. This is not

[12] The proof again is in Appendix B.

at all appreciated in the literature on multi–sector models, which tends to connect growth rates from the model economy which are calculated with current–period numeraires to growth rates from the data which are calculated with chain indexes. Since the growth properties of GDP are dramatically different under the two methods, proceeding in this way can be very misleading.[13]

4.2.2 Even technological progress and unequal taxes

We now turn to the second case in which we can derive analytical results: the growth factors are the same in all sectors but the sectoral taxes differ. This case captures the feature of reality that the levels of sectoral labor productivity differ. Using equation (4), it is then straightforward to show that

Proposition 6 *If Assumptions 1–6 hold and that $\gamma \equiv \gamma_i$, $i = g, m, n$. The growth factor of real GDP per capita is not constant in general:*

$$\widehat{Y}_t^{nj} = \widehat{Y}_t^{ch} = \gamma \frac{\sum_{i=g,m,n}(1+\tau_i)H_{it}}{\sum_{i=g,m,n}(1+\tau_i)H_{it-1}} \tag{16}$$

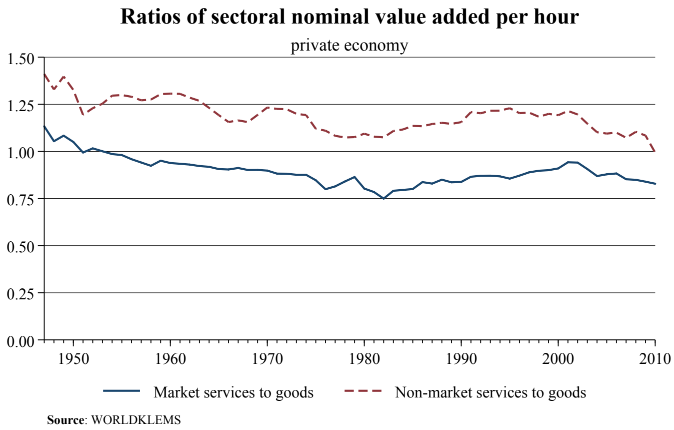
The proposition shows that even if labor productivity growth is the same in all sectors, the growth factor of real GDP per capita will in general not be equal to that rate. This comes about because the tax differences introduce a wedge between the nominal labor productivities of the different sectors; compare (5). Structural transformation then leads to an acceleration (slow down) of the growth of GDP per capita if labor is reallocated towards the sectors with higher (lower) levels of labor productivity. Figure 5 shows that in the data

$$\frac{P_{nt}C_{nt}}{H_{nt}} \geq \frac{P_{gt}C_{gt}}{H_{gt}}$$
$$\frac{P_{mt}C_{mt}}{H_{mt}} \leq \frac{P_{gt}C_{gt}}{H_{gt}}$$

Normalizing $\tau_g = 0$, this implies that $\tau_n > 0$ and $\tau_m < 0$ and that the reallocation to non–market services increase GDP growth whereas the reallocation to market services decreases the growth of real GDP per capita. Nordhaus (2002,2008) called this the "Denison Effect".

[13] Critical readers might have noticed that in our empirical analysis we use aggregate growth rates from the WORLD KLEMS dataset, which are based on the Törnqvist index, whereas in our theoretical analysis we consider the aggregate growth rates calculated by the chain–weighted index (a Fischer index). Although both indexes are conceptually different, they are equal to a second–order approximation. In applied work, they are therefore typically used interchangeably.

Figure 5: Relative Nominal Productivities



5 Calibration and Predictions

In this section, we calibrate our model to match key features of the postwar U.S. economy, such as the sectoral growth rates and the reallocation across sectors. We then use the calibrated model to study how big unbalanced growth slowdown will be in the future. Our model implies that it will not be larger than in the past.

5.1 Calibration

We start with the calibration of the taxes and sectoral TFPs. Normalizing the taxes in the goods sector to zero, $\tau_g = 0$, and defining nominal value added as $VA_{jt} \equiv P_{jt}C_{jt} = P_{jt}A_{jt}H_{jt}$, (4)–(5) give

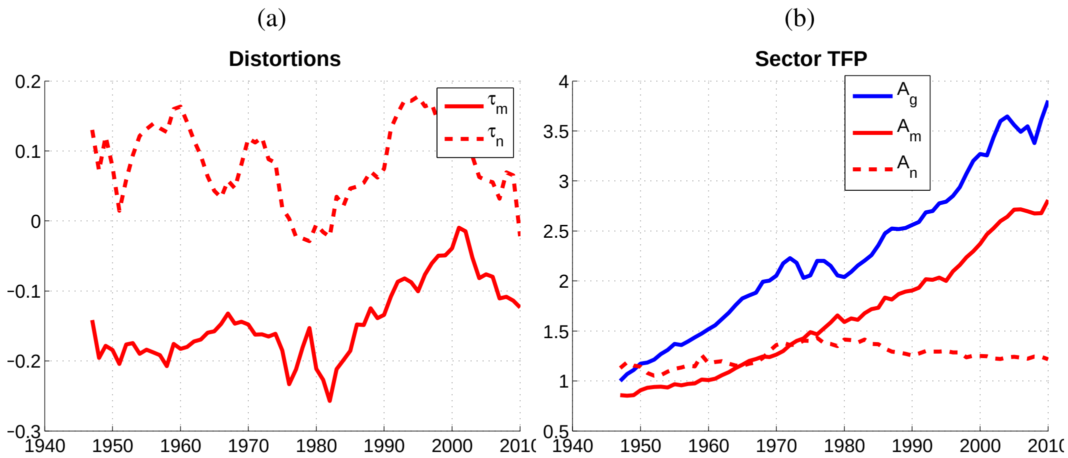
$$1 + \tau_{jt} = \frac{VA_{jt}/H_{jt}}{VA_{gt}/H_{gt}}, \qquad j = m, n$$

Measuring $\{(VA_{jt}/H_{jt})/(VA_{gt}/H_{gt})\}$ for $j = m, n$ and $t = 1947, ..., 2010$ from the data, this relationship gives us the values for $\{\tau_{jt}\}$. Next, we set $P_{g1947} = P_{m1947} = P_{n1947} = A_{g1947} = 1$ and back out the implied A_{j1947}. We then take $\{P_{it}\}$ for $i = g, m, n$ and $t = 1947, ..., 2010$ from the data and calibrate $\{A_{jt}\}$ so as to match the observed changes in real labor productivity, $\{[VA_{jt}/(P_{jt}H_{jt})]/[VA_{gt}/(P_{gt}H_{gt})]\}$, in the data. Figure 6 shows the calibrated taxes and TFPs.

We jointly calibrate the remaining ten parameters $(\alpha_g, \alpha_s, \alpha_m, \alpha_n, \sigma_s, \sigma_c, \varepsilon_g, \varepsilon_s, \varepsilon_m, \varepsilon_n)$. The first targets are the nominal value added of non–market services relative to goods and relative to market services:

$$\left\{\frac{P_{nt}C_{nt}}{P_{jt}C_{jt}}\right\}_{t=1947,...,2010}, \qquad j = g, m \tag{17}$$

Figure 6: Taxes and Sector TFPs



These targets allow us to identify all parameters except for the ε_i. To be precise, we can identify $\varepsilon_g - \varepsilon_s$ and $\varepsilon_m - \varepsilon_n$, but not all four ε_i. Different from **?**, we need these parameter values because we are solving for the equilibrium path of the whole model, instead of for the implied demand system that takes prices and income as given.

To obtain additional targets, we use the fact that the values of ε_i affect how changes in C_t translate into changes in $(P_{st}, C_{st}, P_t, C_t)$. We stress that, in general, it is not appropriate to compare the model–implied $(P_{st}, C_{st}, P_t, C_t)$ directly with the corresponding statistics from the data, because the model aggregates use non–homothetic CES aggregators whereas WORLD KLEMS aggregates use Thörnquivst indexes. Although locally the two are equal to a second–order approximation, over time they may grow far apart. Hence, the model statistics and the data statistics with the same names are really different objects, and it does not make conceptual sense to require them to be close. To find a calibration strategy that makes conceptual sense, we apply the model's non–homothetic CES aggregator to raw quantities from the model and from the data and compare the resulting aggregates. In particular, we first calculate the $\{\tilde{P}_{st}, \tilde{C}_{st}, \tilde{P}_t, \tilde{C}_t\}$ that are implied by the data values of $\{C_{gt}(D), C_{mt}(D), C_{nt}(D)$ (where D indicates data) and the non–homothetic CES aggregators from the model given the model parameters; we then minimize the difference between the $\{\tilde{P}_{st}, \tilde{C}_{st}, \tilde{P}_t, \tilde{C}_t\}$ and the $\{P_{st}, C_{st}, P_t, C_t\}$ that are generated by the model quantities $\{C_{gt}, C_{mt}, C_{nt}\}$.

To implement this, let us start with the definitions of the non–homothetic CES aggregators:

$$\begin{aligned} C_t &- \left(\alpha_g^{\frac{1}{\sigma_c}} C_{gt}^{\frac{\sigma_c-1}{\sigma_c}} C_t^{\frac{\varepsilon_g-1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_{st}^{\frac{\sigma_c-1}{\sigma_c}} C_t^{\frac{\varepsilon_s-1}{\sigma_c}}\right)^{\frac{\sigma_c}{\sigma_c-1}} &= 0 \\ C_{st} &- \left(\alpha_m^{\frac{1}{\sigma_s}} C_{mt}^{\frac{\sigma_s-1}{\sigma_s}} C_t^{\frac{\varepsilon_m-1}{\sigma_s}} + \alpha_n^{\frac{1}{\sigma_s}} C_{nt}^{\frac{\sigma_s-1}{\sigma_s}} C_t^{\frac{\varepsilon_n-1}{\sigma_s}}\right)^{\frac{\sigma_s}{\sigma_s-1}} &= 0 \end{aligned} \tag{18}$$

The first step is to substitute in the consumption quantities from the data so as to obtain the consumption aggregates that are implied by the data given the functional forms and the parameters of the model:

$$\begin{aligned}
\tilde{C}_{st} &- \left[\alpha_m^{\frac{1}{\sigma_s}}\left(\frac{P_m(D)C_m(D)}{P_m(D)}\right)^{\frac{\sigma_s-1}{\sigma_s}}\tilde{C}_t^{\frac{\varepsilon_m-1}{\sigma_s}} + \alpha_n^{\frac{1}{\sigma_s}}\left(\frac{P_n(D)C_n(D)}{P_n(D)}\right)^{\frac{\sigma_s-1}{\sigma_s}}\tilde{C}_t^{\frac{\varepsilon_n-1}{\sigma_s}}\right]^{\frac{\sigma_s}{\sigma_s-1}} &= 0\\
\tilde{C}_t &- \left[\alpha_g^{\frac{1}{\sigma_c}}\left(\frac{P_g(D)C_g(D)}{P_g(D)}\right)^{\frac{\sigma_c-1}{\sigma_c}}\tilde{C}_t^{\frac{\varepsilon_g-1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}}\tilde{C}_{st}^{\frac{\sigma_c-1}{\sigma_c}}\tilde{C}_t^{\frac{\varepsilon_s-1}{\sigma_c}}\right]^{\frac{\sigma_c}{\sigma_c-1}} &= 0
\end{aligned}$$

The next step is to solve this system of equations for $\tilde{C}_{st}$ and $\tilde{C}_t$. Then, we use definition of the price indexes to solve for the price indexes that that are implied by the data given the functional forms and the parameters of the model:

$$\begin{aligned}
\tilde{P}_{st} &- \left[\alpha_n P_{nt}(D)^{1-\sigma_s}\tilde{C}_t^{\varepsilon_n-1} + \alpha_m P_{mt}(D)^{1-\sigma_s}\tilde{C}_t^{\varepsilon_m-1}\right]^{\frac{1}{1-\sigma_s}} &= 0\\
\tilde{P}_t &- \left[\alpha_g P_{gt}(D)^{1-\sigma_c}\tilde{C}_t^{\varepsilon_g-1} + \alpha_s \tilde{P}_{st}^{1-\sigma_c}\tilde{C}_t^{\varepsilon_s-1}\right]^{\frac{1}{1-\sigma_c}} &= 0
\end{aligned} \tag{19}$$

We can now run an OLS regression of $\tilde{P}_s$, $\tilde{C}_s$, or $\tilde{P}$ on $\tilde{C}$ and a constant. The slope coefficient contains the information on ε_i that we are interested in, and so we can use it as a target in the calibration. This way of calibrating is called indirect inference: we use an auxiliary model (the equation that we estimate with OLS) and require the calibrated model to match an auxiliary parameter (here the slope of the estimated equation). Endogeneity is not a problem because we estimate the same equation on actual data and on simulated model data, so possible endogeneity appears in both equations in the same fashion. We tried different combinations of targets and OLS coefficients and many of them worked, i.e., we got a fixed point and all parameters were separately identified. The calibrated parameter values were similar across the different cases that work. Our preferred strategy is target the expenditure shares from (17), $\tilde{C}_{st}/\tilde{C}_t$, $\tilde{P}_{st}/\tilde{P}_t$, and the OLS coefficient in the linear regression of $\tilde{P}_{st}$ on a constant and on $\tilde{C}_t$.

The calibration results are in Table 5. We find the expected parameter constellation for goods and services: they are complements ($\sigma_c < 1$); goods are necessities ($\varepsilon_g < 1$); services are luxuries ($\varepsilon_s > 1$). We find the expected parameter constellation for market and non–market services: they are substitutes ($\sigma_s < 1$); market services are necessities ($\varepsilon_m < 1$); non–market services are luxuries ($\varepsilon_n > 1$). The parameter values satisfy Assumption 1, except for the fact that $\sigma_s < \varepsilon_n$. This does not create a problem though because Assumption 1 was necessary and sufficient for the utility aggregator C_{st} to increase in its arguments for *all possible values* of $C_{jt} \geq 0$ ($j = m, n$). Although that is not the case for our calibrated parameters, we have verified that the utility aggregator C_{st} increases in its arguments for all *equilibrium values* of C_{jt} ($j = m, n$) that the model economy actually assumes.[14]

Figures 7, 8, and 9 show that the calibrated model matches well the targeted moments and several non–targeted moments. The calibrated model also matches that the effect of unbalanced

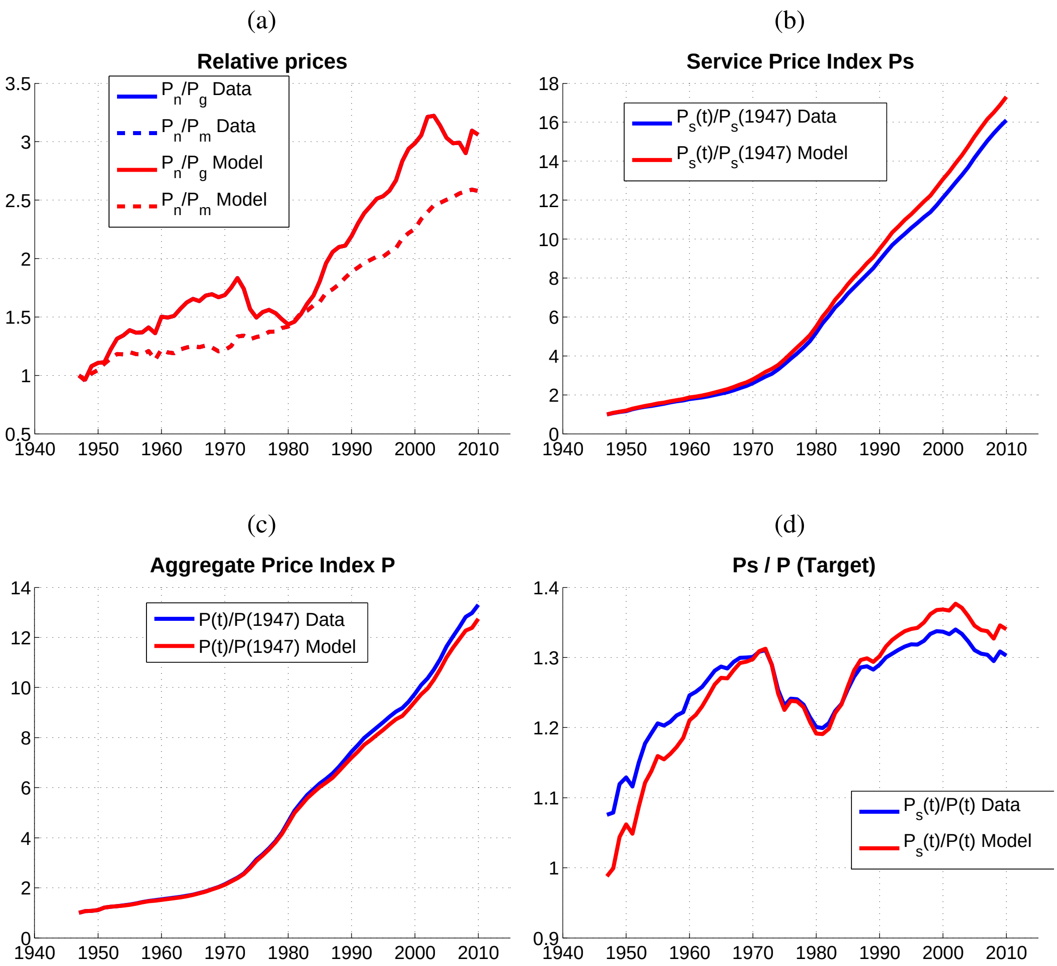
[14]This amounts to verifying that the denominator of (27b) is negative along the actual equilibrium path.

Table 5: Calibrated Parameters

α_g	α_s	α_m	α_n	σ_s	σ_c	ε_g	ε_s	ε_m	ε_n
0.54	0.46	0.66	0.34	1.28	0.17	0.57	1.45	0.42	1.39

growth slowdown among our three sectors is 0.2 percentage points per year as it is in the data.

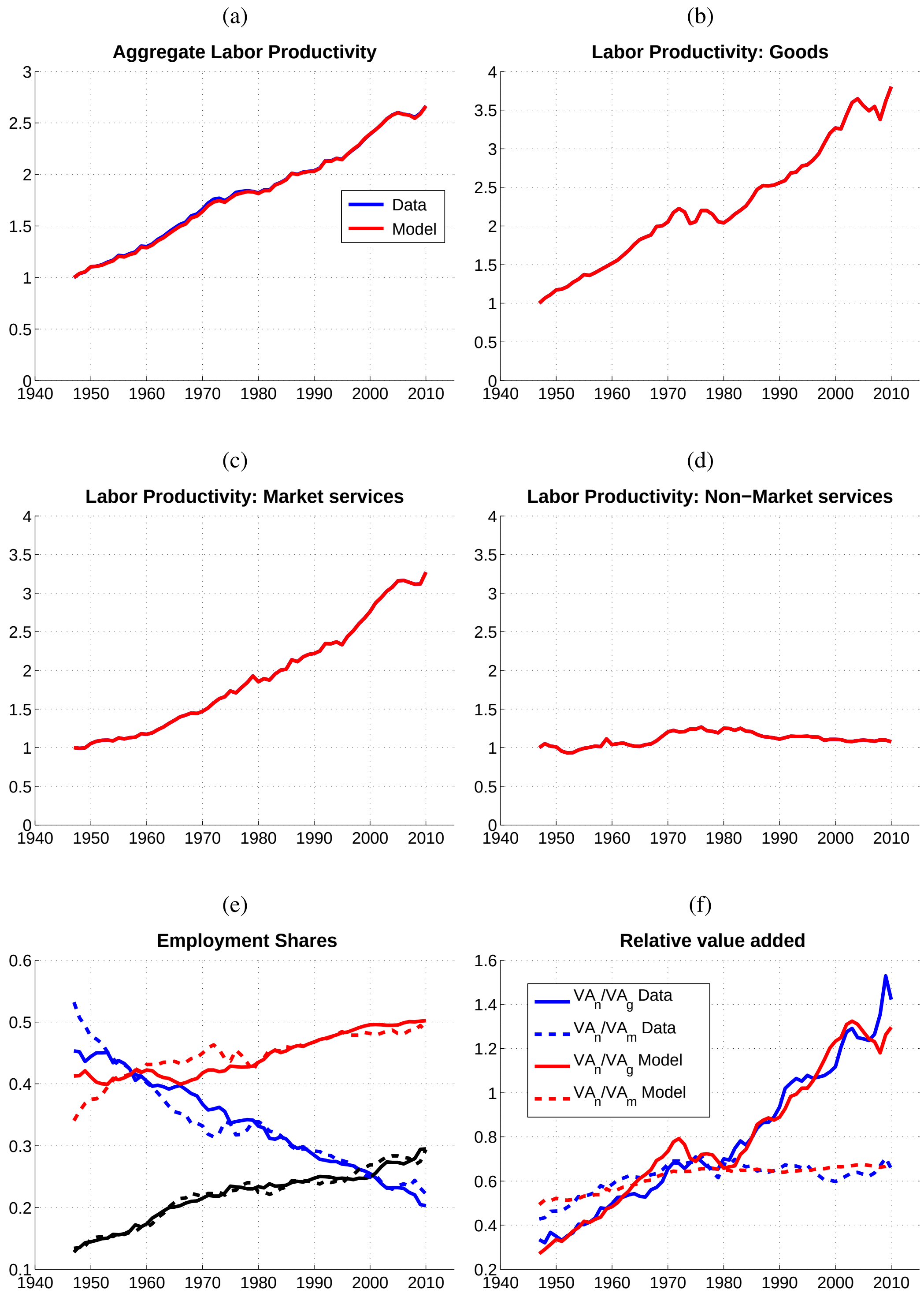
Figure 7: Prices – Model and Data



5.2 Model simulation

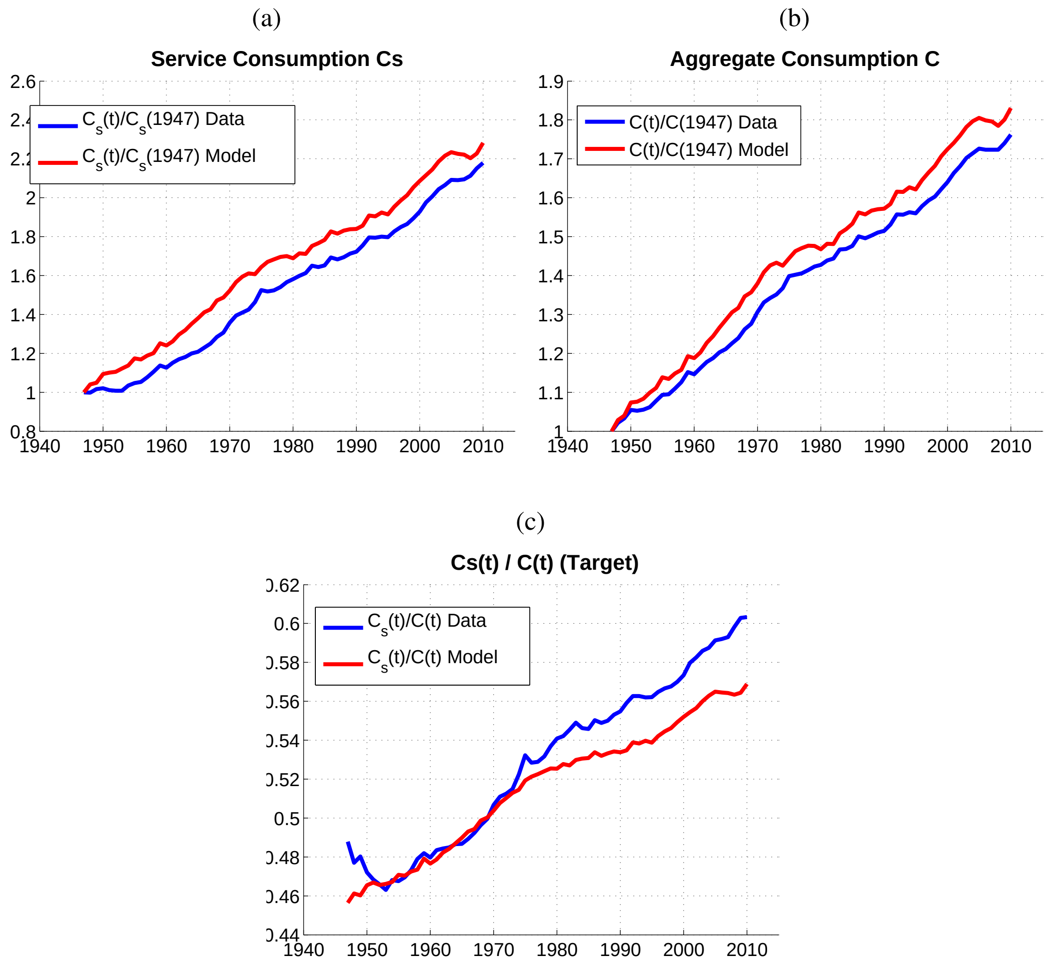
To produce the out–of–sample implications of our model, we simulate it forward, starting in 2010. We assume that between 2010–2010+T, the variables $\{A_{gt}, A_{mt}, A_{nt}, P_{gt})$ grow at the same constant rates on average as they did "in the past". The key issue to settle is what we mean by "in the past". The evidence presented in Section 2 suggests that there was a structural

Figure 8: Productivity, Employment and Value Added – Model and Data



break in the productivity patterns changed at some time around 1980. We therefore do not go further back then 1980. We calculate average growth rates for $\{A_{gt}, A_{mt}, A_{nt}, P_{gt})$ during three periods: 1980–2010, 1990–2010, and 1980–2005. The first period uses information from the

Figure 9: Consumption – Model and Data



entire period. The second period starts somewhat later in 1990 in case the structural break happened after 1980. The third period stops in 2005 to avoid having data from the Great Recession, which is an unusual episode. Table 6 shows the data inputs and the results. The top rows show the average annual growth rate of $\{A_{gt}, A_{mt}, A_{nt}, P_{gt})$ and the average tax rates for different calibration periods. The bottom rows show the implied average annual growth rates of aggregate labor productivity. The model implies that the future unbalanced growth slowdown in productivity growth is most 0.22 percentage points, which is close to the number we calculated for the postwar period.

It is important to realize that the model is essential for making these predictions. If instead we had just run a simple regression on past data and extrapolated the result into the future, we would have gotten different results. For example, during the period from 1990–2010 the linear fit of aggregate labor productivity gives a slope coefficient of -0.021. This implies that a simple extrapolation by 20 years would predict a slowdown from 1.30% to $1.30 - 0.022 \cdot 20 = 0.86\%$

Table 6: Future Unbalanced Growth Slowdown

calibration period	data averages over calibration period					
	$\Delta A_g/A_g$	$\Delta A_m/A_m$	$\Delta A_n/A_n$	$\Delta P_g/P_g$	τ_m	τ_n
1980–2010	1.98	1.71	-0.33	2.32	-0.11	0.09
1990–2010	1.94	1.88	-0.21	1.78	-0.08	0.11
1980–2005	2.11	1.90	-0.32	2.28	-0.11	0.10

	aggregate productivity growth (in %)			
	1990–2010	2010–2030	2030–2050	2050–2150
1980–2010	1.21	1.10	1.05	1.00
1990–2010	1.30	1.22	1.17	1.11
1980–2005	1.34	1.22	1.17	1.12

which is quite far from the 1.22% that the model predicts. This means that the non–linear dynamics that results from the model is not well captured by a simple regression. Hence, the model is needed for making out–of-sample forecasts.

We end this section with providing some intuition for why our model predicts that there will be at most as much future productivity slowdown than there was past productivity slowdown. A first reason is that while the value added and the hours shares of services were of similar size as those of goods in 1947, in 2010 they were almost four times those of goods. Hence, between 1947 and 2010 there was a lot of reallocation from goods with the fastest productivity growth to services with slower productivity growth, which led to productivity slowdown. Given that the goods sector is rather small in 2010, that source of unbalanced growth slow down is bound to be of much less importance in the future. Instead, the center stage is now taken by changes in the composition of the service sector, which lead to the reallocation between market services, which have fast productivity growth, and non–market services, which are stagnating. Since the data suggest that market and non–market services are substitutes, the model predicts that non–market services are not taking over the economy in the limit, which bounds the extent of future productivity slowdown.

Our conclusion differs sharply from that of standard models of structural transformation. These models feature just one elasticity of substitution among the value added of all sectors, which is typically set such that the different sectoral value added are complements. They imply that the sector with the slowest productivity growth takes over in the limit, which are non–market services in our three–sector split. Since non–market services have had no productivity growth in the recent past (and even shrank somewhat), doing our extrapolation exercise with a standard model of structural transformation would predict that in the limit productivity growth falls all the way to zero and then even shrinks somewhat.

6 Conclusion

We have have demonstrated that structural transformation considerably slowed down productivity growth in the post World War II period. We have build a model that accounts for this. Our model implies that future structural transformation will not be more of a drag on productivity growth than it has been in the past. To reach this conclusion it has been crucial that we have disaggregated services into the subcategories market and non–market services. The data suggest that they had very different productivity performances: market services have had steady productivity growth in the post–war U.S. whereas non–market service have had stagnating productivity growth since the 1980s. The data also suggest that the two subcategories of services are substitutes. This implies that the stagnating non–market services will not take over the economy in the limit, which is in sharp contrast to what existing models of structural transformation imply.

In this paper, we have taken the sectoral growth rates as given and we have explored which consequences changes in the sectoral composition have for unbalanced growth slowdown. A first interesting question for future work is why different sectors show different productivity growth. A second interesting question for future work is to study whether the slow growing sectors will continue to grow slowly even when they comprise sizeable shares of the economy. We plan to tackle these questions next.

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Appendix A: Data Work

A Calculations behind Table ??

A.1 Preliminary remarks

Conceptually, the first two columns of the table are the actual and the counterfactual accumulated growth factors where the counterfactual accumulated growth factor are calculated with the counterfactual sector labor shares from 1947:

$$\left[\frac{Y_{2010}}{H_{2010}}\right] \Big/ \left[\frac{Y_{1947}}{H_{1947}}\right] \quad \text{and} \quad \left[\sum_{i=g,m,n} \frac{H_{i,1947}}{H_{1947}} \frac{Y_{i,2010}}{H_{i,2010}}\right] \Big/ \left[\frac{Y_{1947}}{H_{1947}}\right]$$

As in the body of the paper, Y and H denote real value added and hours worked. The third and the fourth column are the average annual growth rates that are calculated with sector labor shares from 2010 and 1947:

$$\left\{\left[\sum_{i=g,m,n}\frac{H_{i,2010}}{H_{2010}}\frac{Y_{i,2010}}{H_{i,2010}}\right]\Big/\left[\sum_{i=g,m,n}\frac{H_{i,2010}}{H_{2010}}\frac{Y_{i,1947}}{H_{i,1947}}\right]\right\}^{1/63} = \left\{\left[\frac{Y_{2010}}{H_{2010}}\right]\Big/\left[\sum_{i=g,m,n}\frac{H_{i,2010}}{H_{2010}}\frac{Y_{i,1947}}{H_{i,1947}}\right]\right\}^{1/63}$$

and

$$\left\{\left[\sum_{i=g,m,n}\frac{H_{i,1947}}{H_{1947}}\frac{Y_{i,2010}}{H_{i,2010}}\right]\Big/\left[\sum_{i=g,m,n}\frac{H_{i,1947}}{H_{1947}}\frac{Y_{i,1947}}{H_{i,1947}}\right]\right\}^{1/63} = \left\{\left[\sum_{i=g,m,n}\frac{H_{i,1947}}{H_{1947}}\frac{Y_{i,2010}}{H_{i,2010}}\right]\Big/\left[\frac{Y_{1947}}{H_{1947}}\right]\right\}^{1/63}$$

In practise, these statistics are unfortunately not as straightforward to calculate as the above formulas suggest. The complications arise from the fact that WORLD KLEMS calculates quantity and price indices according to a Törnqvist aggregation procedure, which implies that the quantity indices are not additive. Below we describe how average annual growth rates and accumulated growth factors can be calculated without imposing additivity.

A.2 Aggregate growth rates as sector aggregates

The growth rate of a variable in period t is defined by the log difference between periods t and $t-1$. The aggregate growth rates of real value added, Y_t, and efficiency hours, $\widetilde{H}_t$, are defined as the weighted averages of the corresponding sectoral growth rates:

$$\Delta\ln(Y_t) \equiv \sum_{i=g,m,n} S(Y_{it})\Delta\ln(Y_{it}) \tag{20a}$$

$$\Delta\ln(\widetilde{H}_t) \equiv \sum_{i=g,m,n} S(\widetilde{H}_{it})\Delta\ln(\widetilde{H}_{it}), \tag{20b}$$

where $S(Y_{it})$ and $S(\widetilde{H}_{it})$ denote the averages over periods t and $t-1$ of the nominal shares of sector i's value added and compensation of labor in the corresponding totals. We use averages here because the shares are used as weights of growth rates from on period to the next.

A.3 Aggregate productivity measures as sector aggregates

We first calculate the growth rate of aggregate value added per hour:

$$\begin{aligned}\Delta\ln\left(LP(H_t)\right) &\equiv \Delta\ln(Y_t) - \Delta\ln(H_t)\\ &= \sum_{i=g,m,n} S(Y_{it})\left[\Delta\ln(Y_{it}) - \Delta\ln(H_{it})\right] + \sum_{i=g,m,n} S(Y_{it})\Delta\ln(H_{it}) - \Delta\ln(H_t).\end{aligned}$$

Since,

$$\Delta\ln(H_t) = \ln\left(\frac{H_t}{H_{t-1}}\right) = \ln\left(\frac{\sum_{i=g,m,n}(H_{it} - H_{it-1} + H_{it-1})}{H_{t-1}}\right)$$
$$= \ln\left(1 + \sum_{i=g,m,n} \frac{H_{it-1}}{H_{t-1}} \frac{H_{it} - H_{it-1}}{H_{it-1}}\right) \approx \sum_{i=g,m,n} \frac{H_{it-1}}{H_{t-1}} \Delta\ln(H_{it}).$$

the growth rate of aggregate value added per hour is approximately equal to

$$\Delta\ln\left(LP(H_t)\right) = \sum_{i=g,m,n} S(Y_{it})\Delta\ln\left(LP(H_{it})\right) + \sum_{i=g,m,n} \left[S(Y_{it}) - \frac{H_{it-1}}{H_{t-1}}\right]\Delta\ln(H_{it}) \tag{21}$$

We now turn to the calculation of the growth rates of aggregate value added per efficiency hour. Applying the definitions (20), we obtain:

$$\Delta\ln\left(LP(\widetilde{H}_t)\right) \equiv \Delta\ln(Y_t) - \Delta\ln(\widetilde{H}_t) = \sum_{i=g,m,n} S(Y_{it})\Delta\ln(Y_{it}) - \sum_{i=g,m,n} S(\widetilde{H}_{it})\Delta\ln(\widetilde{H}_{it})$$
$$= \sum_{i=g,m,n} S(Y_{it})\Delta\ln\left(LP(\widetilde{H}_{it})\right) + \sum_{i=g,m,n} \left[S(Y_{it}) - S(\widetilde{H}_{it})\right]\Delta\ln(\widetilde{H}_{it}). \tag{22}$$

Thus, the growth rate of aggregate value added per efficiency hour is the sum of the weighted average of the growth rates of sector value added per efficiency hour and a correction term, which captures the role of the difference between the sectoral–value–added shares and the sectoral–labor–compensation shares, $[S(Y_{it}) - S(\widetilde{H}_{it})]$.

A.4 Counterfactual experiments

To assess the effect of structural transformation on aggregate productivity growth, we define counterfactual labor productivity measures with fixed period–T sector weights using expressions (21) and (22):

$$\Delta\ln\left(LP(H_t, T)\right) \approx \sum_{i=g,m,n} S(Y_{iT})\Delta\ln\left(LP(H_{it})\right) + \sum_{i=g,m,n} \left[S(Y_{iT}) - \frac{H_{iT-1}}{H_{T-1}}\right]\Delta\ln(H_{it}), \tag{23a}$$

$$\Delta\ln\left(LP(\widetilde{H}_t, T)\right) = \sum_{i=g,m,n} S(Y_{iT})\Delta\ln\left(LP(\widetilde{H}_{it})\right) + \sum_{i=g,m,n} \left[S(Y_T) - S(\widetilde{H}_T)\right]\Delta\ln(\widetilde{H}_{it}). \tag{23b}$$

The first two columns of Tables **??** compare the actual with the counterfactual accumulated growth factors of GDP between 1948 and 2010 where the counterfactual accumulated growth factor is calculated with the counterfactual sectoral weights from 1947/48:[15]

$$\exp\left(\sum_{t=1948}^{2010} \Delta \ln\Big(LP(H_t)\Big)\right) \quad \text{vs} \quad \exp\left(\sum_{t=1948}^{2010} \Delta \ln\Big(LP(H_t, 1948)\Big)\right) \tag{24a}$$

$$\exp\left(\sum_{t=1948}^{2010} \Delta \ln\Big(LP(\widetilde{H}_t)\Big)\right) \quad \text{vs} \quad \exp\left(\sum_{t=1948}^{2010} \Delta \ln\Big(LP(\widetilde{H}_t, 1948)\Big)\right). \tag{24b}$$

The last two columns of Tables **??** compare the average growth rates calculated with the sector weights from 2010 and 1948:[16]

$$\frac{1}{63}\sum_{t=1948}^{2010} \Delta \ln\Big(LP(H_t, 2010)\Big) \quad \text{vs} \quad \frac{1}{63}\sum_{t=1948}^{2010} \Delta \ln\Big(LP(H_t, 1948)\Big) \tag{25a}$$

$$\frac{1}{63}\sum_{t=1948}^{2010} \Delta \ln\Big(LP(\widetilde{H}_t, 2010)\Big) \quad \text{vs} \quad \frac{1}{63}\sum_{t=1948}^{2010} \Delta \ln\Big(LP(\widetilde{H}_t, 1948)\Big) \tag{25b}$$

Appendix B: Derivations and Proofs

Proof of Lemma 1

Rewrite (2a) as

$$1 = \alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g-\sigma_c}{\sigma_c}} C_{gt}^{\frac{\sigma_c-1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s-\sigma_c}{\sigma_c}} C_{st}^{\frac{\sigma_c-1}{\sigma_c}}$$

Differentiating with respect to C_{gt} gives:

$$0 = \alpha_g^{\frac{1}{\sigma_c}} \frac{\varepsilon_g - \sigma_c}{\sigma_c} C_t^{\frac{\varepsilon_g-2\sigma_c}{\sigma_c}} \frac{\partial C_t}{\partial C_{gt}} C_{gt}^{\frac{\sigma_c-1}{\sigma_c}} + \alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g-\sigma_c}{\sigma_c}} \frac{\sigma_c - 1}{\sigma_c} C_{gt}^{\frac{-1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} \frac{\varepsilon_s - \sigma_c}{\sigma_c} C_t^{\frac{\varepsilon_s-2\sigma_c}{\sigma_c}} \frac{\partial C_t}{\partial C_{gt}} C_{st}^{\frac{\sigma_c-1}{\sigma_c}}$$

Solving for the partial derivative gives:

$$\frac{\partial C_t}{\partial C_{gt}} = \frac{(1-\sigma_c)\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g}{\sigma_c}} C_{gt}^{\frac{-1}{\sigma_c}}}{(\varepsilon_g - \sigma_c)\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g-1}{\sigma_c}} \left(\frac{C_{gt}}{C_t}\right)^{\frac{\sigma_c-1}{\sigma_c}} + (\varepsilon_s - \sigma_c)\alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s-1}{\sigma_c}} \left(\frac{C_{st}}{C_t}\right)^{\frac{\sigma_c-1}{\sigma_c}}} \tag{26}$$

[15] Recall that to calculate the sectoral weights we take averages over 1948 and 1947.

[16] Recall that to calculate the sectoral weights we take averages over 2009–2010 and 1947–1948.

Similar derivations yield:

$$\frac{\partial C_t}{\partial C_{it}} = \frac{(1-\sigma_c)\alpha_i^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_i}{\sigma_c}} C_{it}^{\frac{-1}{\sigma_c}}}{(\varepsilon_g - \sigma_c)\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g-1}{\sigma_c}} \left(\frac{C_{gt}}{C_t}\right)^{\frac{\sigma_c-1}{\sigma_c}} + (\varepsilon_s - \sigma_c)\alpha_s^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_s-1}{\sigma_c}} \left(\frac{C_{st}}{C_t}\right)^{\frac{\sigma_c-1}{\sigma_c}}}, \qquad i = g, s \tag{27a}$$

$$\frac{\partial C_{st}}{\partial C_{jt}} = \frac{(1-\sigma_s)\alpha_j^{\frac{1}{\sigma_s}} C_{st}^{\frac{\varepsilon_j}{\sigma_s}} C_{jt}^{\frac{-1}{\sigma_s}}}{(\varepsilon_m - \sigma_s)\alpha_m^{\frac{1}{\sigma_s}} C_{st}^{\frac{\varepsilon_m-1}{\sigma_s}} \left(\frac{C_{mt}}{C_{st}}\right)^{\frac{\sigma_s-1}{\sigma_s}} + (\varepsilon_n - \sigma_s)\alpha_n^{\frac{1}{\sigma_s}} C_{st}^{\frac{\varepsilon_n-1}{\sigma_s}} \left(\frac{C_{nt}}{C_{st}}\right)^{\frac{\sigma_s-1}{\sigma_s}}}, \qquad j = m, n \tag{27b}$$

Clearly, Assumption 1 is sufficient for these partial derivatives to be positive. To see that it is also necessary, let's pick one of them, $\partial C_t/\partial C_{gt}$ and note that the proof for the others is analogous. The right–hand side of equation (26) then has to be positive for all non–negative (C_{gt}, C_{st}). For $\sigma_c > 1$ and $C_{st} = 0$ or for $\sigma_c < 1$ and $C_{st} \to \infty$, this amounts to:

$$\frac{\partial C_t}{\partial C_{gt}} = \frac{(1-\sigma_c)\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g}{\sigma_c}} C_{gt}^{\frac{-1}{\sigma_c}}}{(\varepsilon_g - \sigma_c)\alpha_g^{\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_g-1}{\sigma_c}} \left(\frac{C_{gt}}{C_t}\right)^{\frac{\sigma_c-1}{\sigma_c}}} > 0$$

For $\sigma_c > 1$ this is the case if and only if $\sigma_c > \varepsilon_g$ and for $\sigma_c < 1$ this is the case if and only if $\sigma_c < \varepsilon_g$. Using the same arguments for $\partial C_t/\partial C_{st}$ shows that to have both partial derivatives positive requires either $\sigma_c < \min\{\varepsilon_g, \varepsilon_s\}$ or $\max\{\varepsilon_g, \varepsilon_s\} < \sigma_c$. This is Assumption 1 for $i = g, s$. QED

Derivation of equilibrium conditions

The first–order condition to the inner and outer parts of the household's problem are:

$$P_{jt} = \mu_t \alpha_j^{\frac{1}{\sigma_s}} C_{jt}^{-\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_j-1}{\sigma_s}} C_{st}^{\frac{1}{\sigma_s}}, \quad j = m, n \tag{28a}$$

$$P_{it} = \lambda_t \alpha_i^{\frac{1}{\sigma_c}} C_{it}^{-\frac{1}{\sigma_c}} C_t^{\frac{\varepsilon_i-1}{\sigma_c}} C_t^{\frac{1}{\sigma_c}}, \quad i = g, s \tag{28b}$$

To derive (7b), multiply both sides of (28a) with C_{jt} and adding up the resulting equations, we get

$$P_{mt}C_{mt} + P_{nt}C_{nt} = \mu_t \left(\alpha_m^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_m-1}{\sigma_s}} C_{mt}^{\frac{\sigma_s-1}{\sigma_s}} + \alpha_n^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_n-1}{\sigma_s}} C_{nt}^{\frac{\sigma_s-1}{\sigma_s}}\right) C_{st}^{\frac{1}{\sigma_s}} = \mu_t C_{st}^{\frac{\sigma_s-1}{\sigma_s}} C_{st}^{\frac{1}{\sigma_s}} = \mu_t C_{st} \tag{29}$$

which implies that

$$P_{st} = \frac{P_{mt}C_{mt} + P_{nt}C_{nt}}{C_{st}} = \mu_t \tag{30}$$

Rewriting (28a) again

$$P_{jt}^{1-\sigma_s} = P_{st}^{1-\sigma_s}\alpha_j^{\frac{1-\sigma_s}{\sigma_s}} C_{jt}^{\frac{\sigma_s-1}{\sigma_s}} C_t^{(1-\sigma_s)\frac{\varepsilon_j-1}{\sigma_s}} C_{st}^{\frac{1-\sigma_s}{\sigma_s}}$$

which implies

$$\alpha_j C_{st}^{\varepsilon_j-1} P_{jt}^{1-\sigma_s} = P_{st}^{1-\sigma_s}\alpha_j^{\frac{1}{\sigma_s}} C_{jt}^{\frac{\sigma_s-1}{\sigma_s}} C_t^{\frac{\varepsilon_j-1}{\sigma_s}} C_{st}^{\frac{1-\sigma_s}{\sigma_s}}$$

Adding this up over $j = m, n$ yields

$$\begin{aligned}
\alpha_m C_t^{\varepsilon_m-1} P_{mt}^{1-\sigma_s} + \alpha_n C_t^{\varepsilon_n-1} P_{nt}^{1-\sigma_s} &= P_{st}^{1-\sigma_s}\left(\alpha_m^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_m-1}{\sigma_s}} C_{mt}^{\frac{\sigma_s-1}{\sigma_s}} + \alpha_n^{\frac{1}{\sigma_s}} C_t^{\frac{\varepsilon_n-1}{\sigma_s}} C_{nt}^{\frac{\sigma_s-1}{\sigma_s}}\right) C_{st}^{\frac{1-\sigma_s}{\sigma_s}} \\
&= P_{st}^{1-\sigma_s} C_{st}^{\frac{\sigma_s-1}{\sigma_s}} C_{st}^{\frac{1-\sigma_s}{\sigma_s}} = P_{st}^{1-\sigma_s}
\end{aligned}$$

implying that the price index is given as

$$P_{st} = \left(\alpha_m C_t^{\varepsilon_m-1} P_{mt}^{1-\sigma_s} + \alpha_n C_t^{\varepsilon_n-1} P_{nt}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}$$

which is (7b). Similar steps give (8b) and (8c).

To derive expressions for the expenditure shares, we rewrite the relative expenditure share (8a) as

$$P_{it}C_{it} = \alpha_i C_t^{\varepsilon_i-\sigma_c} P_{it}^{1-\sigma_c} \frac{P_{gt}C_{gt}}{\alpha_g C_t^{\varepsilon_g-\sigma_c} P_{gt}^{1-\sigma_c}} \qquad i = \{g, s\}$$

Summing over $i = \{g, s\}$ yields

$$P_{gt}C_{gt} + P_{st}C_{st} = \left(\alpha_g C_t^{\varepsilon_g-\sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s-\sigma_c} P_{st}^{1-\sigma_c}\right) \frac{P_{gt}C_{gt}}{\alpha_g C_t^{\varepsilon_g-\sigma_c} P_{gt}^{1-\sigma_c}}$$

Hence, the expenditure shares of $i = g, s$ can be expressed as

$$\chi_{it} \equiv \frac{P_{it}C_{it}}{P_tC_t} = \frac{\alpha_i C_t^{\varepsilon_i-\sigma_c} P_{it}^{1-\sigma_c}}{\alpha_g C_t^{\varepsilon_g-\sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s-\sigma_c} P_{st}^{1-\sigma_c}} = \frac{\alpha_i C_t^{\varepsilon_i-1} P_{it}^{1-\sigma_c}}{\alpha_g C_t^{\varepsilon_g-1} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s-1} P_{st}^{1-\sigma_c}} \tag{31a}$$

where $P_tC_t = P_{gt}C_{gt} + P_{st}C_{st}$. A similar derivation for $j = m, n$ yields

$$\chi_{jt} \equiv \frac{P_{jt}C_{jt}}{P_{st}C_{st}} = \frac{\alpha_j C_t^{\varepsilon_j-\sigma_s} P_{jt}^{1-\sigma_s}}{\alpha_m C_t^{\varepsilon_m-\sigma_s} P_{mt}^{1-\sigma_s} + \alpha_n C_t^{\varepsilon_n-\sigma_s} P_{nt}^{1-\sigma_s}} = \frac{\alpha_j C_t^{\varepsilon_j-1} P_{jt}^{1-\sigma_s}}{\alpha_m C_t^{\varepsilon_m-1} P_{mt}^{1-\sigma_s} + \alpha_n C_t^{\varepsilon_n-1} P_{nt}^{1-\sigma_s}} \tag{31b}$$

Combining (8c) and (31a), we obtain

$$\chi_{it} = \alpha_i C_t^{\varepsilon_i-\sigma_c} \left(\frac{P_{it}}{P_tC_t}\right)^{1-\sigma_c} \qquad i = \{g, s\}$$

which is (32a). Similarly, we obtain (32b)

$$\chi_{jt} = \alpha_j C_t^{\varepsilon_j-\sigma_s} \left(\frac{P_{jt}}{P_{st}C_{st}}\right)^{1-\sigma_s} \qquad j = \{m, n\}$$

where $P_{st}C_{st} = P_{mt}C_{mt} + P_{mt}C_{mt}$.

For what follows, it is also sometimes useful to have expressions for the expenditure shares. Appendix B shows that

$$\chi_{it} = \alpha_i C_t^{\varepsilon_i-\sigma_c} \left(\frac{P_{it}}{P_tC_t}\right)^{1-\sigma_c}, \quad i = \{g, s\} \tag{32a}$$

$$\chi_{jt} = \alpha_j C_t^{\varepsilon_j-\sigma_s} \left(\frac{P_{jt}}{P_{st}C_{st}}\right)^{1-\sigma_s}, \quad j = \{m, n\} \tag{32b}$$

where $P_{st}C_{st} = P_{mt}C_{mt} + P_{mt}C_{mt}$. QED

Interpretation of Assumption 5

First we express consumption expenditures as function of prices and C_t. Combining (8c) and (7b), and using the fact that $P_{gt} = 1$, we get

$$E_t \equiv P_tC_t = \left(\alpha_g C_t^{\varepsilon_g-\sigma_c} + \alpha_s C_t^{\varepsilon_s-\sigma_c} \left(\alpha_m C_t^{\varepsilon_m-1} P_{mt}^{1-\sigma_s} + \alpha_n C_t^{\varepsilon_n-1} P_{nt}^{1-\sigma_s}\right)^{\frac{1-\sigma_c}{1-\sigma_s}}\right)^{\frac{1}{1-\sigma_c}} \tag{33}$$

Next we take the derivative of E_t with respect to C_t using (33)

$$\frac{\partial E_t}{\partial C_t} = \frac{1}{1-\sigma_c} \frac{E_t}{\alpha_g C_t^{\varepsilon_g-\sigma_c} P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s-\sigma_c} P_{st}^{1-\sigma_c}} \left[\alpha_g C_t^{\varepsilon_g-\sigma_c} \frac{\varepsilon_g - \sigma_c}{C_t} + \alpha_s C_t^{\varepsilon_s-\sigma_c} P_{st}^{1-\sigma_c} \frac{\varepsilon_s - \sigma_c}{C_t}\right.$$
$$\left. + \frac{1-\sigma_c}{1-\sigma_s} \frac{\alpha_s C_t^{\varepsilon_s-\sigma_c} P_{st}^{1-\sigma_c}}{\alpha_m C_t^{\varepsilon_m-1} P_{mt}^{1-\sigma_s} + \alpha_n C_t^{\varepsilon_n-1} P_{nt}^{1-\sigma_s}} \left(\alpha_m C_t^{\varepsilon_m-1} P_{mt}^{1-\sigma_s} \frac{\varepsilon_m - 1}{C_t} \alpha_n C_t^{\varepsilon_n-1} P_{nt}^{1-\sigma_s} \frac{\varepsilon_n - 1}{C_t}\right)\right]$$

Using the expression for expenditure shares in (31a), we can simplify this as

$$\frac{\partial E_t}{\partial C_t} = \frac{E_t}{1-\sigma_c}\left[\chi_{gt}\frac{\varepsilon_g-\sigma_c}{C_t} + \chi_{st}\frac{\varepsilon_s-\sigma_c}{C_t} + \frac{1-\sigma_c}{1-\sigma_s}\chi_{st}\left(\chi_{mt}\frac{\varepsilon_m-1}{C_t} + \chi_{nt}\frac{\varepsilon_n-1}{C_t}\right)\right]$$

It follows that the elasticity in question is

$$\frac{C_t}{E_t}\frac{\partial E_t}{\partial C_t} = \frac{\varepsilon_g-\sigma_c}{1-\sigma_c}\chi_{gt} + \frac{\varepsilon_s-\sigma_c}{1-\sigma_c}\chi_{st} + \frac{\varepsilon_m-1}{1-\sigma_s}\chi_{st}\chi_{mt} + \frac{\varepsilon_n-1}{1-\sigma_s}\chi_{st}\chi_{nt} \tag{34}$$

We can restate this condition as

$$\frac{C_t}{E_t}\frac{\partial E_t}{\partial C_t} = \frac{\varepsilon_g-\sigma_c}{1-\sigma_c}\chi_{gt} + \left(\frac{\varepsilon_s-\sigma_c}{1-\sigma_c} + \frac{\varepsilon_n-1}{1-\sigma_s} + \frac{\varepsilon_m-\varepsilon_n}{1-\sigma_s}\chi_{mt}\right)\chi_{st}$$

Since $(\varepsilon_m-\varepsilon_n)/(1-\sigma_s) > 0$, we have

$$\frac{C_t}{E_t}\frac{\partial E_t}{\partial C_t} \geq \frac{\varepsilon_g-\sigma_c}{1-\sigma_c}\chi_{gt} + \left(\frac{\varepsilon_s-\sigma_c}{1-\sigma_c} + \frac{\varepsilon_n-1}{1-\sigma_s}\right)\chi_{st}$$

Next we use that $\chi_{gt} = 1-\chi_{st}$.

$$\frac{C_t}{E_t}\frac{\partial E_t}{\partial C_t} \geq \frac{\varepsilon_g-\sigma_c}{1-\sigma_c} + \left(\frac{\varepsilon_s-\sigma_c}{1-\sigma_c} + \frac{\varepsilon_n-1}{1-\sigma_s} - \frac{\varepsilon_g-\sigma_c}{1-\sigma_c}\right)\chi_{st} \geq \frac{\varepsilon_g-\sigma_c}{1-\sigma_c}$$

which says that the elasticity of consumption expenditures with respect to real consumption is non–negative.

Proof of Lemma 2

First rewrite (8c) as

$$(P_tC_t)^{1-\sigma_c} = \alpha_g C_t^{\varepsilon_g-\sigma_c}P_{gt}^{1-\sigma_c} + \alpha_s C_t^{\varepsilon_s-\sigma_c}P_{st}^{1-\sigma_c} \tag{35}$$

Taking into account that the taxes are rebated we have

$$P_tC_t = A_{gt}H_{gt} + P_{mt}A_{mt}H_{mt} + P_{nt}A_{nt}H_{nt} = A_{gt}$$

which implies that

$$\widehat{PC}_{t+1} = \widehat{A}_{gt+1} = \gamma_g$$

Using this and recalling (31a), we can rewrite the growth rate of consumption expenditure as

$$\begin{aligned}\gamma_g^{1-\sigma_c} &= \widehat{PC}_{t+1} \\ &= \chi_{gt}\widehat{P}_{gt+1}^{1-\sigma_c}\widehat{C}_{t+1}^{\varepsilon_g-\sigma_c} + \chi_{st}\widehat{P}_{st+1}^{1-\sigma_c}\widehat{C}_{t+1}^{\varepsilon_s-\sigma_c} \\ &= \chi_{gt}\widehat{C}_{t+1}^{\varepsilon_g-\sigma_c} + \chi_{st}\widehat{P}_{st+1}^{1-\sigma_c}\widehat{C}_{t+1}^{\varepsilon_s-\sigma_c}\end{aligned} \tag{36}$$

Using equations (7b) and (31b) and the equilibrium relation for relative prices, $P_{jt} = A_{gt}/A_{jt}$, we obtain

$$\begin{aligned}\widehat{P}_{st+1}^{1-\sigma_s} &= \chi_{mt}\widehat{P}_{mt+1}^{1-\sigma_s}\widehat{C}_{t+1}^{\varepsilon_m-1} + \chi_{nt}\widehat{P}_{nt+1}^{1-\sigma_s}\widehat{C}_{t+1}^{\varepsilon_n-1} \\ &= \chi_{mt}\left(\frac{\gamma_g}{\gamma_m}\right)^{1-\sigma_s}\widehat{C}_{t+1}^{\varepsilon_m-1} + \chi_{nt}\left(\frac{\gamma_g}{\gamma_n}\right)^{1-\sigma_s}\widehat{C}_{t+1}^{\varepsilon_n-1}\end{aligned} \tag{37}$$

Combining (36) and (37), and rearranging, we get

$$1 = \chi_{gt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_c} + \chi_{st}\left[\chi_{mt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_m-1}{1-\sigma_s}}}{\gamma_m}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_n-1}{1-\sigma_s}}}{\gamma_n}\right)^{1-\sigma_s}\right]^{\frac{1-\sigma_c}{1-\sigma_s}} \tag{38}$$

We first derive a lower bound on $\widehat{C}_{t+1}$. Note that the following inequalities hold

$$\frac{\varepsilon_g-\sigma_c}{1-\sigma_c} < \frac{\varepsilon_s-\sigma_c}{1-\sigma_c} + \frac{\varepsilon_n-1}{1-\sigma_s} < \frac{\varepsilon_s-\sigma_c}{1-\sigma_c} + \frac{\varepsilon_m-1}{1-\sigma_s} \tag{39}$$

The first inequality is our Assumption 5, and the second inequality follows from Assumption 4, $\sigma_s > 1$, $\varepsilon_m < 1 < \varepsilon_n$. Using these inequalities and Assumption 2 that $\gamma_g > \gamma_n, \gamma_m$, equation (38) implies the inequality

$$1 > \chi_{gt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_c} + \chi_{st}\left[\chi_{mt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_s}\right]^{\frac{1-\sigma_c}{1-\sigma_s}}$$

which implies the claimed upper bound:

$$\gamma_g^{\frac{1-\sigma_c}{\varepsilon_g-\sigma_c}} \geq \widehat{C}_{t+1}$$

To derive the lower bound, we use again the inequalities (39) and $\gamma_n < \gamma_g, \gamma_m$. Equation

(38) then implies the following inequality:

$$1 < \chi_{gt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_m-1}{1-\sigma_s}}}{\gamma_n}\right)^{1-\sigma_c} + \chi_{st}\left[\chi_{mt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_m-1}{1-\sigma_s}}}{\gamma_n}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_m-1}{1-\sigma_s}}}{\gamma_n}\right)^{1-\sigma_s}\right]^{\frac{1-\sigma_c}{1-\sigma_s}}$$

which implies the lower bound:

$$\gamma_n^{\frac{(1-\sigma_c)(1-\sigma_s)}{(\varepsilon_m-1)(1-\sigma_c)+(\varepsilon_s-\sigma_c)(1-\sigma_s)}} \leq \widehat{C}_{t+1}$$

QED

Proof of Lemma 3

Using equations (7b) and (31b) and the equilibrium relation for relative prices, $P_{jt} = A_{gt}/A_{jt}$, we obtain

$$\begin{aligned}\widehat{P}_{st+1} &= \left[\chi_{mt}\widehat{P}_{mt+1}^{1-\sigma_s}\widehat{C}_{t+1}^{\varepsilon_m-1} + \chi_{nt}\widehat{P}_{nt+1}^{1-\sigma_s}\widehat{C}_{t+1}^{\varepsilon_n-1}\right]^{\frac{1}{1-\sigma_s}} \\ &= \left[\chi_{mt}\left(\frac{\gamma_g}{\gamma_m}\widehat{C}_{t+1}^{\frac{\varepsilon_m-1}{1-\sigma_s}}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\gamma_g}{\gamma_n}\widehat{C}_{t+1}^{\frac{\varepsilon_n-1}{1-\sigma_s}}\right)^{1-\sigma_s}\right]^{\frac{1}{1-\sigma_s}} \qquad (40)\end{aligned}$$

The term in the square bracket in the (40) is of the form $\chi_{mt}x_{1t}^{1-\sigma_s} + (1-\chi_{mt})x_{2t}^{1-\sigma_s}$. Since $1 < \sigma_s$, we know that $x^{1-\sigma_s}$ is a convex function. Hence,

$$\chi_{mt}x_{1t}^{1-\sigma_s} + (1-\chi_{mt})x_{2t}^{1-\sigma_s} \leq [\chi_{mt}x_{1t} + (1-\chi_{mt})x_{2t}]^{1-\sigma_s}$$

Since the exponent on the square brackets is negative, $1-\sigma_s < 0$, this implies that

$$\widehat{P}_{st+1} \geq \chi_{mt}\frac{\gamma_g}{\gamma_m}\widehat{C}_{t+1}^{\frac{\varepsilon_m-1}{1-\sigma_s}} + \chi_{nt}\frac{\gamma_g}{\gamma_n}\widehat{C}_{t+1}^{\frac{\varepsilon_n-1}{1-\sigma_s}} = \left(\chi_{mt}\frac{\gamma_n}{\gamma_m}\widehat{C}_{t+1}^{\frac{\varepsilon_m-\varepsilon_n}{1-\sigma_s}} + \chi_{nt}\right)\frac{\gamma_g}{\gamma_n}\widehat{C}_{t+1}^{\frac{\varepsilon_n-1}{1-\sigma_s}}$$

The term in the round brackets is increasing in $\widehat{C}_{t+1}$ because $\varepsilon_m - \varepsilon_n < 0$ and $1-\sigma_s < 0$. Hence the inequality is satisfied at the lower bound for $\widehat{C}_{t+1}$ given by (12a). Taking account the parameter restriction from (13a), we conclude that the lower bound on the coefficient of χ_{mt} is larger then 1. Hence the inequality can be restated as

$$\widehat{P}_{st+1} > \frac{\gamma_g}{\gamma_n}\widehat{C}_{t+1}^{\frac{\varepsilon_n-1}{1-\sigma_s}}$$

The right–hand side is larger than one if and only if

$$\left(\frac{\gamma_g}{\gamma_n}\right)^{\frac{\sigma_s-1}{\varepsilon_n-1}} > \widehat{C}_{t+1}$$

A sufficient condition for this is that

$$\left(\frac{\gamma_g}{\gamma_n}\right)^{\frac{\sigma_s-1}{\varepsilon_n-1}} > \bar{C}$$

Using Lemma 2, this is equivalent to:

$$\left(\frac{\gamma_g}{\gamma_n}\right)^{\frac{\sigma_s-1}{\varepsilon_n-1}} > \gamma_g^{\frac{1-\sigma_c}{\varepsilon_g-\sigma_c}}$$

Rearranging gives inequality (13b). QED

Proof of Proposition 1

Using equation (32a), the growth rate of the expenditure share of goods follows as

$$\frac{\chi_{gt+1}}{\chi_{gt}} = \left(\frac{P_{gt+1}}{P_{gt}}\frac{P_t C_t}{P_{t+1}C_{t+1}}\right)^{1-\sigma_c} \widehat{C}_{t+1}^{\varepsilon_g-\sigma_c} \tag{41}$$

Using that $P_{gt} = 1$ and that $P_t C_t$ is growing at rate γ_g, the previous equation can be rewritten as

$$\frac{\chi_{gt+1}}{\chi_{gt}} = \frac{1}{\gamma_g^{1-\sigma_c}}\widehat{C}_{t+1}^{\varepsilon_g-\sigma_c}. \tag{42}$$

Using the upper bound from Lemma 2, we have

$$\frac{\chi_{gt+1}}{\chi_{gt}} < \frac{1}{\gamma_g^{1-\sigma_c}}\left(\gamma_g^{\frac{1-\sigma_c}{\varepsilon_g-\sigma_c}}\right)^{\varepsilon_g-\sigma_c} = 1. \tag{43}$$

It follows now that $\chi_{gt+1} < \chi_{gt}$ for all t.

To see why it is not possible that $\lim_{t\to\infty}\chi_{gt} = \chi_g > 0$, consider equation (42). It implies that χ_{gt} converges to a positive limit if $\widehat{C}_{t+1}^{\varepsilon_g-\sigma_c}$ converges to $\gamma_g^{1-\sigma_c}$. We know from the previous argument that χ_{gt} is decreasing over time and that

$$\widehat{C}_{t+1}^{\varepsilon_g-\sigma_c} < \gamma_g^{1-\sigma_c}.$$

If $\widehat{C}_{t+1}$ decreases when χ_{gt} decreases, then we are done because it implies that $\widehat{C}_{t+1}^{\varepsilon_g-\sigma_c}$ shrinks further away from $\gamma_g^{1-\sigma_c}$ as time evolves.

To show that $\widehat{C}_{t+1}$ decreases when χ_{gt} decreases, recall the equilibrium condition (38)

$$1 = \chi_{gt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_c} + \chi_{st}\left[\chi_{mt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_m-1}{1-\sigma_s}}}{\gamma_m}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_n-1}{1-\sigma_s}}}{\gamma_n}\right)^{1-\sigma_s}\right]^{\frac{1-\sigma_c}{1-\sigma_s}}$$

If

$$\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_c} < \left[\chi_{mt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_m-1}{1-\sigma_s}}}{\gamma_m}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_n-1}{1-\sigma_s}}}{\gamma_n}\right)^{1-\sigma_s}\right]^{\frac{1-\sigma_c}{1-\sigma_s}} \tag{44}$$

for all $\chi_{mt}, \chi_{nt} = 1 - \chi_{mt}$, then decreasing χ_{gt} – and thereby increasing $\chi_{st} = 1 - \chi_{gt}$ – increases the right–hand side. To restore the equality with 1, the right–hand side must decrease. Since the assumptions imply that

$$0 < \frac{\varepsilon_g - \sigma_c}{1-\sigma_c} < \frac{\varepsilon_s - \sigma_c}{1-\sigma_c} + \frac{\varepsilon_n - 1}{1-\sigma_s} < \frac{\varepsilon_s - \sigma_c}{1-\sigma_c} + \frac{\varepsilon_m - 1}{1-\sigma_s}. \tag{45}$$

implies that both terms on the right–hand side increase in $\widehat{C}_{t+1}$, the right–hand side decreases if $\widehat{C}_{t+1}$ decreases.

To complete the proof, we have to show that Assumptions 1–5 implies (44). To see this, rearrange (44) as

$$\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_s} > \chi_{mt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_m-1}{1-\sigma_s}}}{\gamma_m}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_n-1}{1-\sigma_s}}}{\gamma_n}\right)^{1-\sigma_s}$$

where we used that $\sigma_c < 1$ and $\sigma_s > 1$. To derive a sufficient condition for this inequality to hold, increase the right–hand side by replacing γ_n with γ_m and ε_m with ε_n (recall $1 - \sigma_s < 0$):

$$\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_g-\sigma_c}{1-\sigma_c}}}{\gamma_g}\right)^{1-\sigma_s} > \chi_{mt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_n-1}{1-\sigma_s}}}{\gamma_m}\right)^{1-\sigma_s} + \chi_{nt}\left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_n-1}{1-\sigma_s}}}{\gamma_m}\right)^{1-\sigma_s} = \left(\frac{\widehat{C}_{t+1}^{\frac{\varepsilon_s-\sigma_c}{1-\sigma_c}+\frac{\varepsilon_n-1}{1-\sigma_s}}}{\gamma_m}\right)^{1-\sigma_s}$$

Since $\gamma_g > \gamma_m$ and

$$\frac{\varepsilon_g - \sigma_c}{1-\sigma_c} < \frac{\varepsilon_s - \sigma_c}{1-\sigma_c} + \frac{\varepsilon_n - 1}{1-\sigma_s},$$

this inequality holds. Since this sufficient condition does not feature χ_{gt}, χ_{st}, inequality (44) holds for all expenditure shares. QED

Proof of Proposition 3

The claim is implied by the proof of Proposition 1. QED

Proof of Lemma 4

It is straightforward to show that:

$$\widehat{Y}_t^{la} = \frac{\widehat{P_t Y_t}}{\widehat{P}_t^{pa}}$$
$$\widehat{Y}_t^{pa} = \frac{\widehat{P_t Y_t}}{\widehat{P}_t^{la}}$$

Hence,

$$\widehat{Y}_t^{ch} = \sqrt{\widehat{Y}_t^{la} \times \widehat{Y}_t^{pa}} = \sqrt{\frac{\widehat{P_t Y_t}}{\widehat{P}_t^{pa}} \times \frac{\widehat{P_t Y_t}}{\widehat{P}_t^{la}}} = \frac{\widehat{P_t Y_t}}{\widehat{P}_t^{ch}}$$

QED

Proof of Proposition 4

We want to show that the growth factor in units of the current–period numeraire C_{jt} is given by $\widehat{Y}_t^{nj} = \gamma_j$. Using (3b) and (4), we have

$$\begin{aligned}
\widehat{Y}_t^{nj} &= \frac{\sum_{i=g,m,n} A_{it} H_{it} \frac{P_{it}}{P_{jt}}}{\sum_{i=g,m,n} A_{it-1} H_{it-1} \frac{P_{it-1}}{P_{jt-1}}} \\
&= \frac{A_{jt}}{A_{jt-1}} \frac{\sum_{i=g,m,n} \frac{A_{it}}{A_{jt}} \frac{P_{it}}{P_{jt}} H_{it}}{\sum_{i=g,m,n} \frac{A_{it-1}}{A_{jt-1}} \frac{P_{it-1}}{P_{jt-1}} H_{it-1}} \\
&= \frac{A_{jt}}{A_{jt-1}} \frac{\sum_{i=g,m,n} H_{it}}{\sum_{i=g,m,n} H_{it-1}} \\
&= \gamma_j
\end{aligned}$$

where we used that $\sum_{i=g,m,n} H_{it} = \sum_{i=g,m,n} H_{it-1} = 1$. QED

Proof of Proposition 5

That the growth factor with a current–period numeraire is constant follows from Lemma 1.

To show that $\Delta\widehat{Y}_t^{ch} < 0$, we will show that $\Delta\widehat{Y}_t^{la} < 0$ and $\Delta\widehat{Y}_t^{pa} < 0$. We start with the Laspeyres Quantity Index with period t–1 as the base year for the prices:

$$
\begin{aligned}
\widehat{Y}_t^{la} &= \frac{P_{gt-1}C_{gt} + P_{mt-1}C_{mt} + P_{nt-1}C_{nt}}{P_{gt-1}C_{gt-1} + P_{mt-1}C_{mt-1} + P_{nt-1}C_{nt-1}} \\
&= \frac{P_{gt-1}C_{gt-1}}{P_{t-1}C_{t-1}}\frac{C_{gt}}{C_{gt-1}} + \frac{P_{mt-1}C_{mt-1}}{P_{t-1}C_{t-1}}\frac{C_{mt}}{C_{mt-1}} + \frac{P_{nt-1}C_{nt-1}}{P_{t-1}C_{t-1}}\frac{C_{nt}}{C_{nt-1}} \\
&= \frac{H_{gt-1}}{H_{t-1}}\frac{A_{gt}H_{gt}}{A_{gt-1}H_{gt-1}} + \frac{H_{mt-1}}{H_{t-1}}\frac{A_{mt}H_{mt}}{A_{mt-1}H_{mt-1}} + \frac{H_{nt-1}}{H_{t-1}}\frac{A_{nt}H_{nt}}{A_{nt-1}H_{nt-1}} \\
&= \gamma_g H_{gt} + \gamma_m H_{mt} + \gamma_n H_{nt} \\
&= \gamma_g(1 - H_{st}) + \gamma_m(H_{st} - 1) + \gamma_n H_{nt} \\
&= \gamma_g - (\gamma_g - \gamma_m)H_{st} - (\gamma_m - \gamma_n)H_{nt}
\end{aligned}
$$

where we used (5) and that $H_{t-1} = 1$, $H_{gt} = 1 - H_{st}$, and $H_{mt} = H_{st} - H_{nt}$.

$\Delta\widehat{Y}_t^{la} < 0$ iff

$$(\gamma_g - \gamma_m)\Delta H_{st} - (\gamma_m - \gamma_n)\Delta H_{nt} < 0$$

Since $\gamma_g > \gamma_m > \gamma_n$ and we know that $\Delta H_{st} < 0$, a sufficient condition is that $\Delta H_{st} \leq 0$.

We continue with the Paasche Quantity Index with period t as the base year for prices:

$$
\begin{aligned}
\frac{1}{\widehat{Y}_t^{pa}} &= \frac{P_{gt}C_{gt-1} + P_{mt}C_{mt-1} + P_{nt}C_{nt-1}}{P_{gt}C_{gt} + P_{mt}C_{mt} + P_{nt}C_{nt}} \\
&= \frac{P_{gt}C_{gt}}{P_tC_t}\frac{C_{gt-1}}{C_{gt}} + \frac{P_{mt}C_{mt}}{P_tC_t}\frac{C_{mt-1}}{C_{mt}} + \frac{P_{nt}C_{nt}}{P_tC_t}\frac{C_{nt-1}}{C_{nt}} \\
&= \frac{H_{gt}}{H_t}\frac{A_{gt-1}H_{gt-1}}{A_{gt}H_{gt}} + \frac{H_{mt}}{H_t}\frac{A_{mt-1}H_{mt-1}}{A_{mt}H_{mt}} + \frac{H_{nt}}{H_t}\frac{A_{nt-1}H_{nt-1}}{A_{nt}H_{nt}} \\
&= \frac{1}{\gamma_g}H_{gt-1} + \frac{1}{\gamma_m}H_{mt-1} + \frac{1}{\gamma_n}H_{nt-1} \\
&= \frac{1}{\gamma_g} + \left(\frac{1}{\gamma_m} - \frac{1}{\gamma_g}\right)H_{st-1} - \left(\frac{1}{\gamma_n} - \frac{1}{\gamma_m}\right)H_{nt-1}
\end{aligned}
$$

where we used (5) and that $H_t = 1$, $H_{gt-1} = 1 - H_{st-1}$, and $H_{mt-1} = H_{st-1} - H_{nt-1}$.

$\Delta\widehat{Y}_t^{pa} < 0$ iff

$$\left(\frac{\gamma_g - \gamma_n}{\gamma_g\gamma_n}\right)\Delta H_{st-1} + \left(\frac{\gamma_m - \gamma_n}{\gamma_m\gamma_n}\right)\Delta H_{nt-1} > 0$$

Since $\gamma_g > \gamma_m > \gamma_n$ and we know that $\Delta H_{st} < 0$, a sufficient condition again is that $\Delta H_{st-1} \leq 0$.

QED

Appendix C: Details of the Calibration

Equilibrium conditions

$$
\begin{array}{lcll}
C & - & \left[\alpha_g^{\frac{1}{\sigma_c}} C_g^{\frac{\sigma_c-1}{\sigma_c}} C^{\frac{\varepsilon_g-1}{\sigma_c}} + \alpha_s^{\frac{1}{\sigma_c}} C_s^{\frac{\sigma_c-1}{\sigma_c}} C^{\frac{\varepsilon_s-1}{\sigma_c}}\right]^{\frac{\sigma_c}{\sigma_c-1}} & = 0 \\
C_s & - & \left[\alpha_m^{\frac{1}{\sigma_s}} C_m^{\frac{\sigma_s-1}{\sigma_s}} C^{\frac{\varepsilon_m-1}{\sigma_s}} + \alpha_n^{\frac{1}{\sigma_s}} C_n^{\frac{\sigma_s-1}{\sigma_s}} C^{\frac{\varepsilon_n-1}{\sigma_s}}\right]^{\frac{\sigma_s}{\sigma_s-1}} & = 0 \\
1 & - & L_n - L_g - L_m & = 0 \\
C_g & - & A_g L_g & = 0 \\
C_m & - & A_m L_m & = 0 \\
C_n & - & A_n L_n & = 0 \\
P_g & - & w/A_g & = 0 \\
P_m & - & (1+\tau_m)w/A_m & = 0 \\
P_n & - & (1+\tau_n)w/A_n & = 0 \\
P_s & - & \left[\alpha_n P_n^{1-\sigma_s} C^{\varepsilon_n-1} + \alpha_m P_m^{1-\sigma_s} C^{\varepsilon_m-1}\right]^{\frac{1}{1-\sigma_s}} & = 0 \\
\left(\frac{P_g}{P_s}\right) & - & \left(\frac{\alpha_g}{\alpha_s}\right)^{\frac{1}{\sigma_c}} \left(\frac{C_g}{C_s}\right)^{-\frac{1}{\sigma_c}} C^{\frac{\varepsilon_g-\varepsilon_s}{\sigma_c}} & = 0 \\
\left(\frac{P_m}{P_n}\right) & - & \left(\frac{\alpha_m}{\alpha_n}\right)^{\frac{1}{\sigma_s}} \left(\frac{C_m}{C_n}\right)^{-\frac{1}{\sigma_s}} C^{\frac{\varepsilon_m-\varepsilon_n}{\sigma_s}} & = 0
\end{array}
\tag{46}
$$

We solve this system of 12 equations in the following 12 unknowns:

$$C_g, C_m, C_n, C, C_s \quad L_n, L_g, L_m \quad w \quad P_m, P_n, P_s$$

Notice that in the calibration, P_g is set equal to the price of goods that we observe in the data.

C.1 Construction of quality–adjusted labor hours

The levels of efficiency hours for 1995 are expressed in terms of quality adjusted hours. Then using the levels we use the changes in the index for efficiency hours from the WORLD KLEMS data set to calculate the levels for each year.

The quality–adjusted hours for the year 1995 were constructed as follows. The USA data in the WORLD KLEMS data set comes with a so called labor input file, which provides for each year and each industry labor compensation per hour worked in current \$, total number of persons engaged and the average number of hours worked per week for 96 different types of labour. These types are defined by sex (2 types), class of worker (2 types, employed, self-employed), age (8 types), educational level (6 types), $2 \times 2 \times 8 \times 6 = 96$.

We proceed in four steps. Step 1: we calculate the economy wide average of the labor compensation per hour worked for the lowest educational group (completed 8th grade or less). Step 2: we express the labor compensation per hour worked for each type of labor relative to the average labor compensation of the lowest educational group. Step 3: we calculate quality adjusted hours for each type of labor as total hours worked per week of that type of labor scaled by the relative labor compensation we calculated in step 2. Step 4: we add up the quality adjusted hours across labor types for each level of aggregation to obtain the efficiency hour levels.

D Facts for Other Classifications

Figure 10: Postwar U.S. Structural Transformation – Efficiency Hours Worked